Fall 2019

微分方程
Differential Equations

Unit 03.3
Modeling with Systems of DEs

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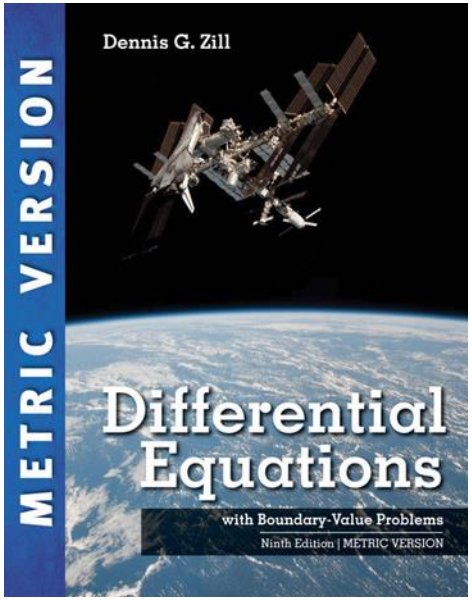
Sep19 – Jan20

$$\frac{dx}{dt} = g_1(t, x, y)$$
$$\frac{dy}{dt} = g_2(t, x, y)$$



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- 2 interacting/competing species: rabits & foxes

$$\frac{dx}{dt} = g_1(t, x, y)$$

$$\frac{dy}{dt} = g_2(t, x, y)$$

- Radioactive series:

U-238 $\longrightarrow$ T-234 $\longrightarrow$ $\cdots$ $\longrightarrow$ Pb-260

$$\frac{dx}{dt} =$$

$$\frac{dy}{dt} =$$

$$\frac{dz}{dt} =$$

- Predator and Prey

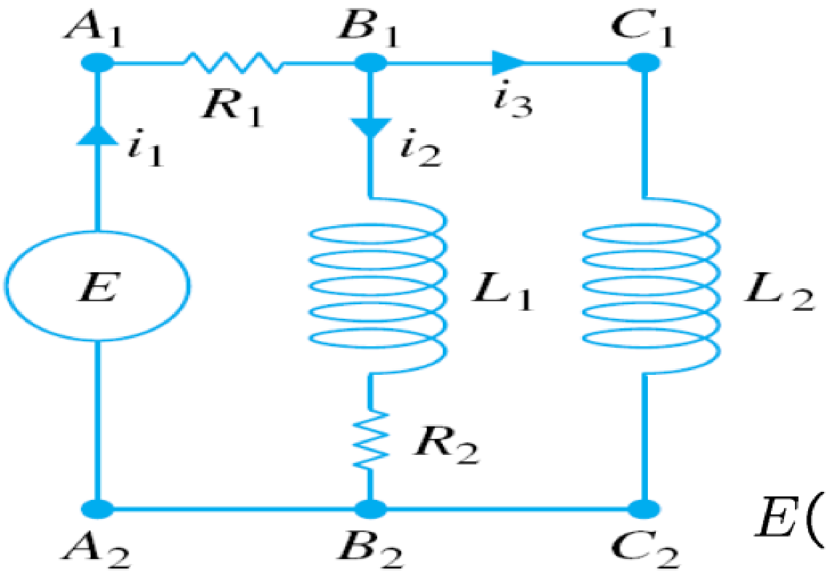
foxes rabbits

$$\frac{dx}{dt} =$$

$$\frac{dy}{dt} =$$

Lotka-Volterra predator-prey model

3.3: Networks



$$i_1(t) = i_2(t) + i_3(t)$$

$$E(t) = i_1 R_1 + L_1 \frac{di_2}{dt} + R_2 i_2$$

$$E(t) = i_1 R_1 + L_2 \frac{di_3}{dt}$$

$$L_1 \frac{di_2}{dt} + (R_1 + R_2) i_2 + R_1 i_3 = E(t)$$

$$L_2 \frac{di_3}{dt} + R_1 i_2 + R_1 i_3 = E(t)$$

Summary

- Linear Models:

$$\frac{dP(t)}{dt} = k\, P(t), \qquad P(0) = P_0$$

- Nonlinear Models:

$$\frac{dP(t)}{dt} = f\,(t, P) \qquad P(0) = P_0$$

$$e.g., \quad = \quad P(a - bP)$$

$$or, \quad = \quad k(a - P)(b - P)$$

$$and, \quad = \quad k(P - a)(P - b)(P - c) \cdots$$

Summary

$$\frac{dP}{dt} = k(P-a)(P-b)$$

$$\Rightarrow \quad \frac{dP}{(P-a)(P-b)} = k\,dt$$

$$\Rightarrow \quad \left(\frac{A}{(P-a)} + \frac{B}{(P-b)}\right) dP = k\,dt$$

$$\Rightarrow \quad A \ln|P-a| + B \ln|P-b| = k\,t + c_1$$

$$\Rightarrow \quad P = F(t)$$

Summary

- 2 interacting/competing species: rabits & foxes

$$\frac{dx}{dt} = g_1(t, x, y)$$

$$\frac{dy}{dt} = g_2(t, x, y)$$

- Predator and Prey foxes rabbits

$$\frac{dx}{dt} = -\ a\ x \ +\ b\ xy$$

$$\frac{dy}{dt} = +\ c\ y \ -\ e\ xy$$

Lotka-Volterra predator-prey model