Fall 2019

微分方程
Differential Equations

Unit 07.5
The Dirac Delta Function

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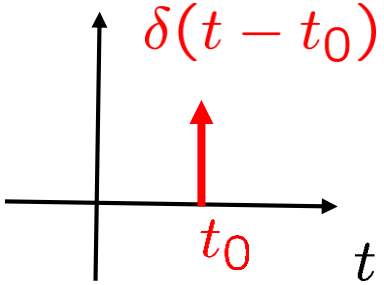
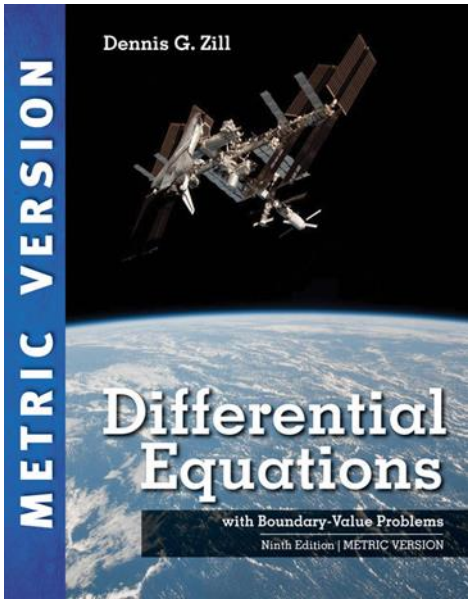
Sep19 – Jan20





Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- Unit Impulse:

$$\delta_a(t - t_0) = \begin{cases} \quad , & \leq t < \\ \quad , & \end{cases}$$

$$\Rightarrow \int_0^{\infty} \delta_a(t - t_0)\, dt =$$

- Dirac Delta Function:

$$\delta(t-t_0) = \qquad\qquad \delta_a(t-t_0)$$

$$\Rightarrow \delta(t-t_0) = \begin{cases} & , \quad t = \\ & , \quad t = \end{cases}$$

$$\Rightarrow \int_0^\infty \delta(t-t_0)\, dt =$$

$$\Rightarrow \mathcal{L}\left\{\delta_a(t-t_0)\right\} = ?$$

$$\delta_a(t-t_0) =$$

$$\Rightarrow \mathcal{L}\left\{\delta_a(t-t_0)\right\} =$$

$$=$$

$$\Rightarrow \lim_{a\to 0} \mathcal{L}\left\{\delta_a(t-t_0)\right\} =$$

$$=$$

$$\Rightarrow \mathcal{L}\left\{\delta(t-t_0)\right\} =$$

Summary - 7.5: Dirac Delta Function

$$\Rightarrow \mathcal{L}\left\{\mathcal{U}(t)\right\} =$$

$$\Rightarrow \mathcal{L}\left\{\frac{d}{dt}\mathcal{U}(t)\right\} =$$

$$\Rightarrow \mathcal{L}\left\{\delta(t)\right\} =$$

$\Rightarrow$ $\delta(t)$ $\frac{d}{dt}\mathcal{U}(t)$