Fall 2019

微分方程
Differential Equations

Unit 02.2
Separable Equations

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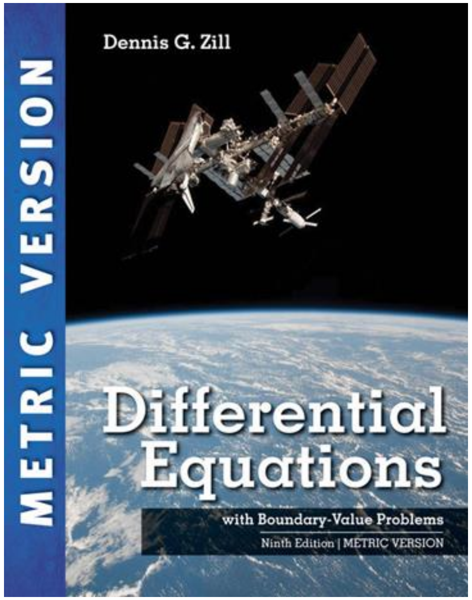
$$\frac{dy}{dx} = g(x)\,h(y)$$



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- 2.1: Solution Curves without a Solution
  - 2.1.1: Direction Fields
  - 2.1.2: Autonomous First-Order DEs
- **2.2: Separable Equations**
- 2.3: Linear Equations
- 2.4: Exact Equations
- 2.5: Solutions by Substitutions
- 2.6: A Numerical Method

Solution by Integration

- $\dfrac{dy}{dx} = f(\,x,\ y\,) \qquad = \quad g(\,x\,)$ continuous

$$\rightarrow \quad \frac{dy}{dx} = g(\,x\,)$$

$$\rightarrow \quad \int \frac{dy}{dx}\,dx = \int g(\,x\,)\,dx$$

$$\rightarrow \quad y(x) = G(x) + c$$

Definition 2.2.1: Separable Equations

- A first-order DE of the form:

$$\frac{dy}{dx} = f(x, y) = g(x)\, h(y)$$

- is said to be separable or to have separable variables.

IF $h(y) \neq 0, \quad \forall y \in I$

$$\Rightarrow \frac{1}{h(y)}\, dy = g(x)\, dx$$

$$\Rightarrow H(y) = G(x) + C$$

$$\Rightarrow y = \phi(x)$$

Definition 2.2.1: Separable Equations

$$\Rightarrow \frac{1}{h(y)}\, dy = g(x)\, dx$$

$$\Rightarrow p(y)\, dy = g(x)\, dx$$

IF $y(x) = \phi(x)$ a solution of the DE

$$\Rightarrow dy = \phi'(x)\, dx$$

$$\Rightarrow p(\phi(x))\, \phi'(x)\, dx = g(x)\, dx$$

$$\Rightarrow \int p(\phi(x))\, \phi'(x)\, dx = \int g(x)\, dx$$

$$\Rightarrow \int p(y)\, dy = \int g(x)\, dx$$

$$\Rightarrow H(y) + C_1 = G(x) + C_2$$

$$\Rightarrow H(y) = G(x) + C$$

$$\Rightarrow y(x) = \phi(x)$$

Example 1: Solving a Separable DE

$$(1 + x)\,dy - y\,dx = 0$$

Example 2: Solution Curve

$$\frac{dy}{dx} = -\frac{x}{y} \qquad y(4) = -3$$

Example 3: Losing a Solution

$$\frac{dy}{dx} = y^2 - 4$$

Example 5: Solution by Integral-Defined Function

$$\frac{dy}{dx} = e^{-x^2} \qquad y(3) = 5$$

Summary

$$\frac{dy}{dx} = f(\,x,\ y\,) \quad \Rightarrow \quad \frac{dy}{dx} = g(x)\,h(y)$$

$$\Rightarrow \frac{1}{h(y)}\,dy = g(x)\,dx$$

$$\Rightarrow H(y) = G(x) + C$$

$$\Rightarrow y = \Phi(x)$$