

Fall 2019

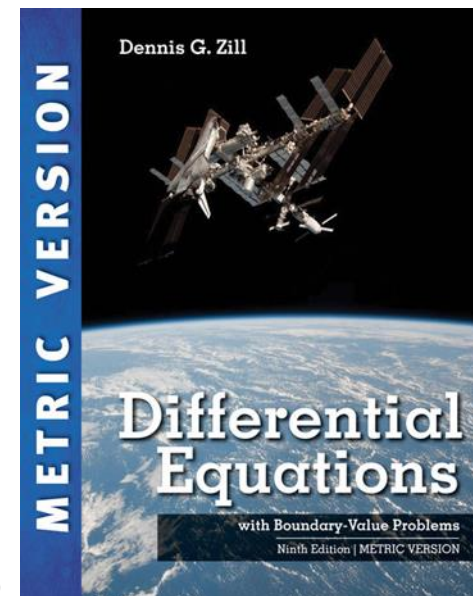
微分方程 Differential Equations

Unit 04.4 Undetermined Coefficients – Superposition

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- 4.1: Linear Differential Equations: Basic Theory
 - 4.1.1: Initial-Value and Boundary-Value Problems
 - 4.1.2: Homogeneous Equations
 - 4.1.3: Nonhomogeneous Equations
- 4.2: Reduction of Order
- 4.3: Homogeneous Linear Eqns w/ Constant Coefficients
- **4.4: Undetermined Coefficients– Superposition Approach**
- 4.5: Undetermined Coefficients – Annihilator Approach
- 4.6: Variation of Parameters
- 4.7: Cauchy-Euler Equations
- 4.8: Solving Systems of Linear Equations by Elimination
- 4.9: Nonlinear Differential Equations

4.4: Undetermined Coefficients – Superposition Approach

$$y'' + 4y' - 2y = 2x^2 - 6 + e^{-3x} - 4\cos(5x)$$

$$\Rightarrow y = y_c + y_p \quad \Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p$$

$$y'' + 4y' - 2y = 6 \quad \Rightarrow y_p =$$

$$y'' + 4y' - 2y = 2x^2 \quad \Rightarrow y_p =$$

$$y'' + 4y' - 2y = e^{-3x} \quad \Rightarrow y_p =$$

$$y'' + 4y' - 2y = \cos(5x) \quad \Rightarrow y_p =$$

$$y'' + 4y' - 2y = \sin(5x)$$

$$y'' + 4y' - 2y = \ln x, \frac{1}{x}, \tan(x), \sin^{-1}(x), \dots$$

4.4: Example 1

$$y'' + 4y' - 2y = 2x^2 - 3x + 6$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p$$

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = g(x)$$

$$\Rightarrow y =$$

4.4: Example 2

$$y'' - y' + y = 2 \sin(3x)$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p =$$

$$y_p' =$$

$$y_p'' =$$

$$y_p'' - y_p' + y_p =$$

4.4: Example 3

$$y'' - 2y' - 3y = 6e^{2x}$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p =$$

$$y_p' =$$

$$y_p'' =$$

$$y_p'' - y_p' + y_p =$$

4.4: Example 3

$$y'' - 2y' - 3y = 6xe^{2x}$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p =$$

$$y_p' =$$

$$y_p'' =$$

$$y_p'' - y_p' + y_p =$$

4.4: Example 4: But,

$$y'' - 5y' + 4y = 8e^x$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p =$$

$$y_p' =$$

$$y_p'' =$$

$$y_p'' - y_p' + y_p =$$

4.4: Example 4: So,

$$y'' - 5y' + 4y = 8e^x$$

$$\Rightarrow y = y_c + y_p$$

$$\Rightarrow \text{AHE} \rightarrow y_c$$

$$\Rightarrow \text{RHS} \rightarrow y_p =$$

$$y_p' =$$

$$y_p'' =$$

$$y_p'' - y_p' + y_p =$$

4.4: Case I

- Case I: Candidate particular solution
is NOT a complementary function

TABLE 4.1 Trial Particular Solutions

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

4.4: Case II & Example 7

- Case II: Candidate particular solution
is ALSO a complementary function

$$y'' - 2y' + y = e^x$$

- y_c

- y_p

$$y'' + y = 4x + 10 \sin x$$

$$y(\pi) = 0$$

$$y'(\pi) = 2$$

- y_c

- y_p

$$y'' + 4y' - 2y = 2x^2 + 6 + 10\sin 3x - 3e^{-2x}$$

$$\Rightarrow y = y_c + y_p$$

$$\underline{4.4:} \Rightarrow y_p = (Ax^2 + Bx + C) + (E \cos 3x + F \sin 3x) + (Ge^{-2x})$$

$$\begin{aligned} \underline{4.5:} \Rightarrow P(D)(y'' + 4y' - 2y) \\ = P(D)(2x^2 + 6 + 10\sin 3x - 3e^{-2x}) = 0 \end{aligned}$$

$$\text{e.g., } \Rightarrow y^{(4)} - 9y^{(3)} + 7y'' - 6y' + 12y = 0$$

$$\Rightarrow y = y_c + y_p$$