

Fall 2019

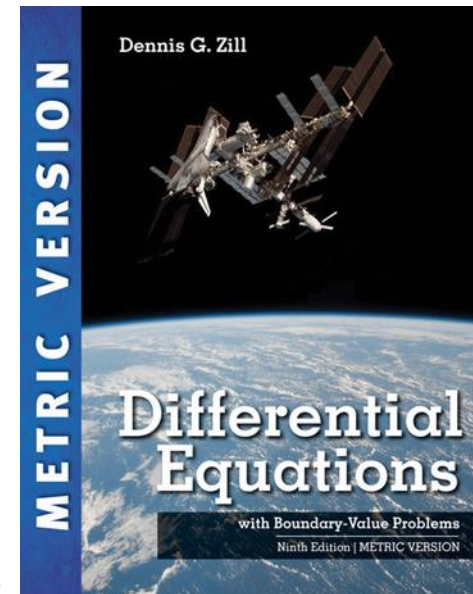
微分方程 Differential Equations

Unit 04.5 Undetermined Coefficients – Annihilator

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- 4.1: Linear Differential Equations: Basic Theory
 - 4.1.1: Initial-Value and Boundary-Value Problems
 - 4.1.2: Homogeneous Equations
 - 4.1.3: Nonhomogeneous Equations
- 4.2: Reduction of Order
- 4.3: Homogeneous Linear Eqns w/ Constant Coefficients
- 4.4: Undetermined Coefficients – Superposition Approach
- **4.5: Undetermined Coefficients – Annihilator Approach**
- 4.6: Variation of Parameters
- 4.7: Cauchy-Euler Equations
- 4.8: Solving Systems of Linear Equations by Elimination
- 4.9: Nonlinear Differential Equations

$$y'' + 3y' + 2y = 4x^2$$

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$$y'' + 3y' + 2y = 4x^2$$

$$y'' + 3y' + 2y = e^{2x}$$

$$y'' + 3y' + 2y =$$

$$y'' + 3y' + 2y =$$

⇒ purpose:

- **Annihilator Operator:** $D =$

$$L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0$$

- L : a linear differential operator with constant coefficients
- $f(x)$: a sufficiently differentiable function
- **IF**
- **THEN** L is said to be an $\quad\quad\quad$ of function $f(x)$

4.5: Examples

$$(1) \quad 1 \quad x \quad x^2 \quad \dots \quad x^{n-1}$$

$$(2) \quad e^{ax} \quad xe^{ax} \quad x^2e^{ax} \quad \dots \quad x^{n-1}e^{ax}$$

$$(3) \quad \begin{array}{lll} e^{ax} \cos bx & xe^{ax} \cos bx & \dots x^{n-1}e^{ax} \cos bx \\ e^{ax} \sin bx & xe^{ax} \sin bx & \dots x^{n-1}e^{ax} \sin bx \end{array}$$

4.5: Example 3

$$y'' + 3y' + 2y = 4x^2$$

$$L =$$

$$P(D) =$$

$$\left\{ \begin{array}{l} L(f(x)) = g(x) \\ L(f(x)) = \end{array} \right.$$

- y_c

4.5: Example 5

$$y'' + y = x \cos x$$

- y_c

- y_p

4.5: Example 6

$$y'' - 2y' + y = 10e^{-2x} \cos(x)$$

$$y'' + 4y' - 2y = 2x^2 + 6 + 10\sin 3x - 3e^{-2x}$$

$$\Rightarrow y = y_c + y_p$$

$$\underline{4.4:} \Rightarrow y_p = (Ax^2 + Bx + C) + (E \cos 3x + F \sin 3x) + (Ge^{-2x})$$

$$\begin{aligned} \underline{4.5:} \Rightarrow P(D)(y'' + 4y' - 2y) \\ = P(D)(2x^2 + 6 + 10\sin 3x - 3e^{-2x}) = 0 \end{aligned}$$

$$\text{e.g., } \Rightarrow y^{(4)} - 9y^{(3)} + 7y'' - 6y' + 12y = 0$$

$$\Rightarrow y = y_c + y_p$$