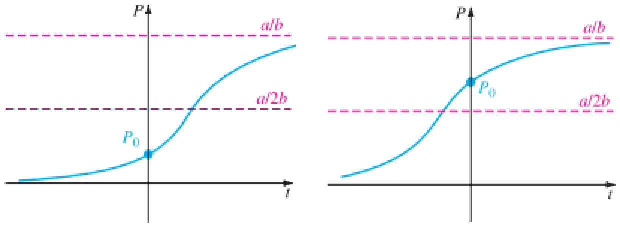
Fall 2019

微分方程
Differential Equations

Unit 03.2
Nonlinear Models



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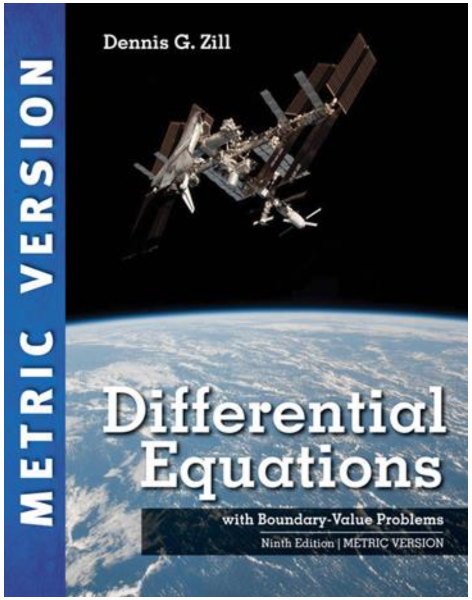
Sep19 – Jan20



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- 3.1: Linear Models
- **3.2: Nonlinear Models**
- 3.3: Modeling with Systems of DEs

$P(t)$: size of population at time t

$$\Rightarrow \quad \frac{dP(t)}{dt} = k\,P(t), \qquad k > 0, \qquad P(0) = P_0$$

$$\Rightarrow \quad P(t) = P_0\, e^{kt}, \qquad t \geq 0$$

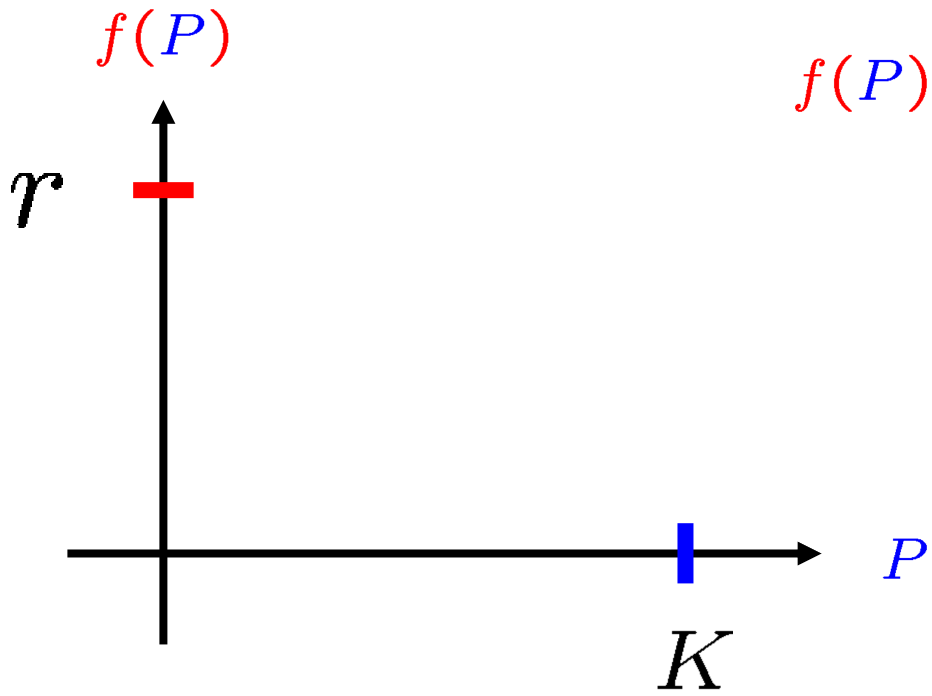
- density-dependent hypothesis:

$$\frac{dP(t)}{dt}$$

$$f(P)$$

$$P(t)$$

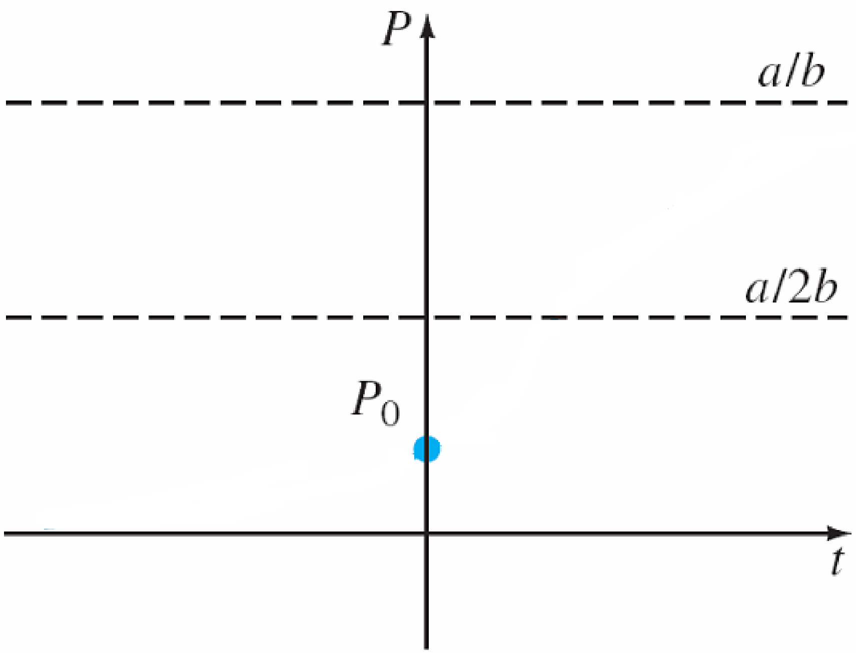
- for example

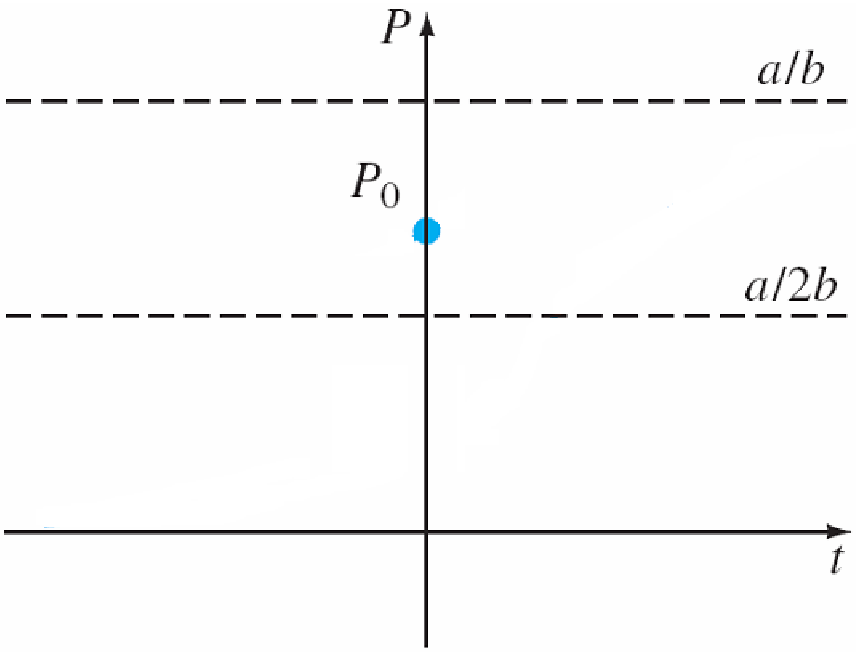


$$f(P) = r - \frac{r}{K} P =$$

$$\Rightarrow \quad \frac{dP(t)}{dt} =$$

$\Rightarrow$ equilibrium points:


P
a/b
a/2b
P_0
t


P
a/b
P_0
a/2b
t

3.2: Modifications of Logistic Equation

$$\frac{dP(t)}{dt} = P\,(a - b\,P)$$

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3.2: Chemical Reactions

$$A \quad + \quad B \quad \longrightarrow \quad C$$

time $= 0$

time $= t$

- Law: $\dfrac{dX(t)}{dt} \quad \propto$

$$\Rightarrow \quad \frac{dX(t)}{dt} =$$

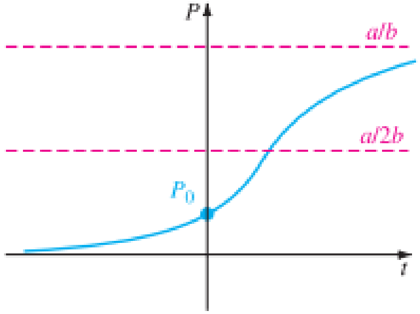
$$\Rightarrow$$

Summary

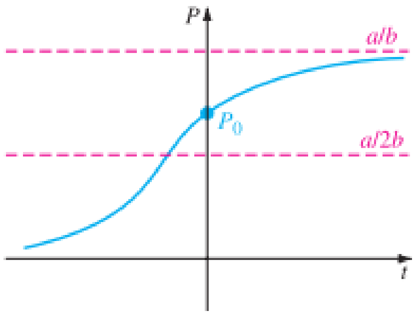
$$\frac{dP(t)}{dt} = P\,(a - b\,P)$$

$$\frac{dP(t)}{dt} = P\,(a - b\,P) - h$$

$$\frac{dP(t)}{dt} = P\,(a - b\,P) + h$$



$$\frac{dP(t)}{dt} = P\,(a - b\,P) - c\,P$$



$$\frac{dP(t)}{dt} = P\,(a - b\,P) + c\,e^{-kP}$$

$$\frac{dP(t)}{dt} = P\,(a - b\,\ln P)$$