Control System: Homework 10 for Units 6I-6K: Bode Plot

Assigned: Dec 9, 2022

Due: Dec 15, 2022 (11:59pm)

1. (Lead compensation)

46. For the system shown in Fig. 6.100, suppose that

$$G(s) = \frac{5}{s(s+1)(s/5+1)}.$$

Design a lead compensation $D(s)$ with unity DC gain so that PM $\geq 40°$ using Bode plot sketches. What is the approximate bandwidth of the system?

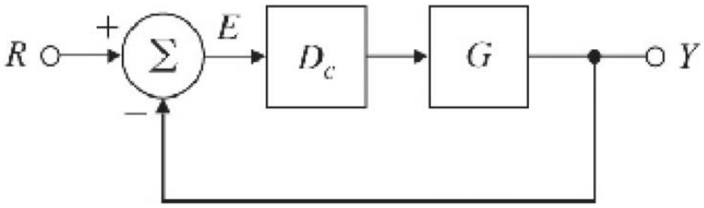


Figure 6.89: Fig. 6.100 Control system for Problem 6.46

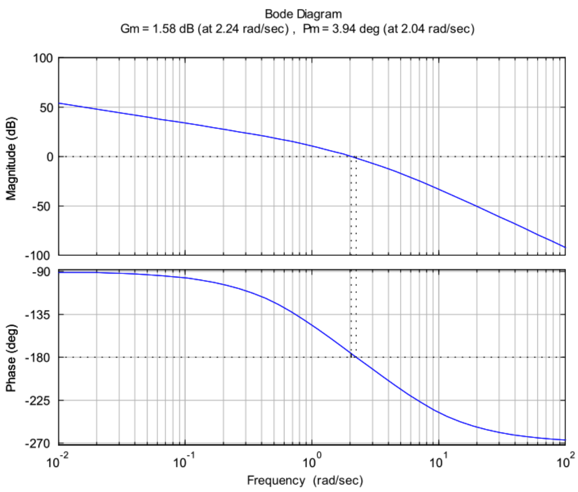
Solution :

Start with a lead compensator design with :

$$D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

which has unity DC gain with $\alpha < 1$.

The Bode plot of the given system is :



Since $PM = 3.9°$, let's add phase lead $\geq 60°$. From Fig. 6.53,

$$\frac{1}{\alpha} \simeq 20 \Longrightarrow \text{choose } \alpha = 0.05$$

To apply maximum phase lead at $\omega = 10$ rad/sec,

$$\omega = \frac{1}{\sqrt{\alpha}T} = 10 \Longrightarrow \frac{1}{T} = 2.2, \ \frac{1}{\alpha T} = 45$$

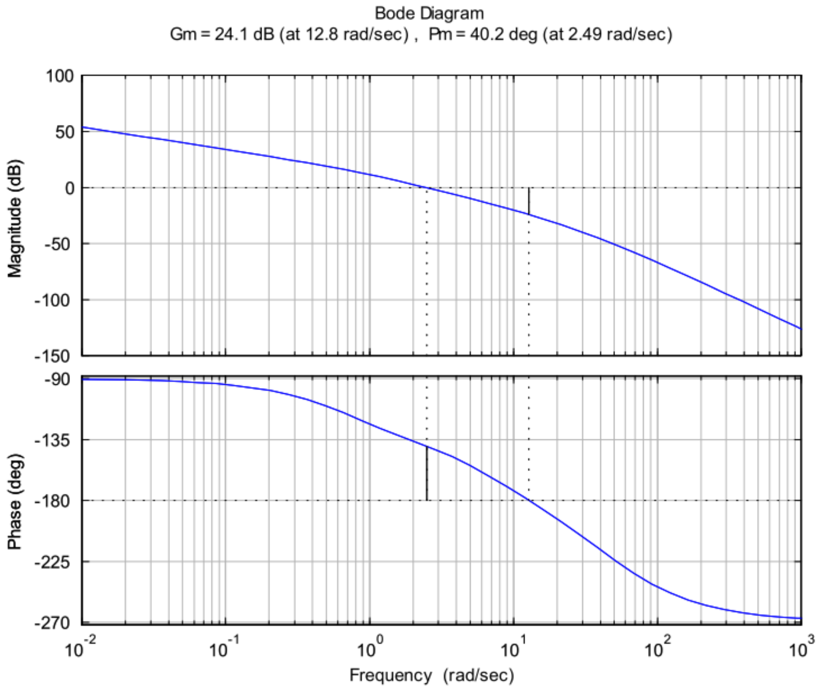
Therefore by applying the lead compensator :

$$D(s) = \frac{\frac{s}{2.2} + 1}{\frac{s}{45} + 1}$$

we get the compensated system with :

$$PM = 40°, \ \omega_c = 2.5$$

The Bode plot with designed compensator is :



From Fig. 6.50, we see that $\omega_{BW} \simeq 2 \times \omega_c \simeq 5$ rad/sec.

2. (Lag compensation)

56. For a system with open-loop transfer function $_c$

$$G(s) = \frac{10}{s[(s/1.4)+1][(s/3)+1]},$$

design a lag compensator with unity DC gain so that PM $\geq 35°$. What is the approximate bandwidth of this system?

Solution :

Lag compensation design :

Use

$$D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

K=1 so that DC gain of $D_c(s) = 1$.

(a) Find the stability margins of the plant without compensation by plotting the Bode, find that:

$$\begin{aligned} PM &= -20° \ (\omega_c = 3.0 \text{ rad/sec}) \\ GM &= 0.44 \ (\omega = 2.05 \text{ rad/sec}) \end{aligned}$$

(b) The lag compensation needs to lower the crossover frequency so that a $PM \simeq 35°$ will result, so we see from the uncompensated Bode that we need the crossover at about

$$\Longrightarrow \omega_{c,new} = 1.$$

where

$$|G(j\omega_c)| \simeq 8.5$$

so the lag needs to lower the gain at $\omega_{c,new}$ from 7.5 to 1.

(c) Pick the zero breakpoint of the lag to avoid influencing the phase at $\omega = \omega_{c,new}$ by picking it a factor of 20 below the crossover, so

$$\frac{1}{T_D} = \frac{\omega_{c,new}}{20}$$

$$\Longrightarrow T_D = 20$$

(d) Choose α :

Since $D_c(j\omega) \cong \frac{1}{\alpha}$ for $\omega >> \frac{1}{T}$, let

$$\frac{1}{\alpha} = \frac{1}{|G(j\omega_{c,new})|}$$

$$\alpha = |G(j\omega_{c,new})| = 8.5$$

(e) Compensation :

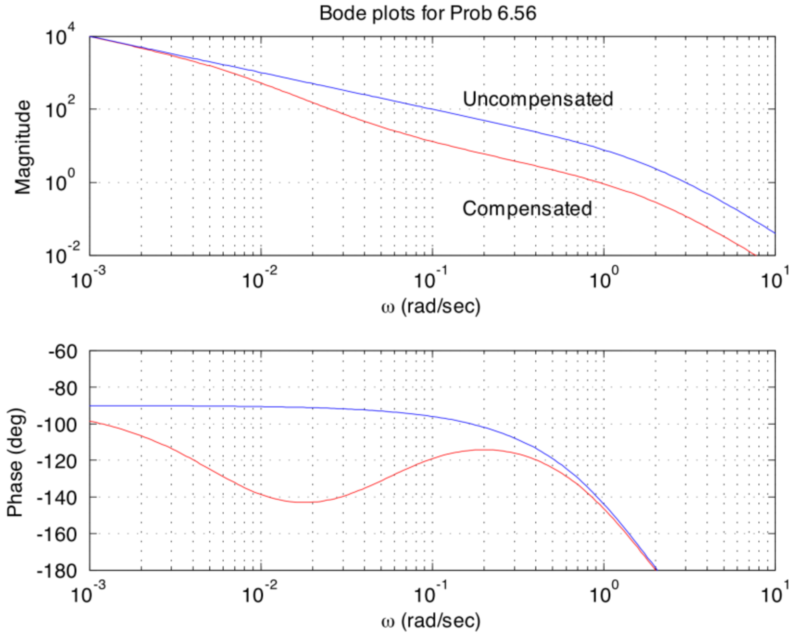
$$D_c(s) = \frac{\frac{s}{0.05} + 1}{\frac{s}{0.0059} + 1}$$

(f) Stability margins of the compensated system :

$$\begin{aligned} PM &= 36^\circ \ (\omega_c = 0.8 \text{ rad/sec}) \\ GM &= 3.6 \ (\omega = 2.0 \text{ rad/sec}) \end{aligned}$$

Approximate bandwidth ω_{BW}:

$$PM \cong 42^\circ \implies \omega_{BW} \cong 2\omega_c = 2 \text{ (rad/sec)}$$



3. (Lead-Lag compensation)

62. Consider the system in Fig. 6.100 with the plant transfer function

$$G(s) = \frac{10}{s(s/10+1)}.$$

We wish to design a compensator $D(s)$ that satisfies the following design specifications:

(a) i. $K_v = 100$,

ii. PM $\geq 45°$,

iii. sinusoidal inputs of up to 1 rad/sec to be reproduced with $\leq 2\%$ error,

iv. sinusoidal inputs with a frequency of greater than 100 rad/sec to be attenuated at the output to $\leq 5\%$ of their input value.

(b) Create the Bode plot of $G(s)$, choosing the open-loop gain so that $K_v = 100$.

(c) Show that a *sufficient* condition for meeting the specification on sinusoidal inputs is that the magnitude plot lies outside the shaded regions in Fig. 6.102. Recall that

$$\frac{Y}{R} = \frac{KG}{1+KG} \quad \text{and} \quad \frac{E}{R} = \frac{1}{1+KG}.$$

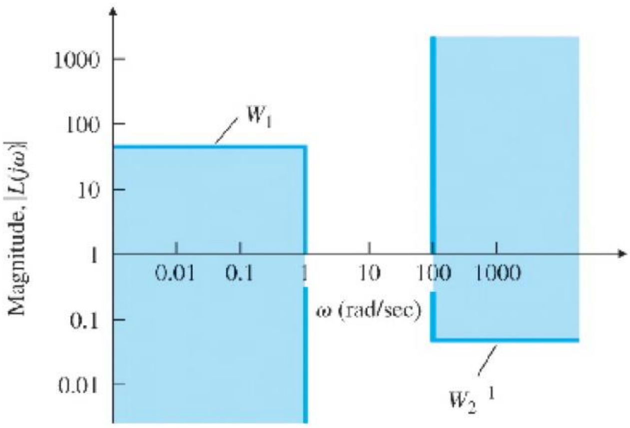


Fig. 6.102 Control system constraints for Problem 62

(d) Explain why introducing a lead network alone cannot meet the design specifications.

(e) Explain why a lag network alone cannot meet the design specifications.

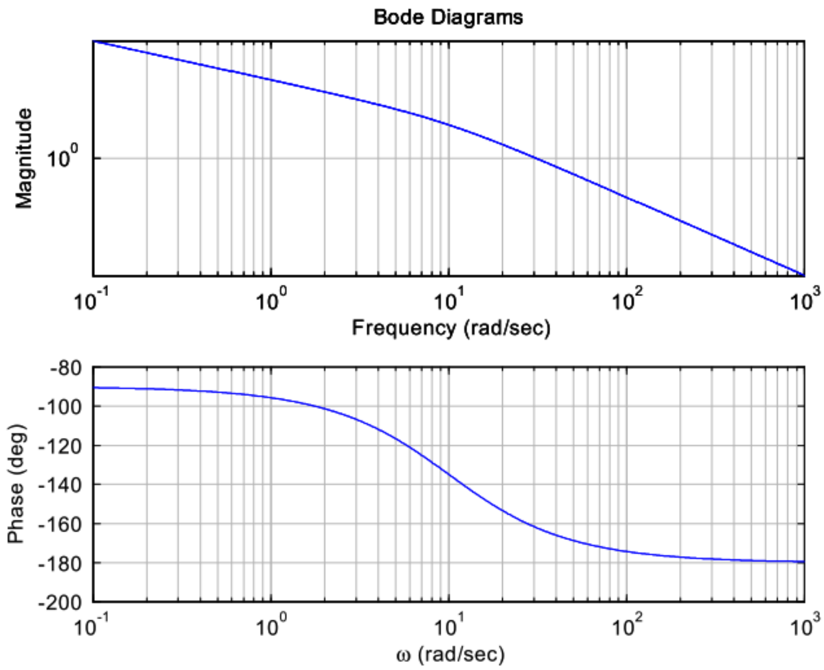
(f) Develop a full design using a lead-lag compensator that meets all the design specifications, without altering the previously chosen low frequency open-loop gain.

Solution :

(a) To satisfy the given velocity constant K_v,

$$K_v = \lim_{s\to 0} sKG(s) = 10K = 100$$
$$\Longrightarrow K = 10$$

(b) The Bode plot of $G(s)$ with the open-loop gain $K = 10$ is :



(c) From the 3rd specification,

$$\left|\frac{E}{R}\right| = \left|\frac{1}{1+KG}\right| < 0.02 \ (2\%)$$
$$\Longrightarrow |KG| > 49 \ (\text{at } \omega < 1 \text{ rad/sec})$$

From the 4th specification,

$$\left|\frac{Y}{R}\right| = \left|\frac{KG}{1+KG}\right| < 0.05 \ (5\%)$$
$$\Longrightarrow |KG| < 0.0526 \ (\text{at } \omega > 100 \text{ rad/sec})$$

which agree with the figure.

(d) A lead compensator may provide a sufficient PM, but it increases the gain at high frequency so that it violates the specification above.

(e) A lag compensator could satisfy the PM specification by lowering the crossover frequency, but it would violate the low frequency specification, W_1.

(f) One possible lead-lag compensator is :

$$D_c(s) = 100\frac{\frac{s}{8.52}+1}{\frac{s}{22.36}+1}\frac{\frac{s}{4.47}+1}{\frac{s}{0.568}+1}$$

which meets all the specification :

$$\begin{aligned} K_v &= 100 \\ PM &= 47.7^\circ \text{ (at } \omega_c = 12.9 \text{ rad/sec)} \\ |KG| &= 50.45 \text{ (at } \omega = 1 \text{ rad/sec)} > 49 \\ |KG| &= 0.032 \text{ (at } \omega = 100 \text{ rad/sec)} < 0.0526 \end{aligned}$$

The Bode plot of the compensated open-loop system $D_c(s)G(s)$ is