Control System: Homework 10 for Units 6I-6K: Bode Plot

Assigned: Dec 24, 2021

Due: Dec 30, 2021 (11:59pm)

1. (Lead compensation)

46. For the system shown in Fig. 6.100, suppose that

$$G(s) = \frac{5}{s(s+1)(s/5+1)}.$$

Design a lead compensation $D(s)$ with unity DC gain so that PM $\geq 40°$ using Bode plot sketches. What is the approximate bandwidth of the system?

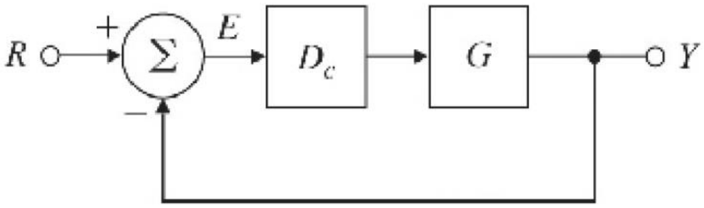


Figure 6.89: Fig. 6.100 Control system for Problem 6.46

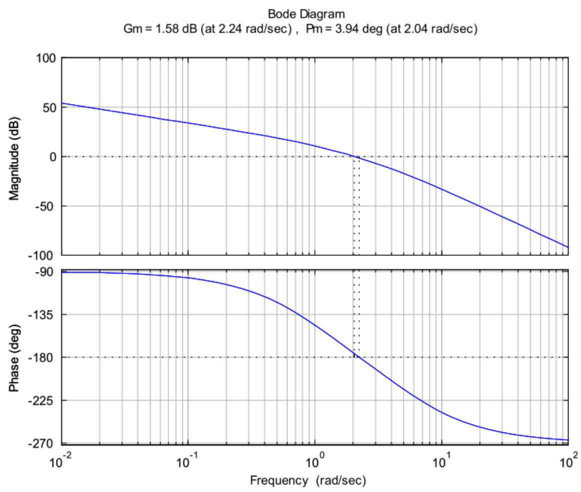
Solution :

Start with a lead compensator design with :

$$D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

which has unity DC gain with $\alpha < 1$.

The Bode plot of the given system is :



Since $PM = 3.9°$, let's add phase lead $\geq 60°$. From Fig. 6.53,

$$\frac{1}{\alpha} \simeq 20 \Longrightarrow \text{choose } \alpha = 0.05$$

To apply maximum phase lead at $\omega = 10$ rad/sec,

$$\omega = \frac{1}{\sqrt{\alpha}T} = 10 \Longrightarrow \frac{1}{T} = 2.2, \ \frac{1}{\alpha T} = 45$$

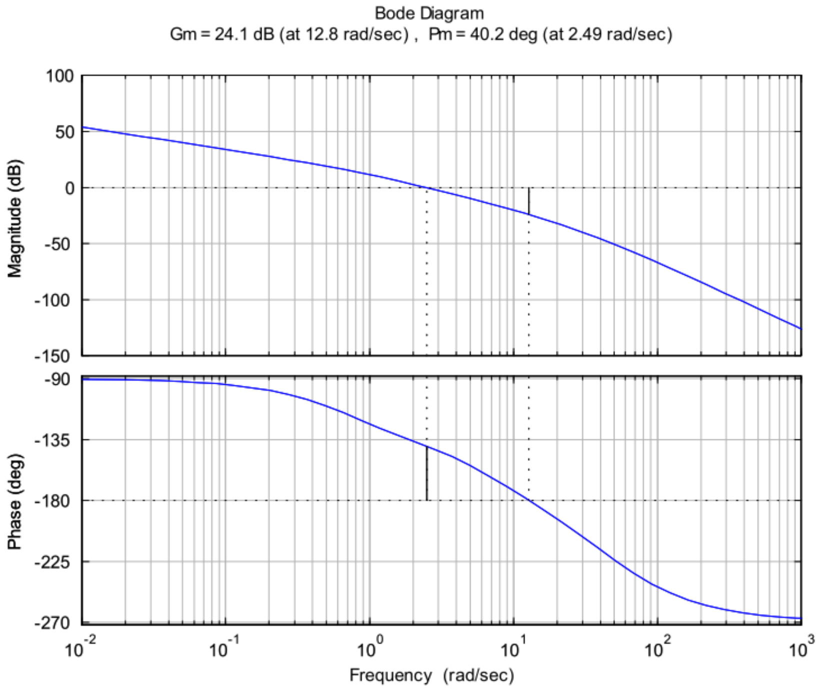
Therefore by applying the lead compensator :

$$D(s) = \frac{\frac{s}{2.2} + 1}{\frac{s}{45} + 1}$$

we get the compensated system with :

$$PM = 40°, \ \omega_c = 2.5$$

The Bode plot with designed compensator is :



From Fig. 6.50, we see that $\omega_{BW} \simeq 2 \times \omega_c \simeq 5$ rad/sec.

2. (Lag compensation)

56. For a system with open-loop transfer function $_c$

$$G(s) = \frac{10}{s[(s/1.4)+1][(s/3)+1]},$$

design a lag compensator with unity DC gain so that PM $\geq 35°$. What is the approximate bandwidth of this system?

Solution :

Lag compensation design :

Use

$$D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

K=1 so that DC gain of $D_c(s) = 1$.

(a) Find the stability margins of the plant without compensation by plotting the Bode, find that:

$$\begin{aligned} PM &= -20° \ (\omega_c = 3.0 \text{ rad/sec}) \\ GM &= 0.44 \ (\omega = 2.05 \text{ rad/sec}) \end{aligned}$$

(b) The lag compensation needs to lower the crossover frequency so that a $PM \simeq 35°$ will result, so we see from the uncompensated Bode that we need the crossover at about

$$\Longrightarrow \omega_{c,new} = 1.$$

where

$$|G(j\omega_c)| \simeq 8.5$$

so the lag needs to lower the gain at $\omega_{c,new}$ from 7.5 to 1.

(c) Pick the zero breakpoint of the lag to avoid influencing the phase at $\omega = \omega_{c,new}$ by picking it a factor of 20 below the crossover, so

$$\frac{1}{T_D} = \frac{\omega_{c,new}}{20}$$

$$\Longrightarrow T_D = 20$$

(d) Choose α :

Since $D_c(j\omega) \cong \frac{1}{\alpha}$ for $\omega >> \frac{1}{T}$, let

$$\frac{1}{\alpha} = \frac{1}{|G(j\omega_{c,new})|}$$

$$\alpha = |G(j\omega_{c,new})| = 8.5$$

(e) Compensation :

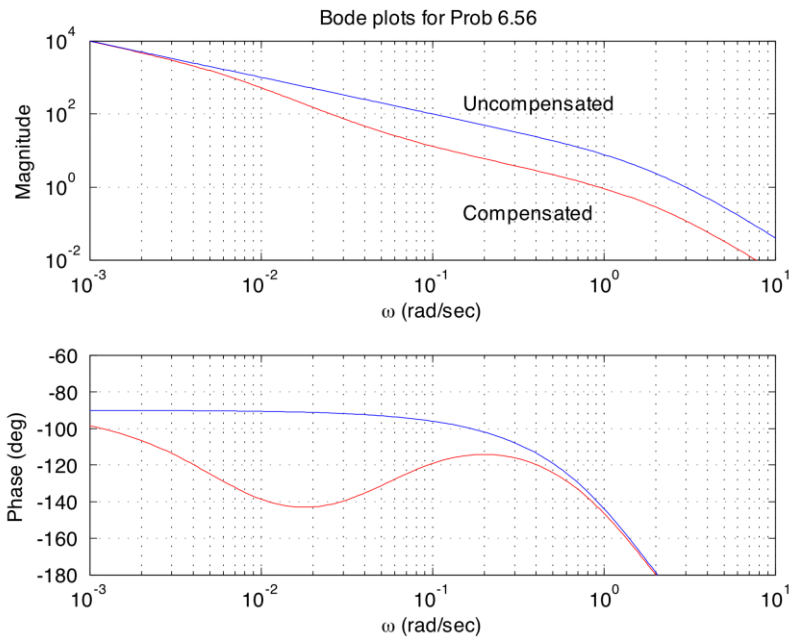
$$D_c(s) = \frac{\frac{s}{0.05}+1}{\frac{s}{0.0059}+1}$$

(f) Stability margins of the compensated system :

$$\begin{aligned} PM &= 36^\circ \ (\omega_c = 0.8 \text{ rad/sec}) \\ GM &= 3.6 \ (\omega = 2.0 \text{ rad/sec}) \end{aligned}$$

Approximate bandwidth ω_{BW}:

$$PM \cong 42^\circ \implies \omega_{BW} \cong 2\omega_c = 2 \text{ (rad/sec)}$$



3. (Lead-Lag compensation)

62. Consider the system in Fig. 6.100 with the plant transfer function

$$G(s) = \frac{10}{s(s/10+1)}.$$

We wish to design a compensator $D(s)$ that satisfies the following design specifications:

(a) i. $K_v = 100$,

ii. PM $\geq 45°$,

iii. sinusoidal inputs of up to 1 rad/sec to be reproduced with $\leq 2\%$ error,

iv. sinusoidal inputs with a frequency of greater than 100 rad/sec to be attenuated at the output to $\leq 5\%$ of their input value.

(b) Create the Bode plot of $G(s)$, choosing the open-loop gain so that $K_v = 100$.

(c) Show that a *sufficient* condition for meeting the specification on sinusoidal inputs is that the magnitude plot lies outside the shaded regions in Fig. 6.102. Recall that

$$\frac{Y}{R} = \frac{KG}{1+KG} \quad \text{and} \quad \frac{E}{R} = \frac{1}{1+KG}.$$

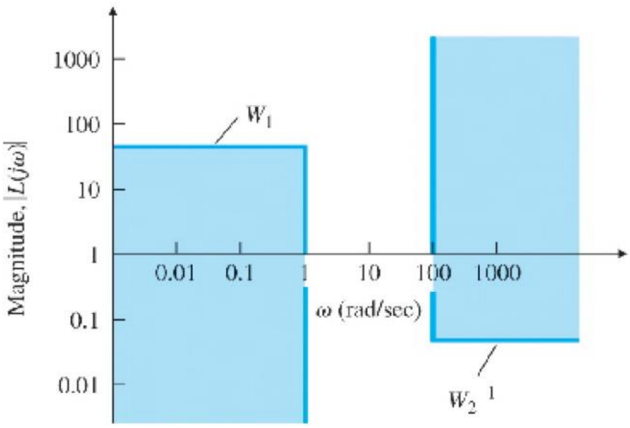


Fig. 6.102 Control system constraints for Problem 62

(d) Explain why introducing a lead network alone cannot meet the design specifications.

(e) Explain why a lag network alone cannot meet the design specifications.

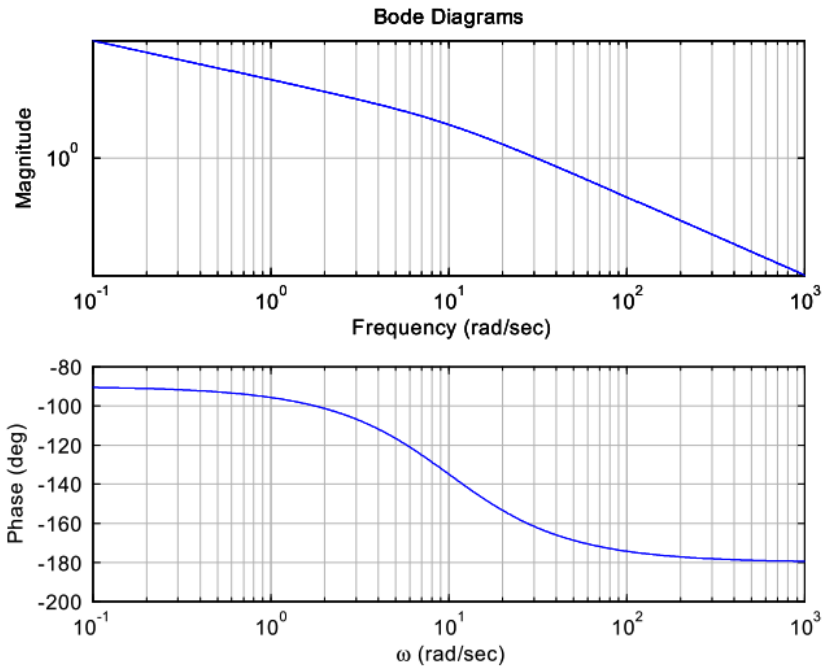
(f) Develop a full design using a lead-lag compensator that meets all the design specifications, without altering the previously chosen low frequency open-loop gain.

Solution :

(a) To satisfy the given velocity constant K_v,

$$\begin{aligned} K_v &= \lim_{s\to 0} sKG(s) = 10K = 100 \\ &\implies K = 10 \end{aligned}$$

(b) The Bode plot of $G(s)$ with the open-loop gain $K = 10$ is :



(c) From the 3rd specification,

$$\begin{aligned} \left|\frac{E}{R}\right| &= \left|\frac{1}{1+KG}\right| < 0.02\ (2\%) \\ &\implies |KG| > 49\ (\text{at } \omega < 1 \text{ rad/sec}) \end{aligned}$$

From the 4th specification,

$$\begin{aligned} \left|\frac{Y}{R}\right| &= \left|\frac{KG}{1+KG}\right| < 0.05\ (5\%) \\ &\implies |KG| < 0.0526\ (\text{at } \omega > 100 \text{ rad/sec}) \end{aligned}$$

which agree with the figure.

(d) A lead compensator may provide a sufficient PM, but it increases the gain at high frequency so that it violates the specification above.

(e) A lag compensator could satisfy the PM specification by lowering the crossover frequency, but it would violate the low frequency specification, W_1.

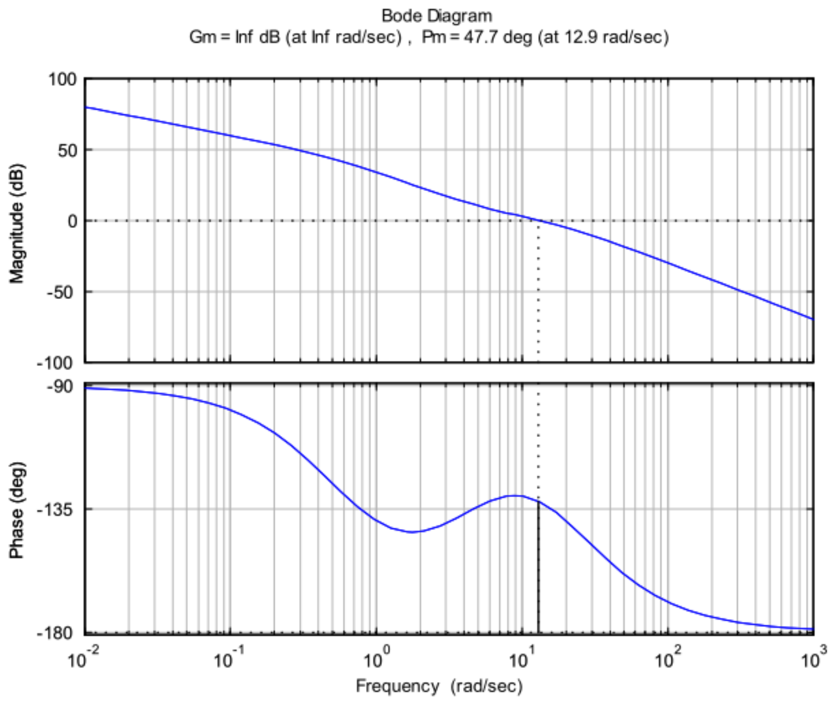
(f) One possible lead-lag compensator is :

$$D_c(s) = 100\frac{\frac{s}{8.52}+1}{\frac{s}{22.36}+1}\frac{\frac{s}{4.47}+1}{\frac{s}{0.568}+1}$$

which meets all the specification :

$$\begin{aligned} K_v &= 100 \\ PM &= 47.7^\circ \text{ (at } \omega_c = 12.9 \text{ rad/sec)} \\ |KG| &= 50.45 \text{ (at } \omega = 1 \text{ rad/sec)} > 49 \\ |KG| &= 0.032 \text{ (at } \omega = 100 \text{ rad/sec)} < 0.0526 \end{aligned}$$

The Bode plot of the compensated open-loop system $D_c(s)G(s)$ is



B08901190 楊子毅

HW 10: Units 6I-6K: Bode Plot	Digital Control Systems, Fall 2019, NTU-EE
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修改自 Problem 1

3. (Lead compensation)

46. For the system shown in Fig. 6.100, suppose that

$$G(s) = \frac{5}{s(s+1)(s/5+1)}.$$

Design a lead compensation $D(s)$ with unity DC gain so that PM $\geq 40°$ using Bode plot sketches. What is the approximate bandwidth of the system?

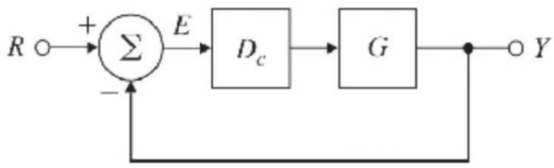


Figure 6.89: Fig. 6.100 Control system for Problem 6.46

We already know that to meet the PM$\geq$ 40°

For D_c in the form of $\frac{T_D s+1}{\alpha T_D s+1} = D_1$

$T_D = \frac{1}{2.25}$ $\alpha = \frac{1}{20}$

Where $D_c = \frac{\frac{s}{2.25}+1}{\frac{s}{45}+1}$ 此時之 PM = 40.2° GM = 24.1dB

(a) What if D_c in the form of $(\frac{T_2 s+1}{\alpha_2 T_2 s+1})^2 = (D_2)^2$

(b) What if D_c in the form of $(\frac{T_3 s+1}{\alpha_3 T_3 s+1})^3 = (D_3)^3$

To choose T_2, α_2, D_2 and T_3, α_3, D_3

That meet the same requirement and find their approximate ω_{BW}

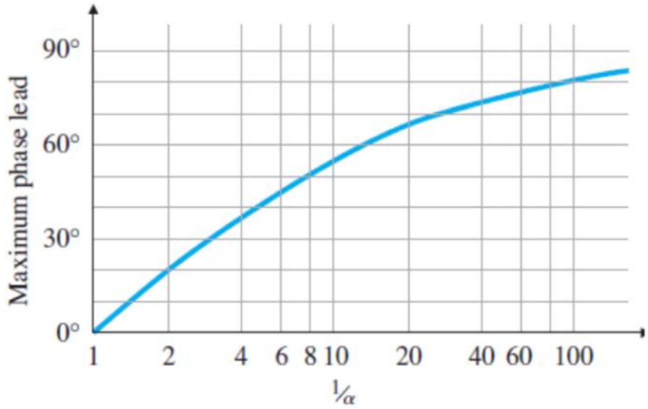
(c) Compare the difference between D_1 D_2 D_3

What advantage we can get from changing D_1 to D_2 *or* D_3

Try to consider it in time domain and in real world.

Solution

For qastion (a)
We can use the same method as original problem

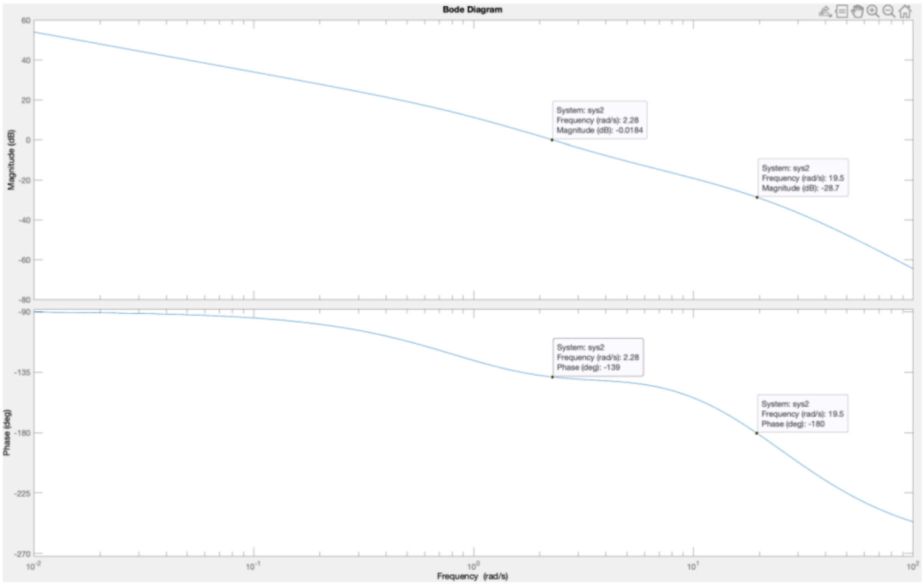


Without copensation , only have PM 3.9°
For $(D_2)^2$ add phase lead ≥ 60°
D_2 add phase lead 40° ，查上表
$\alpha_2 = \frac{1}{5}$

To apply maximum phase at 10 rad/sec
$\omega = \sqrt{z \times p} = \frac{1}{T_2\sqrt{\alpha_2}} = 10$
$T_2 = \frac{1}{4.5}$

having the T_2 and α_2 we can now get
$$D_2 = \frac{T_2 s + 1}{\alpha_2 T_2 s + 1} = \frac{\frac{s}{4.5} + 1}{\frac{s}{22.5} + 1}$$

將$(D_2)^2 \times G(s)$之 bode plot 用 Matlab 畫出來



可以看見結果為 PM = 41° GM = 28.7dB
滿足題目要求之 PM
(note:GM 比原來題目解答還要多)

$$\omega_{BW} \cong 2.28 \times 2 = 4.56$$

For question (b)

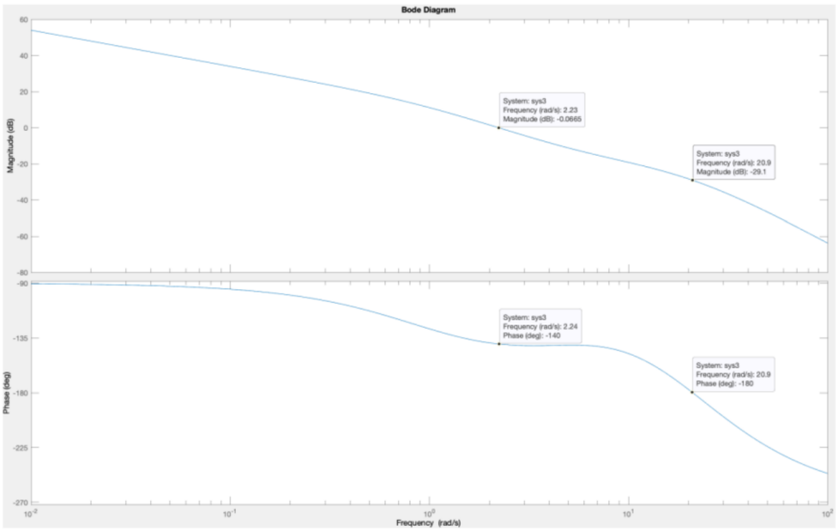
For $(D_3)^3$ add phase lead $\geq 60°$
D_2 add phase lead $30°$ ，查上表
$\alpha_2 = \frac{1}{3}$

To apply maximum phase at 10 rad/sec
$\omega \ = \sqrt{z \times p} = \frac{1}{T_3\sqrt{\alpha_3}} = 10$
$T_3 = \frac{1}{6}$

having the T_3 and α_3 we can now get

$$D_3 \ = \frac{T_3 s + 1}{\alpha_3 T_3 s + 1} = \frac{\frac{s}{6} + 1}{\frac{s}{18} + 1}$$

將$(D_3)^3 \times G(s)$之 bode plot 用 Matlab 畫出來



可以看見結果為 PM = 40° GM = 29.1dB
滿足題目要求之 PM
(note:GM 比原來題目解答還要多)

$$\omega_{BW} \ \cong \ 2.23 \times 2 = 4.46$$

For question (c)

$$D_c = \frac{\frac{s}{2.25}+1}{\frac{s}{45}+1} \qquad D_2 = \frac{\frac{s}{4.5}+1}{\frac{s}{22.5}+1} \qquad D_3 = \frac{\frac{s}{6}+1}{\frac{s}{18}+1}$$

Compare three D(s),可以明顯看出D_{2or3} pole 的位置明顯更靠近原點

又以知頻域上 $\frac{1}{s+a}$ 在時域為 e^{-at}

當 a 很大時，需要在極短時間內使信號急劇下降

在現實中要做出此效果是困難的

由此可知，若無法做出，如D_c高強度的補償系統，

可以利用較低強度的D_{2or3} 多個串接，

以達到相同甚至更好的 PM or GM

（like 三個臭皮匠勝過一個諸葛亮ＸＤ）

追根究底發生原因：

當$\omega = \frac{1}{T\sqrt{\alpha}}$ 為定值 （T 反比$\sqrt{\alpha}$ ）

α 越大 αT就會越大 （αT 正比$\sqrt{\alpha}$ ）

而當利用串接的方式時，

α 所需要的值就可以越大（查表可看見 $\frac{1}{\alpha}$ 可以越小）

$\frac{T_D s+1}{\alpha T_D s+1}$ pole 所在的位置就能更靠近原點

(做圖之 Matlab code)

```
s = tf( 's' );
G = 5/(s*(s+1)*(s/5+1));
a2 = 1/5;
T2 = 1/4.5;
D2 = (T2*s+1)/(a2*T2*s+1);
sys2 = G*D2*D2;

w=logspace(-2,2,10000);
figure(1);
bode(sys2,w);
hold on;

a3 = 1/3;
T3 = 1/6;
D3 = (T3*s+1)/(a3*T3*s+1);
sys3 = G*D3*D3*D3;

figure(2);
bode(sys3,w);
hold on;
```

Homework 10 Units 6I-6K: Bode Plot Digital Control System, Fall 2021, NTU-EE
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Problem 2 (revised)

For a system with open loop transfer function

$$G(s) = \frac{20}{s\left[\left(\frac{s}{7}\right)+1\right]\left[\left(\frac{s}{10}\right)+1\right]}$$

1. Design a lag compensation D(s) with unity DC gain so that PM ≥ 60° using Bode plot sketches, then verify and refine your design using Matlab. What is the approximate bandwidth of the system?
2. Sketch the step response of the closed-loop system before and after adding the lag compensator. Does your design increase its stability?.

Solutions:

1.

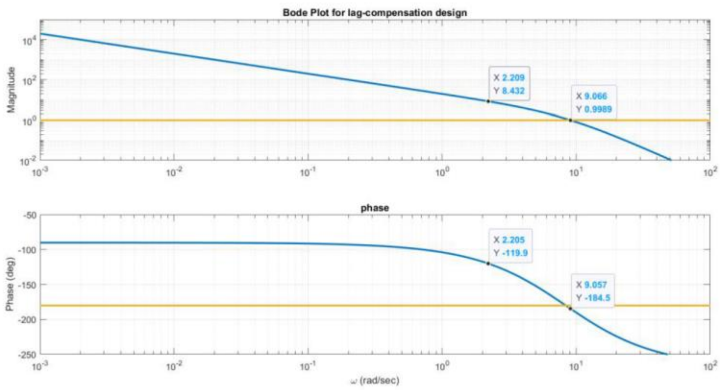
Lag compensation design :

$$D(s) = \frac{T_s+1}{\alpha T_s+1}$$

so that DC gain of D(s) = 1.

(a) The stability margins of the plant without compensation (by Bode plot):

$$PM = -4.5° \text{ (at } \omega = 9.06 \text{ rad/sec)}$$
$$GM = 0.85 \text{ dB (at } \omega = 8.36 \text{ rad/sec)}$$



(b) The lag compensation needs to lower the crossover frequency so that a PM $\geq 60°$ will result, so we see from the uncompensated Bode that we need the crossover at about

$$\omega_{c,new} = 2.2 \text{ rad/sec}$$

where

$$|G(j\omega_c)| \cong 8.42$$

so the lag needs to lower the gain at $\omega_{c,new}$ from 8.42 to 1.

(c) Pick the zero breakpoint of the lag to avoid influencing the phase at $\omega = \omega_{c,new}$ by picking it a factor of 20 below the crossover, so

$$\frac{1}{T} = \frac{\omega_{c,new}}{20} \quad \rightarrow \quad \mathrm{T} = 9$$

(d) Choose α :

Since $D_c(j\omega) \cong \frac{1}{\alpha}$ for $\omega \gg \frac{1}{T}$, let

$$\frac{1}{\alpha} = \frac{1}{|G(j\omega_{c,new})|}$$

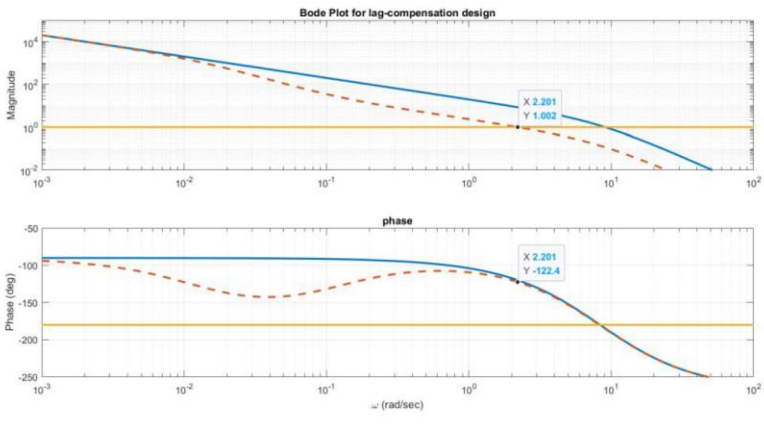
$$\rightarrow \quad \alpha = |G(j\omega_{c,new})| = 8.42$$

(e) Compensation:

$$D_c(s) = \frac{\frac{s}{0.11} + 1}{\frac{s}{0.013} + 1}$$

(f) Stability margins of the compensated system:

$$\mathrm{PM} = 57.6° \text{ (at } \omega = 2.20 \text{ rad/sec)}$$
$$\mathrm{GM} = 7.02 \text{ dB (at } \omega = 8.26 \text{ rad/sec)}$$



(g) Use Matlab to refine the system to meet the requirement of PM ≥ 60°.

When modifying, we can try changing the factor we have selected in (c).

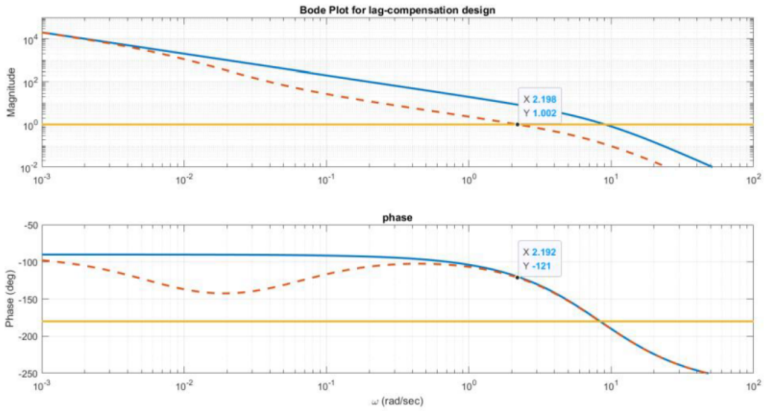
- First try: change the factor from 20 to 40, T = 18.

  Then, the compensation would be

$$D_c(s) = \frac{\frac{s}{0.055} + 1}{\frac{s}{0.0065} + 1}$$

Stability margins of the compensated system:

$$\text{PM} = 59.0° \text{ (at } \omega = 2.2 \text{ rad/sec)}$$
$$\text{GM} = 7.11 \text{ dB (at } \omega = 8.32 \text{ rad/sec)}$$



It becomes nearer to our goal.   Let's move on to the second try.

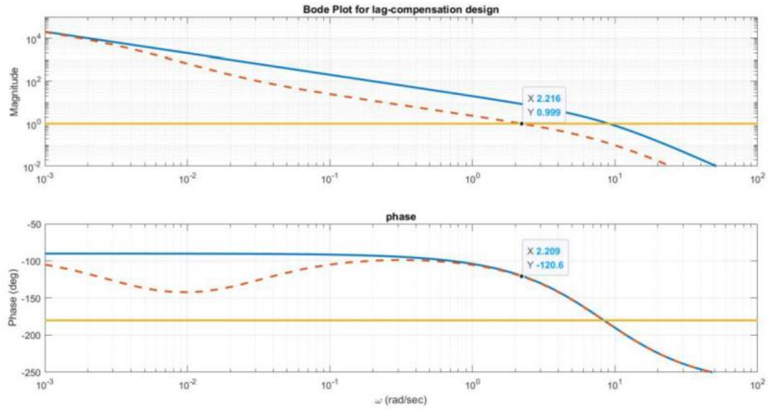
- Second try: change the factor from 20 to 80, T = 36.

  Then, the compensation would be

$$D_c(s) = \frac{\frac{s}{0.0275} + 1}{\frac{s}{0.00327} + 1}$$

Stability margins of the compensated system:

$$\text{PM} = 59.4° \text{ (at } \omega = 2.2 \text{ rad/sec)}$$
$$\text{GM} = 7.11 \text{ dB (at } \omega = 8.34 \text{ rad/sec)}$$



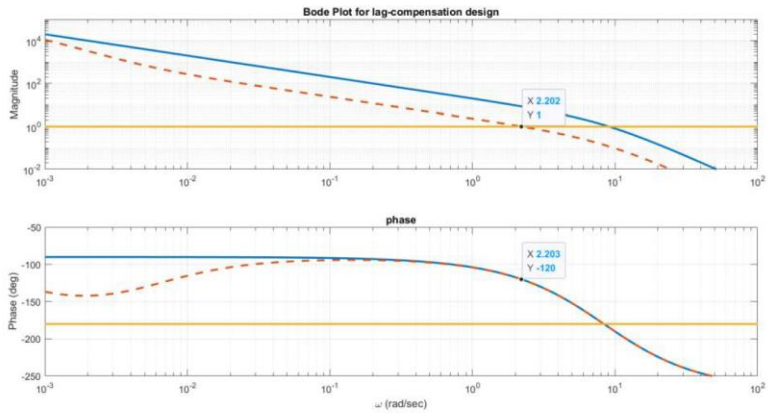
It becomes nearer to our goal. Let's move on to the third try.

- Third try: change the factor from 20 to 400, T = 180.
  Then, the compensation would be

$$D_c(s) = \frac{\frac{s}{0.0055} + 1}{\frac{s}{0.00065} + 1}$$

Stability margins of the compensated system:

$$\text{PM} = 60° \text{ (at } \omega = 2.20 \text{ rad/sec)}$$
$$\text{GM} = 7.18 \text{ dB (at } \omega = 8.36 \text{ rad/sec)}$$



In this case, we reach approximately to our goal for PM ≥ 60°.

- In this part, we slightly improve PM by moving left the location of the zero and pole of the compensator, which indicates the increase of T.

(h) Approximate bandwidth ω_{BW}:

$$\text{PM} \cong 60° \rightarrow \omega_{BW} \cong 2\omega_c = 4.40 \text{ rad/sec}$$

(i) The method we use above is to adjust our design of lag compensator by rough observation using DataTips in the Matlab figures.
While, there is a convenient function in Matlab for us to get all the margins simply and also more accurately.

```
[mag,phas] = bode(num,den,w);
[OLGM,OLPM,OLWcg,OLWcp] = margin(mag,phas,w);
data0 = [OLGM,OLPM,OLWcg,OLWcp];
```

In the last case which we have done in (g), the output of margins is organized as below.

	GM (dB)	PM (degree)	ω_{cg}	ω_{cp}
Uncompensated	0.8501	-4.4910	8.3666	9.0608
Compensated	7.1843	59.9926	8.3617	2.2019

Using the result above, we can check our design and find that the PM of the system after compensation is very near to 60°. For further improvement, we may need to make T much larger.

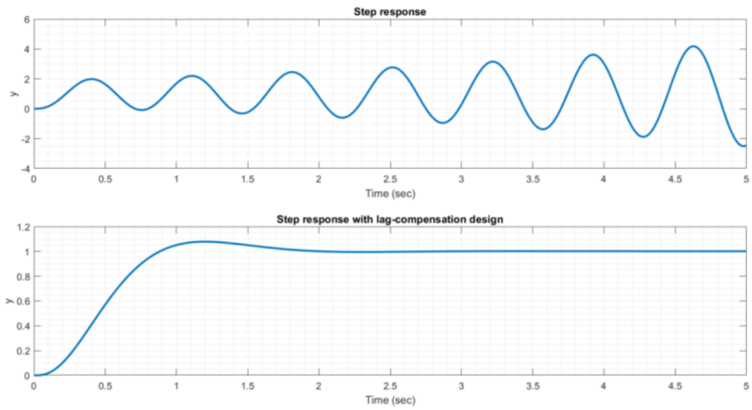
(j) Matlab commands:

```
w = logspace(-3,2,500);
num = 20;
den = conv([1 0],[1/10 1]);
den = conv(den,[1/7 1]);

[mag,phas] = bode(num,den,w);
[OLGM,OLPM,OLWcg,OLWcp] = margin(mag,phas,w);
data0 = [OLGM,OLPM,OLWcg,OLWcp]

%Lag compensator
numl = 1*[1/0.0055 1];
denl = [1/0.00065 1];
numc = conv(num,numl);
denc = conv(den,denl);
```

```
[magc,phasc] = bode(numc,denc,w);
[D1GM,D1PM,D1Wcg,D1Wcp] = margin(magc,phasc,w);
data1 = [D1GM,D1PM,D1Wcg,D1Wcp]

dencl = denc+[0 0 0 numc];
t = 0:.1:20;
y = step(numc,dencl,t);
figure()

subplot(2,1,1)

loglog(w,mag,'-',w,magc,'--',w,ones(500,1),'-','LineWidth',2);

axis([.001 100 .01 100000])
grid on;
%xlabel('\omega (rad/sec)');
ylabel('Magnitude');
title('Bode Plot for lag-compensation design');

subplot(2,1,2)

semilogx(w,phas,'-',w,phasc,'--',w,-180*ones(500,1),'LineWidth',2);

axis([.001 100 -250 -50])
grid on;
xlabel('\omega (rad/sec)');
ylabel('Phase (deg)');
title('phase');
```

2.

(a) Use Matlab to sketch the step response.



With this result, we could confirm that our lag compensation design makes the system more stable.

(b) Matlab commands:

```
k = 20;
s = tf('s');

sysG = 1/(s*(s/7+1)*(s/10+1));
sysD = 1;
sysT = feedback(k*sysD*sysG,1);

t = 0:.01:5;
y = step(sysT,t);

subplot(2,1,1)

plot(t,y,'LineWidth',2);
grid minor
xlabel('Time (sec)');
ylabel('y');
title('Step response');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
k = 20;
s = tf('s');

sysG = 1/(s*(s/7+1)*(s/10+1));
sysD = (s/0.0055+1)/(s/0.00065+1);
sysT = feedback(k*sysD*sysG,1);

t = 0:.01:5;
y = step(sysT,t);

subplot(2,1,2)

plot(t,y,'LineWidth',2);
grid minor
xlabel('Time (sec)');
ylabel('y');
title('Step response with lag-compensation design');
```

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Problem :

For a system with open-loop transfer function

$$G(s) = \frac{53}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{20}+1\right)},$$

(a) Please obtain the magnitude and phase Bode Plots and find the gain margin and phase margin of $G(s)$.

(b) If we choose to use the lead compensation,

$D_c(s) = \frac{T_D s+1}{\alpha T_D s+1}$, $\alpha<1$, please find the gain margin and phase margin of $D_c(s)G(s)$ when $(\alpha, T_D) =$ (i) $(0.1, 1)$, and (ii) $(0.001, 1)$.

(c) If we choose to use the lag compensation,

$D_c(s) = \frac{T_I s+1}{\alpha T_I s+1}$, $\alpha>1$, please find the gain margin and phase margin of $D_c(s)G(s)$ when $(\alpha, T_I) =$ (i) $(2, 1)$, and (ii) $(10, 1)$.

(d) Compare the gain margin and phase margin in (a) and (b).

(e) Compare the gain margin and phase margin in (a) and (c).

(f) Please obtain the Root locus plot of

(i) $D_{c1}(s)G(s) = \dfrac{K_1}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{20}+1\right)} \cdot \dfrac{s+1}{2s+1}$ and

(ii) $D_{c2}(s)G(s) = \dfrac{K_2}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{20}+1\right)} \cdot \dfrac{s+1}{10s+1}$.

(g) Please find the stable range of positive K_1 and K_2 in (f).

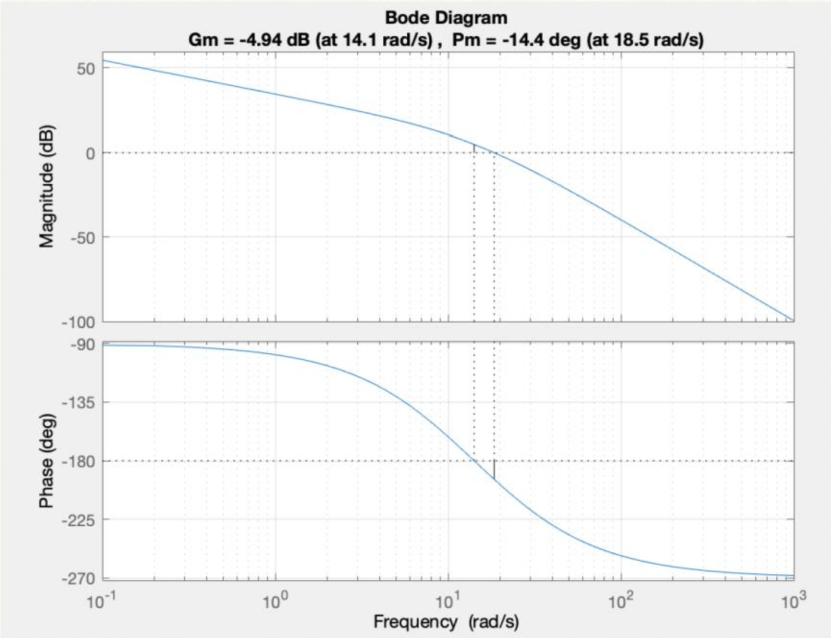
Solution:

(a) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));


figure(1)
bode( sysG );
set(findall(gcf,'type','line'),'linewidth',3);
margin ( sysG );
grid
```

Output:



Gain margin : 0.57 , phase crossover frequency : 14.1 rad/s

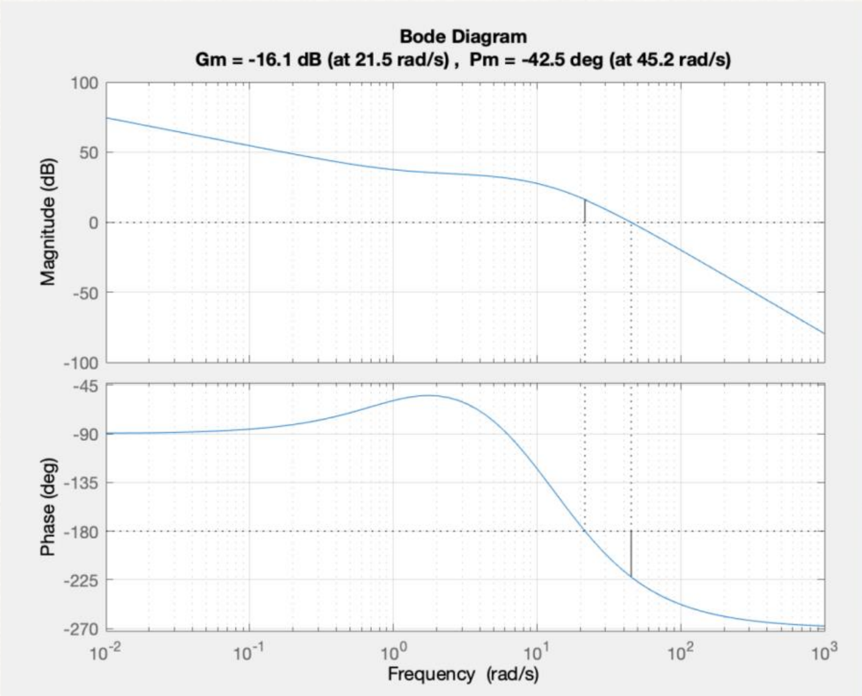
Phase margin : -14.4°, gain crossover frequency : 18.5 rad/s

(b) (i) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(0.1*s+1);

figure(2)
bode( sysG*sysD );
set(findall(gcf,'type','line'),'linewidth',3);
margin ( sysG*sysD );
grid
```

Output:



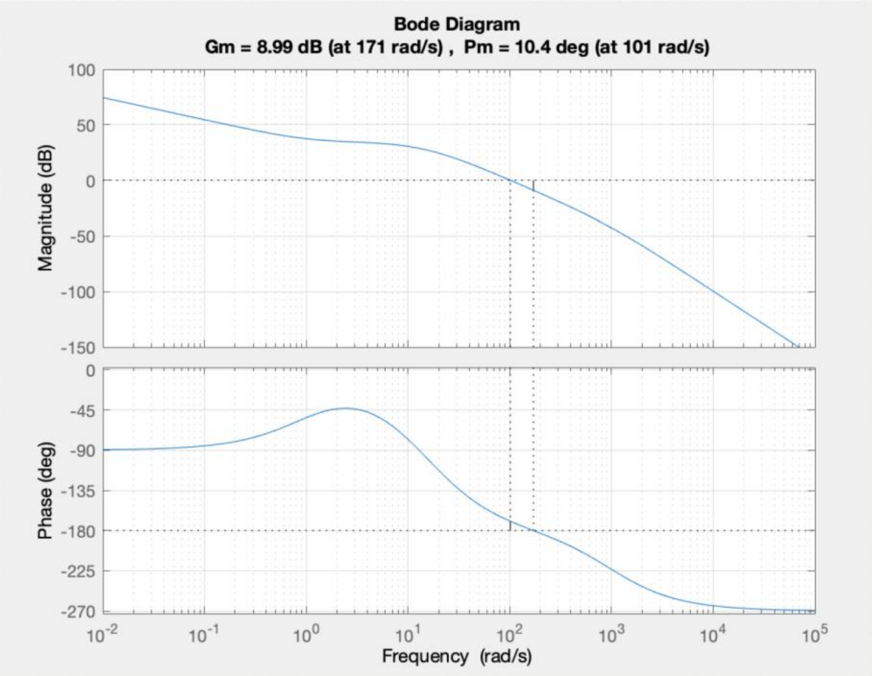
Gain margin: 0.16, phase crossover frequency: 21.5 rad/s

Phase margin: $-42.3°$, gain crossover frequency: 45.2 rad/s (b)(ii) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(0.001*s+1);

figure(3)
bode( sysG*sysD );
set(findall(gcf,'type','line'),'linewidth',3);
margin ( sysG*sysD );
grid
```

Output:



Gain margin : 2.82 , phase crossover frequency : 171 rad/s

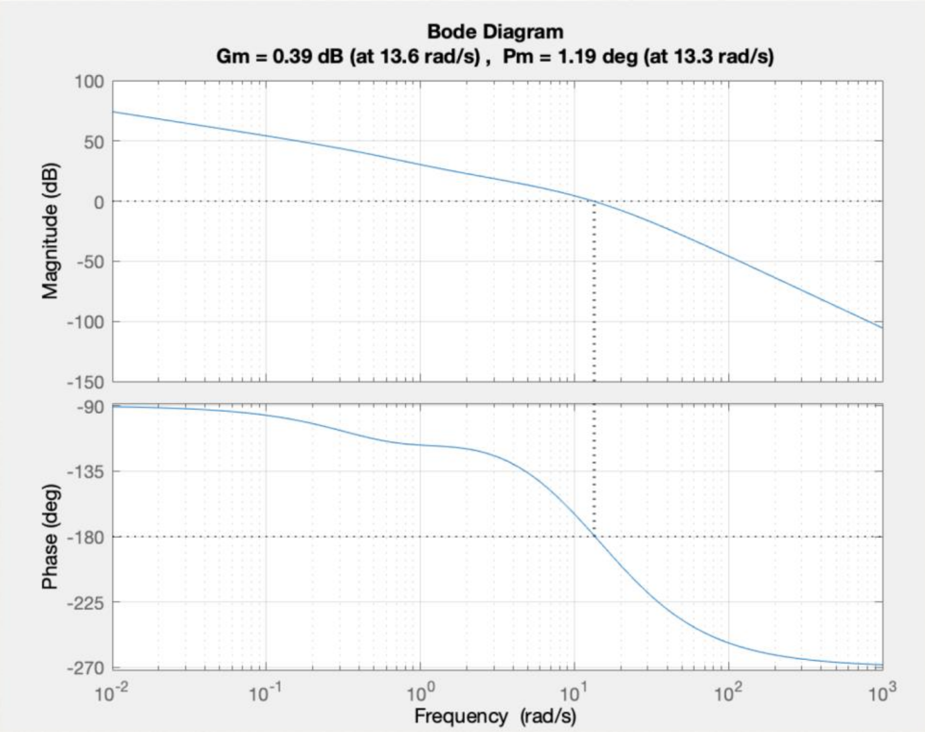
Phase margin : 10.4° , gain crossover frequency : 101 rad/s

(C)(i) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(2*s+1);

figure(4)
bode( sysG*sysD );
set(findall(gcf,'type','line'),'linewidth',3);
margin ( sysG*sysD );
grid
```

Output:



Gain margin : 1.05 , phase crossover frequency : 13.6 rad/s

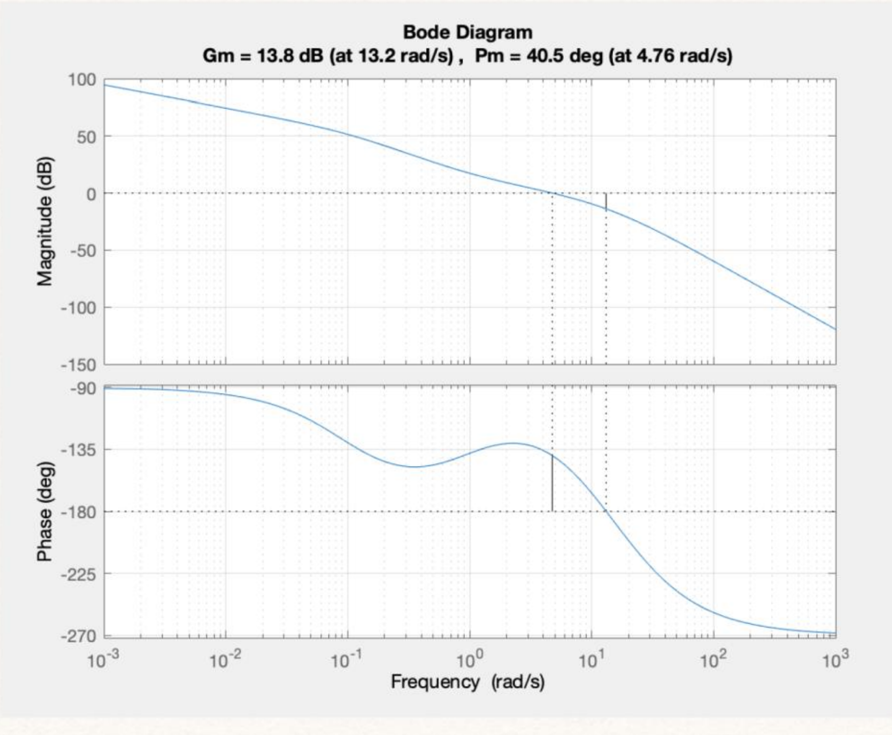
Phase margin : 1.19 , gain crossover frequency : 13.3 rad/s

(C) (ii) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(10*s+1);

figure(5)
bode( sysG*sysD );
set(findall(gcf,'type','line'),'linewidth',3);
margin ( sysG*sysD );
grid
```

Output:



Gain margin : 4.90 , phase crossover frequency : 13.2 rad/s

Phase margin : 40.5 , gain crossover frequency : 4.76 rad/s

(d) In (b)(i), GM becomes worse,

and PM becomes worse

In (b)(ii), GM becomes better,

and PM becomes better

(e) In (C)(i), GM becomes better,

and PM becomes better

In (C)(ii), GM becomes much better,

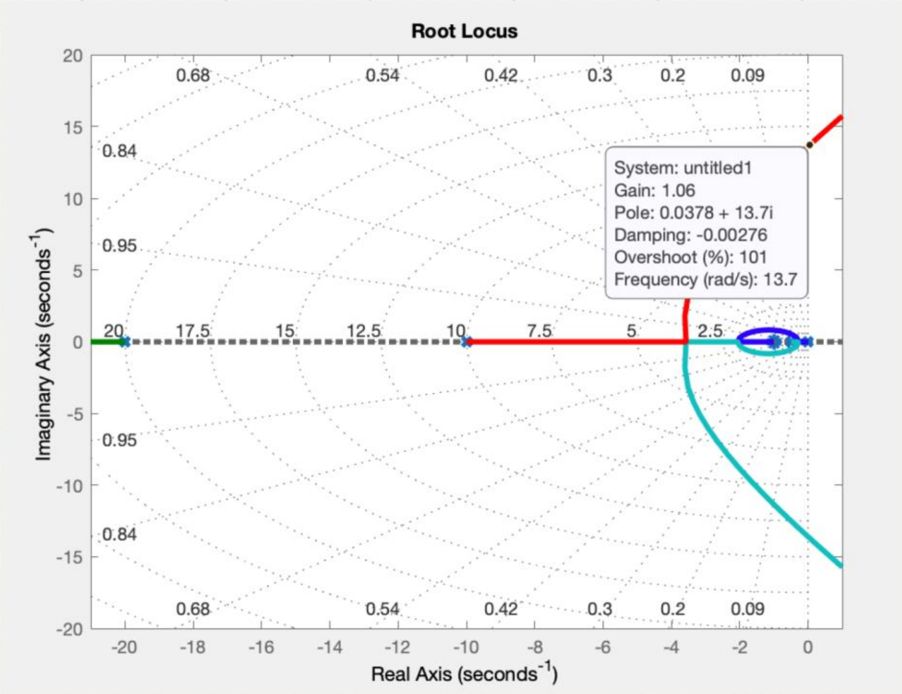
and PM becomes much better

(f) (i) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(2*s+1);

figure(6)
rlocus( sysG*sysD );
axis( [ -21 1, -20 20 ]);
set(findall(gcf,'type','line'),'linewidth',3);
grid
```

Output:

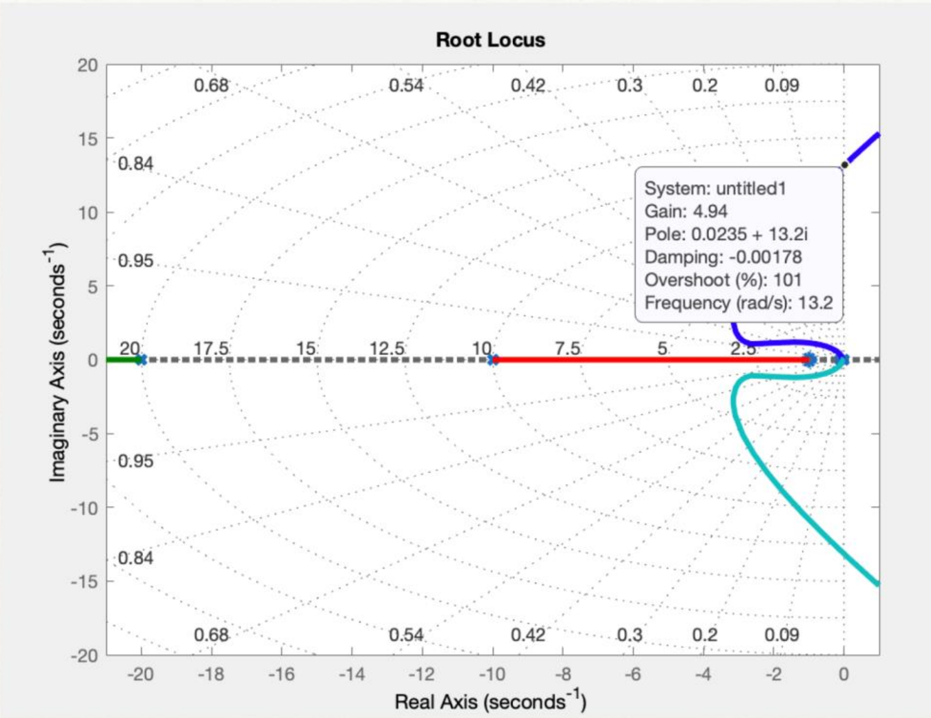


(f) (ii) Matlab Code:

```
clear all; close all
s = tf( 's' )
sysG = 53/(s*(s/10+1)*(s/20+1));
sysD =(s+1)/(10*s+1);

figure(7)
rlocus( sysG*sysD );
axis( [ -21 1, -20 20 ]);
set(findall(gcf,'type','line'),'linewidth',3);
grid
```

Output:



(g) From (f), we obtain that the stable range of positive K_1 is $K_1 < 1.06$, and the stable range of positive K_2 is $K_2 < 4.94$

HW10 電機三 B08502048 陳杰伸

(Revised from problem 1)

1. For the unity feedback system with

$$\mathrm{KG(s)} = \frac{K}{s(s+1)(\frac{s}{5}+1)}$$

(a) Design a lead compensator such that $\mathrm{K}_v = 5$ and $\mathrm{PM} \geq 40°$.

(b) Design a lag compensator with same specifications in (a)

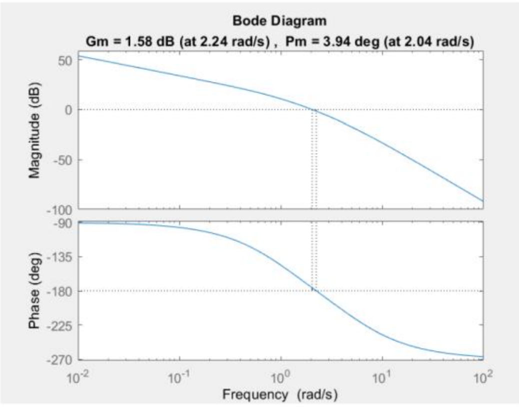
(c) Use MATLAB to plot the Bode plot of $G(s), KG(s)D_C(s)$ in (a) and (b) together. Use the plot to support your target for K and $D_C(s)$ in (a) and (b).

Sol.

(a)

We have $\mathrm{K}_v = \lim_{s\to 0} \mathrm{sKG(s)} = K$. Therefore, $K = 5$.

Use MATLAB to plot the Bode plot of KG(s).



We have $PM = 3.9°$ at $\omega_c = 2.04\ (\mathrm{rad/s})$.

Determine phase lead $= 40° + 10° + 3.9° = 53.9°$

Choose phase lead $= 55°$, $\frac{1}{\alpha} = 10$ for lead compensator from figure in textbook.

Then, choose zero at $\omega_c\sqrt{\alpha} = 0.64 \approx 1$.

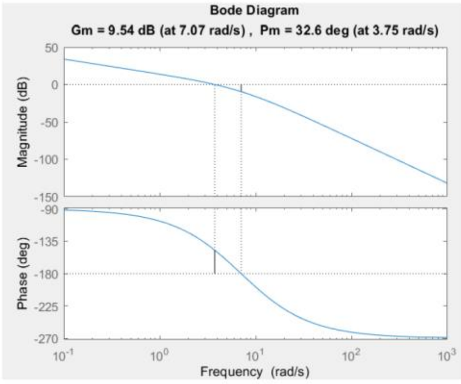
Also, pole at $\frac{1}{\alpha} \times zero = 10$.

Finally,

$$D_C(\mathrm{s}) = \frac{\frac{s}{1}+1}{\frac{s}{10}+1} = 10\frac{s+1}{s+10}$$

1/4

However, $PM = 32.6° < 40°$ as shown in follows.

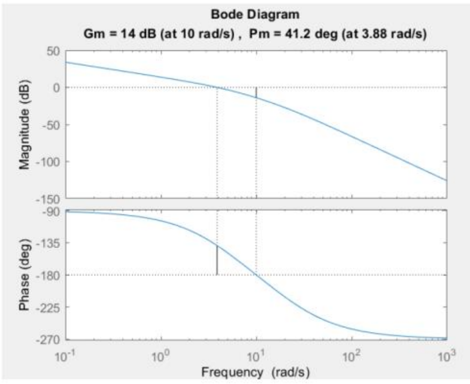


Now, choose $\frac{1}{\alpha} = 20$.

We choose zero at 1. Then, pole at 20.
Finally,

$$D_C(s) = \frac{\frac{s}{1}+1}{\frac{s}{20}+1} = 20\frac{s+1}{s+20}$$

Check that $PM = 41.2° \geq 40°$. This $D_C(s)$ is the design of the lead compensator.



※ MATLAB Code for (a).

```
s = tf('s');
KG = 5/(s*(s+1)*(s/5+1));
figure
margin(KG);

DC = (s/1+1)/(s/10+1);
figure
margin(KG*DC);

DC = (s/1+1)/(s/20+1);
figure
margin(KG*DC);
```

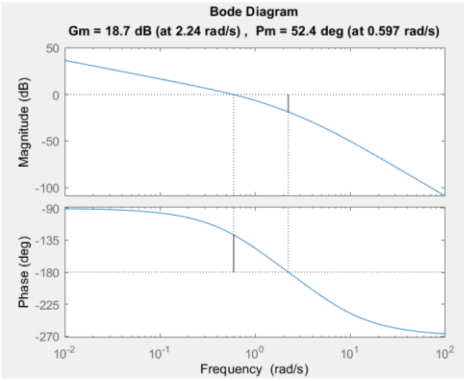
(b)

Recall the Bode plot from (a) when $K = 5$.

We have at $\omega = 0.8$ (rad/s) with potential $\mathrm{PM} = 50°$ and gain $= 17.2$(dB).

Therefore, to match $\mathrm{PM} = 50° \geq 40°$. We want $K = \frac{5}{10^{\frac{17.2}{20}}} = 0.7$ to reduce gain.

Use MATLAB to check our calculations. We have $\mathrm{PM} = 52.4° \geq 40°$.



To match $\mathrm{K}_v = 5$. We need $\alpha = \frac{5}{0.7} = 7.14$

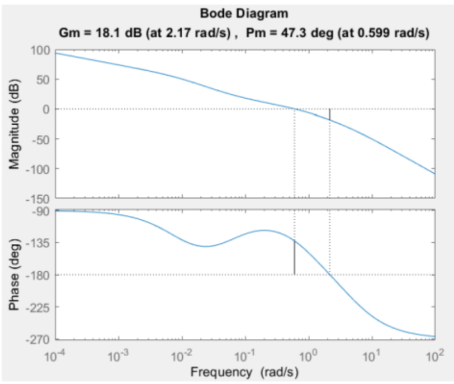
Then, choose zero at $\frac{\omega_c}{10} = \frac{0.597}{10} = 0.06$

Also, pole at $\frac{1}{\alpha} \times zero = 0.0084$

Finally,

$$D_C(s) = 7.14 \frac{\frac{s}{0.06} + 1}{\frac{s}{0.0084} + 1}$$

Check that $PM = 47.3° \geq 40°$. This $D_C(s)$ is the design of the lag compensator.



※ MATLAB Code for (b).

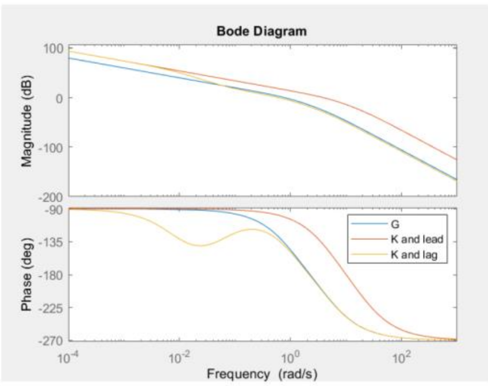
```
KG = 0.7/(s*(s+1)*(s/5+1));
figure
margin(KG);

DC = 7.14*(s/0.06+1)/(s/0.0084+1);
figure
margin(KG*DC);
```

(c)

For (a), use K to match K_v and use $D_C(s)$ to match PM.
We can see that K create displacement in $G(s)$ to have higher gain to match K_v.
And, $D_C(s)$ creates huge phase lead in $G(s)$ in some range and some gain in high ω.
$D_C(s)$ in total maintains the PM affected by K. Note that $G(s)$ has $\text{PM} = 43.2°$.

For (b), use K to match PM and use $D_C(s)$ to match K_v.
We can see that K create displacement in $G(s)$ to have slightly higher PM.
And, $D_C(s)$ improves gain to match K_v and overlap the plot for (a) in low ω.
Though it also creates huge phase lag in $G(s)$ in some range, the pole and zero are far from ω_c. Hence, there is little effect to PM.



※ MATLAB Code for (b).

```
figure
hold on;
G = 1/(s*(s+1)*(s/5+1));
bode(G);
K = 5;
DC = (s/1+1)/(s/20+1);
bode(K*G*DC);
K = 0.7;
DC = 7.14*(s/0.06+1)/(s/0.0084+1);
bode(K*G*DC);
legend('G','K and lead','K and lag');
hold off;
```

HW 10_Unit 6 Control Systems, Fall 2021, NTU-EE

Name: 林佩穎 b07901102 Date: 12/28, 2021

10.1 Problem 1

(Question)

For a system having plant transfer function:

$$G(s) = \frac{10}{s(s+1)(\frac{s}{10}+1)}$$

(a) We wish to design a compensator D(s) that satisfies the following design specifications:

i. $K_v = 2$

ii. PM $\geq 45°$

iii. Unconditional stability (PM $\geq 0°$ for all $\omega \leq \omega_c$)

iv. Maximum phase lead occurs at 10 rad/sec

(b) Draw the Bode plot for the system, and find GM and PM.

(c) What type of compensator should we choose? Why?

(d) Use your choice of compensator to meet all design specifications. Discuss the effect of the compensator.

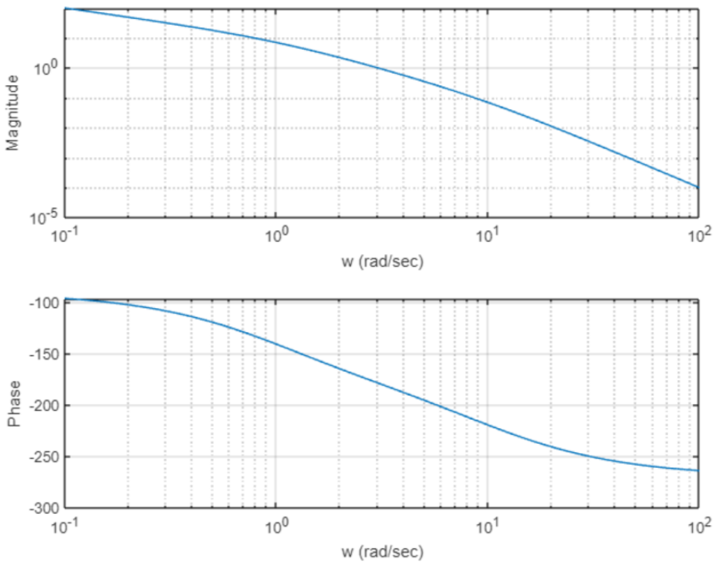
(Solution)

(a) Given K_v, we can solve the K of the compensator.

$$K_v = \lim_{s\to 0} sKG(s) = 10K = 2$$

$$\mathrm{K} = 0.2$$

(b) The Bode plot of the original system with $\mathrm{K} = 0.2$. Since it is difficult to directly read the PM and the crossover frequency, we can use the Matlab to find the margin.



We find the uncompensated system has $\mathrm{PM} = 1.58°, \mathrm{GM} = 1.1$

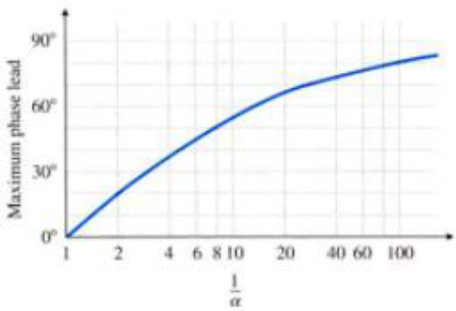
(c) Since the desired PM is $45°$, also allowing for $10°$ extra margin, we wish to have $45° + 10° - 1° \approx 55°$ at the crossover frequency.

Since here a substantial improvement in damping of the system is required, a lead compensator may be a good choice. Although it may increase the gain at high frequency, but the design specification doesn't have strict requirement on that.

$$D_c(s) = \frac{T_D s + 1}{\alpha T_D s + 1}$$

(d) From the below figure, we choose $1/\alpha \cong 10$ for approximately $55°$ phase lead between zero and pole.

$$\alpha = 0.1$$



We can have the greatest benefit from the compensation if the maximum phase lead occurs at the crossover frequency. And the desired crossover frequency $\omega_c = 10$ rad/sec.

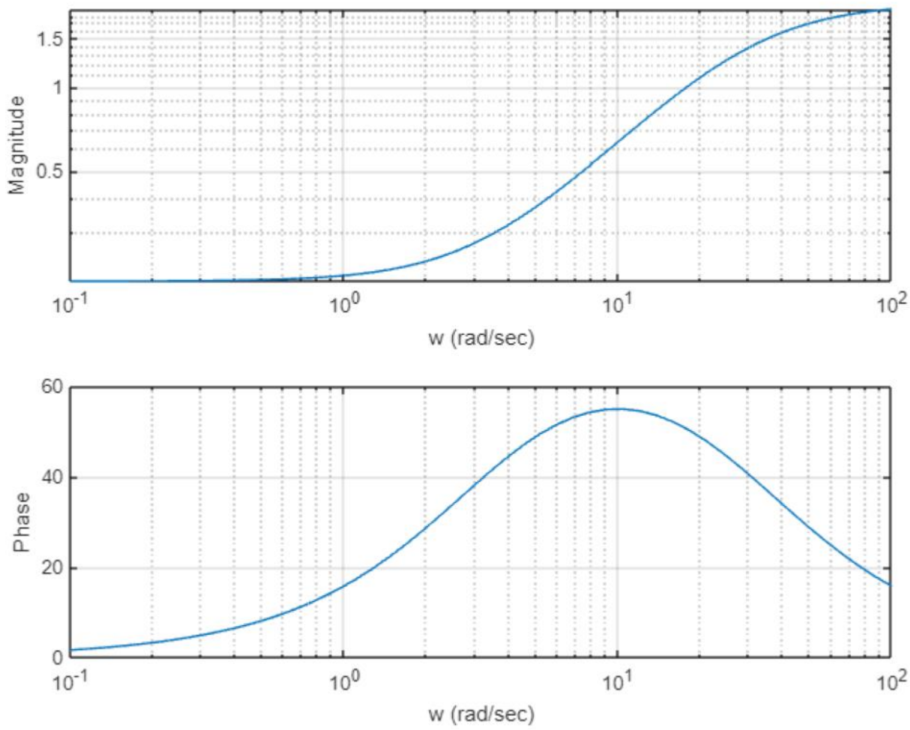
$$\omega = \frac{1}{\sqrt{\alpha T}} = 10$$

We can derive the possible choice of zero and pole for the lead compensator.

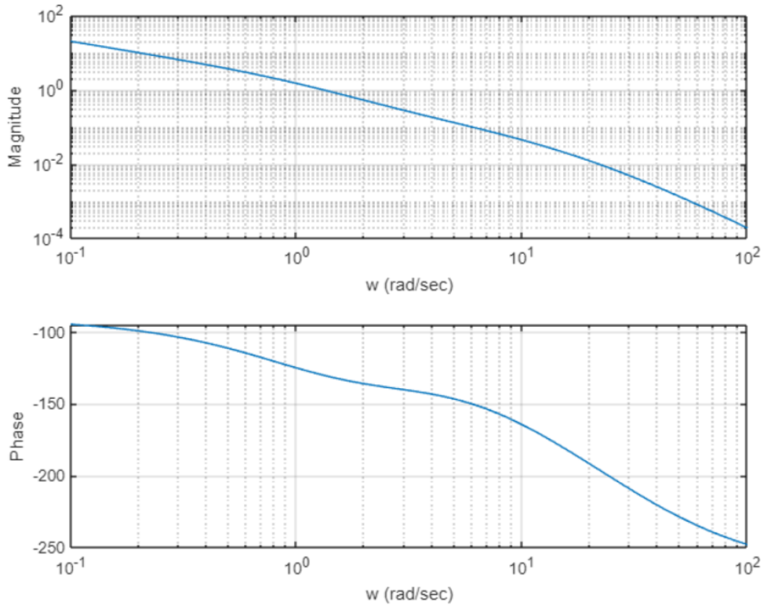
$$\frac{1}{\mathrm{T}} = 3.16, \frac{1}{\alpha\mathrm{T}} = 31.62$$

$$D_c(s) = 0.2 \times \frac{\frac{s}{3.16}+1}{\frac{s}{31.62}+1}$$

The Bode plot of the compensator is shown below, we can see that the maximum phase lead occurs at $\omega = 10$ rad/sec. Note that there is also a gain at higher frequency.



After applying the compensation, the bode plot becomes:



After adding the compensation, $\mathrm{PM} = 50.14°, \mathrm{GM} = 46.89$, both satisfies the design specification. And the approximate bandwidth of the system is $\omega_{BW} \cong 2 \times \omega_c \cong 2 \times 1.3 = 2.6$ rad/sec.

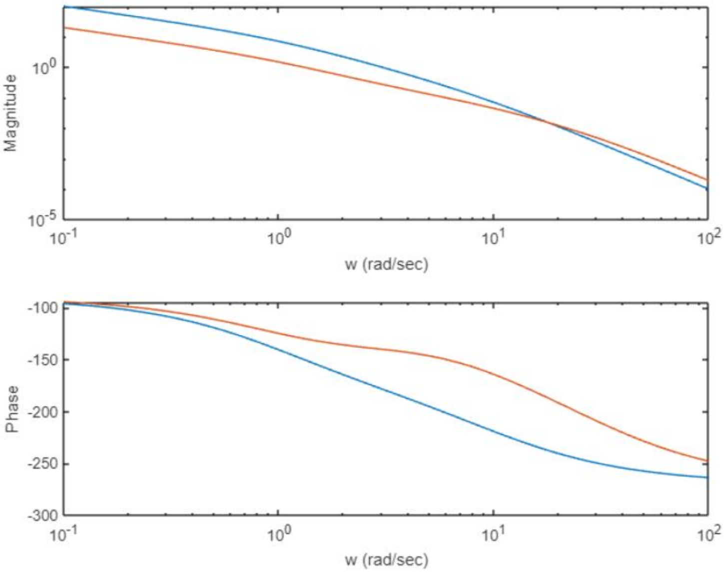
Note 1: Although we wish to set the crossover frequency at 10 rad/sec, the resulting crossover frequency for PM is still 1.3 rad/sec, which is quite different from the expectation. Why?

Note 2: Originally, I want to set the desired bandwidth to be the design specification, however after some experiment, I found it is hard to trace back to set the crossover frequency to meet the desired value…

Comparison:

The blue line is the bode plot for the uncompensated system, and the orange line is the compensated one. We can see that the lead compensator has add phase lead to the system, and thus having larger PM than the original system. The maximum phase lead occurs around 10 rad/sec.

Also, since the gain of the compensator K = 0.2, we can see that the compensated system has smaller DC gain than the uncompensated system. On the other hand, since the lead compensator gain would increase as the frequency grows higher, the compensated system has larger gain than the uncompensated system at the higher frequency.



Matlab code:

```
clf;
s=tf('s');
K=0.2;
D= (s/3.16+1)/(s/31.62+1);
sysG=10/s/(s+1)/(s/10+1);
w=logspace(-1,2);
[mag,phase]=bode(sysG, w);
[mag2,phase2]=bode(K*D*sysG, w);
[GM, PM, Wcg, Wcp] = margin(mag2, phase2, w);
figure(1)
subplot(2,1,1)
loglog(w, squeeze(mag)),grid;
hold on
loglog(w, squeeze(mag2)),grid;
xlabel('w (rad/sec)');
ylabel('Magnitude');
subplot(2,1,2)
semilogx(w, squeeze(phase)),grid;
hold on
semilogx(w, squeeze(phase2)),grid;
xlabel('w (rad/sec)');
ylabel('Phase');
```