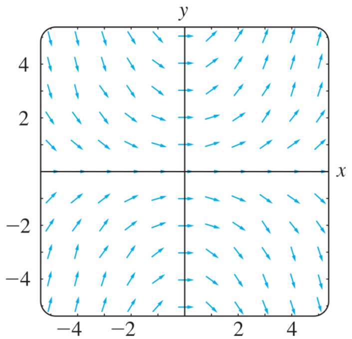
Fall 2019

微分方程
Differential Equations

Unit 02.1
Solution Curves Without a Solution



(a) direction field for $dy/dx = 0.2xy$

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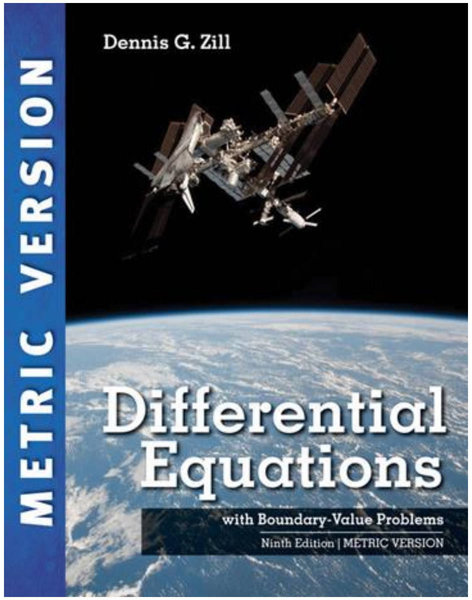
Sep19 – Jan20



Figures and images used in these lecture notes are adopted from
Differential Equations with Boundary-Value Problems, 9th Ed., D.G. Zill, 2018 (Metric Version)

Outline

- **2.1: Solution Curves without a Solution**
  - 2.1.1: Direction Fields
  - 2.1.2: Autonomous First-Order DEs
- 2.2: Separable Equations
- 2.3: Linear Equations
- 2.4: Exact Equations
- 2.5: Solutions by Substitutions
- 2.6: A Numerical Method

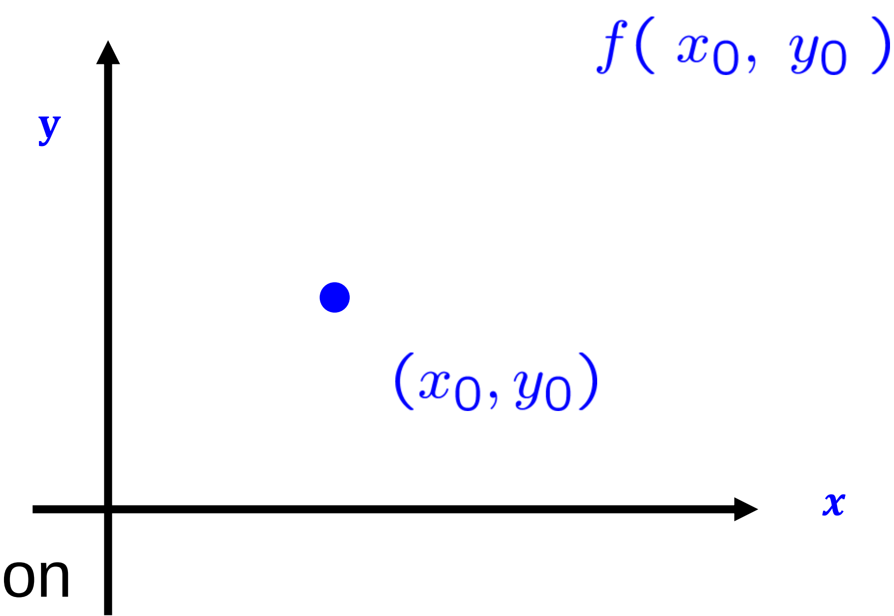
- Qualitative questions about properties of solutions:
  - How does a solution behave near a certain point?
  - How does a solution behave as $x \to \infty$?

- A derivative dy/dx of a differentiable function $y = y(x)$ gives slopes of tangent lines at points on its graph.

• $\dfrac{dy}{dx} = f(\,x,\; y\,)$

$f(\,x,\; y\,)$

$f(\,x,\; y(x)\,)$



- the slope function or rate function

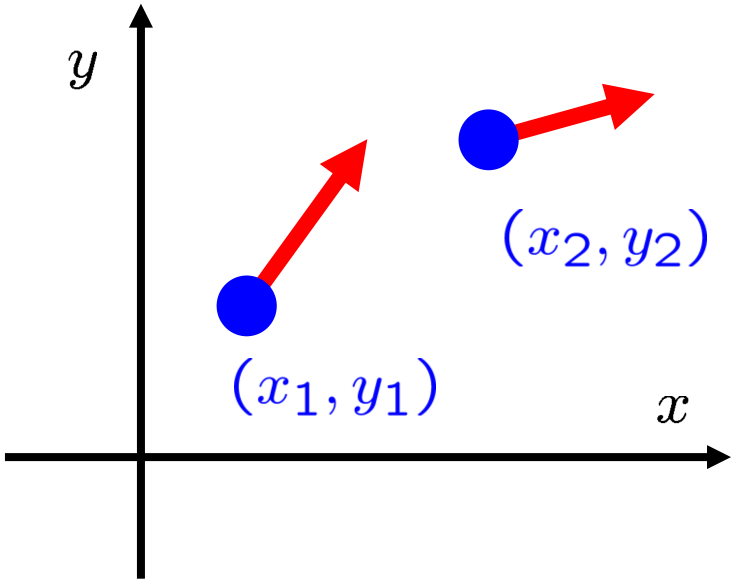
- $\dfrac{dy}{dx} = f(\,x,\ y\,)$

- Existence & Uniqueness: (2.1)

$f(\,x,\ y\,) \longrightarrow f(x,y)$

continuous

$f(\,x,\ y(x)\,) \longrightarrow \dfrac{\partial f(x,y)}{\partial y}$

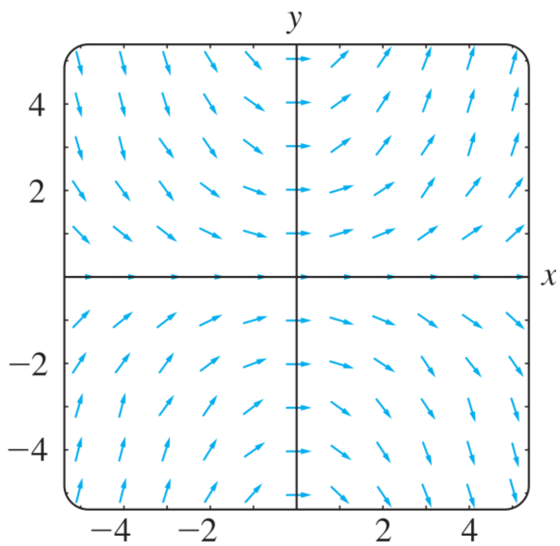


$$\left.\frac{dy}{dx}\right|_{(x_i,y_i)} = f(x_i, y_i)$$

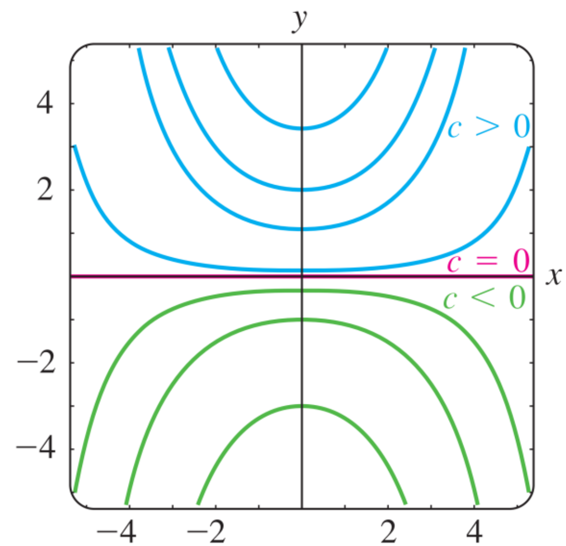
- the slope function or rate function

Direction Fields

- $\dfrac{dy}{dx} = f(x, y) = 0.2\, x\, y \quad \rightarrow \quad y = c\, e^{0.1\, x^2}$



(a) direction field for $dy/dx = 0.2xy$



(b) some solution curves in the family $y = ce^{0.1x^2}$

- $\dfrac{dy}{dx} = f(y)$

$$\begin{cases} \dfrac{dy}{dx} = 0.2\,x\,y \\ \dfrac{dy}{dx} = 1 + y^2 \end{cases}$$

$$\begin{cases} \dfrac{dP}{dt} = k\,P \\ \dfrac{dT}{dt} = k\,(T - T_m) \end{cases}$$

- $\dfrac{dy}{dx} = f(y) = 0$

$\Rightarrow f(c) = 0$

$\Rightarrow y = c$ ■ The critical, equilibrium, stationary point

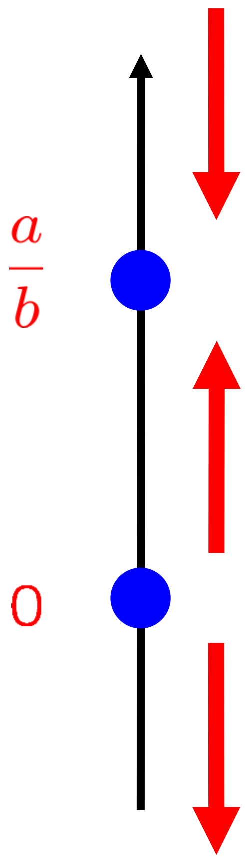
$\Rightarrow y(x) = c$ ■ The equilibrium solution

Autonomous First-Order DEs

- $\dfrac{dP}{dt} = P\,(\,a - b\,P\,), \quad a > 0, b > 0$

$\Rightarrow f(P) = 0 \qquad \Rightarrow P = 0 \text{ OR } \dfrac{a}{b}$

- 2 critical points



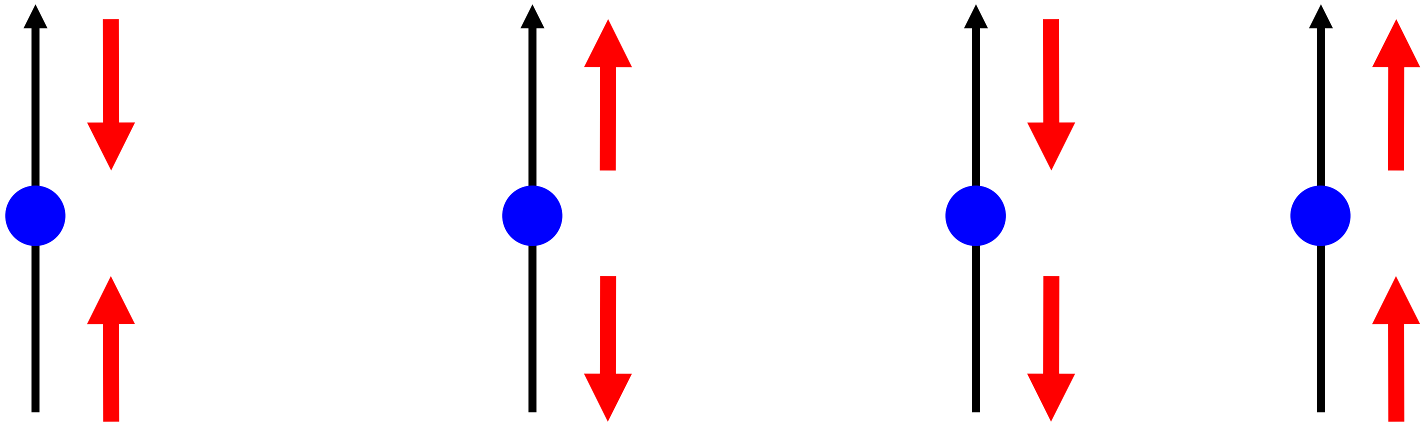
$$f(P)\Big|_{P>\frac{a}{b}} < 0$$

$$f(P)\Big|_{0<P<\frac{a}{b}} > 0$$

$$f(P)\Big|_{P<0} < 0$$

- Phase Portrait

Autonomous First-Order DEs



Asymptotically Stable | Unstable | Semi-Stable

Attractor | Repeller | Saddle

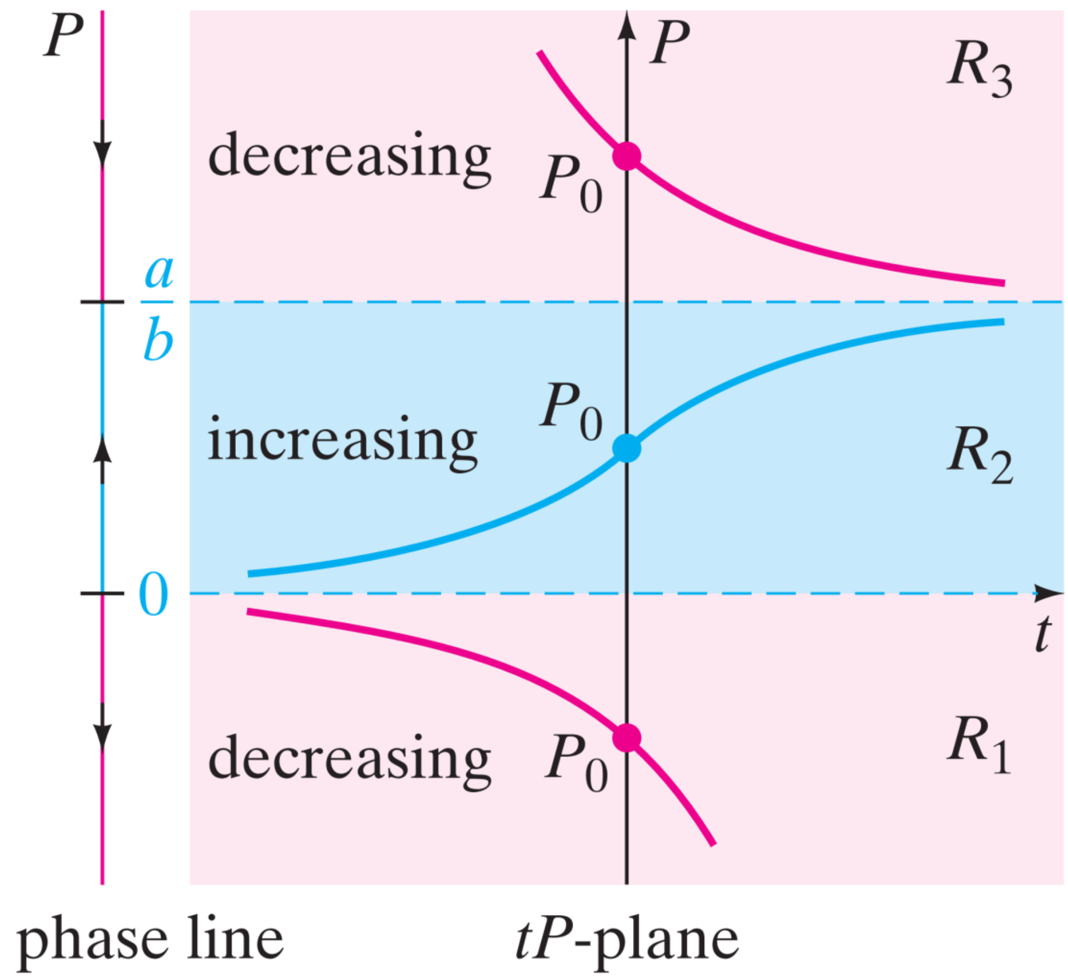
- $\dfrac{dP}{dt} = P\,(a - b\,P), \quad a > 0, b > 0$

$\Rightarrow f(P) = 0 \quad \Rightarrow P = 0 \text{ OR } \dfrac{a}{b}$ ▪ 2 critical points

$f(P)\big|_{P>\frac{a}{b}} < 0$

$f(P)\big|_{0<P<\frac{a}{b}} > 0$

$f(P)\big|_{P<0} < 0$



Autonomous First-Order DEs

- $\dfrac{dy}{dx} = y\,(\,3 - y\,)$