Control System: Homework 09 for Units 6D-6G: Bode Plot

Assigned: Dec 17, 2021

Due: Dec 23, 2021 (11:59pm)

1. (Nyquist plot)

22. (a) For $\omega = 0.1$ to 100 rad/sec, sketch the phase of the minimum-phase system

$$\left| G(s) = \frac{s+1}{s+10} \right|_{s=j\omega}$$

and the nonminimum-phase system

$$\left| G(s) = -\frac{s-1}{s+10} \right|_{s=j\omega},$$

noting that $\angle(j\omega - 1)$ decreases with ω rather than increasing.

(b) Does a RHP zero affect the relationship between the -1 encirclements on a polar plot and the number of unstable closed-loop roots in Eq. (6.28)?

(c) Sketch the phase of the following unstable system for $\omega = 0.1$ to 100 rad/sec:

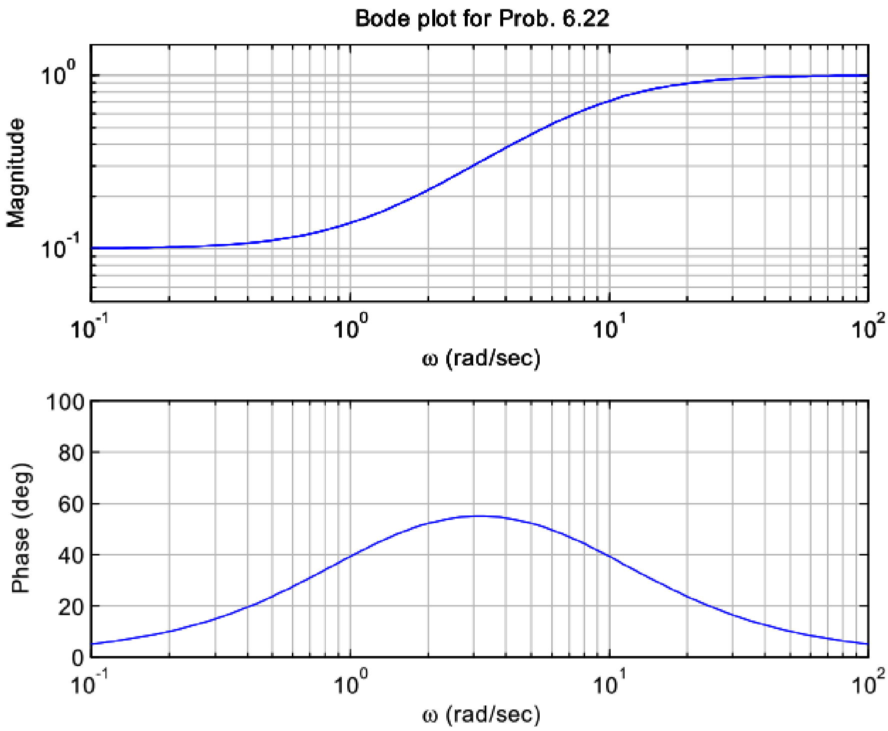
$$G(s) = \left| \frac{s+1}{s-10} \right|_{s=j\omega}.$$

(d) Check the stability of the systems in (a) and (c) using the Nyquist criterion on $KG(s)$. Determine the range of K for which the closed-loop system is stable, and check your results qualitatively using a rough root-locus sketch.

Solution :

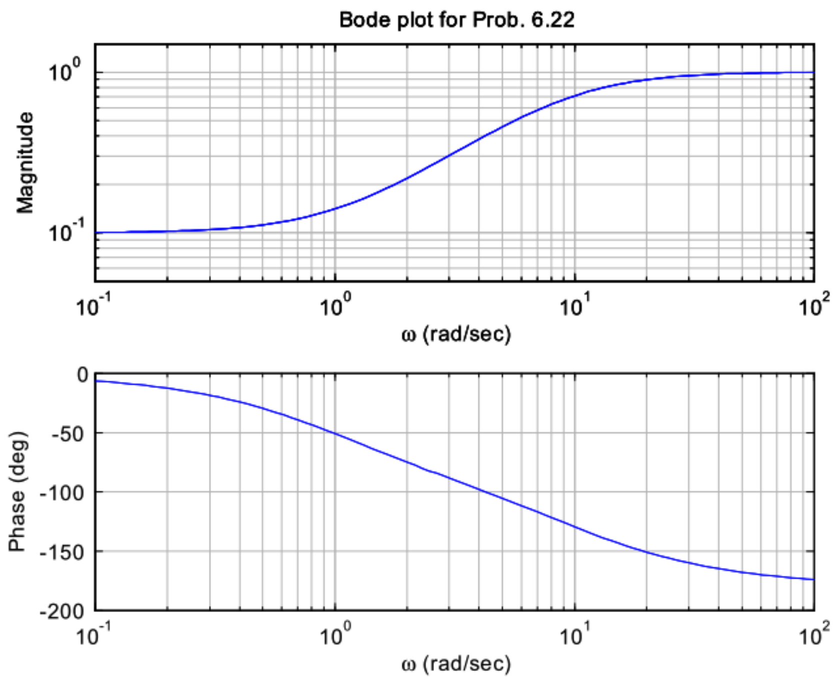
(a) Minimum phase system,

$$G_1(j\omega) = \frac{s+1}{s+10}|_{s=j\omega}$$



Non-minimum phase system,

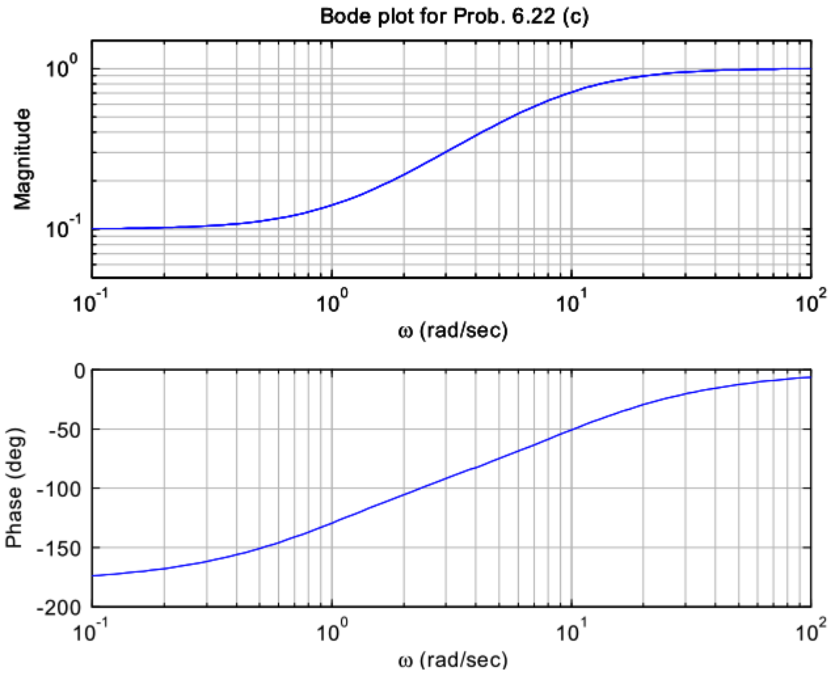
$$G_2(j\omega) = -\frac{s-1}{s+10}|_{s=j\omega}$$



(b) No, a RHP zero doesn't affect the relationship between the -1 encirclements on the Nyquist plot and the number of unstable closed-loop roots in Eq. (6.28).

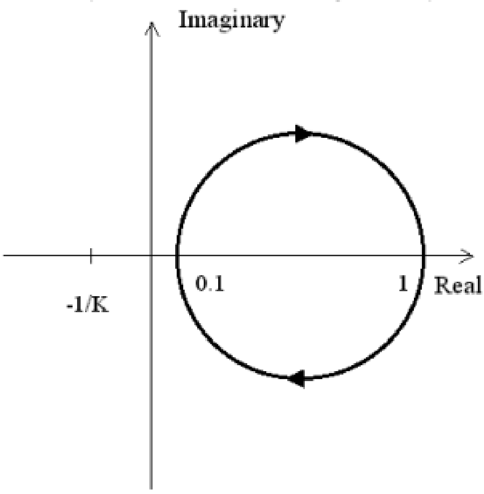
(c) Unstable system:

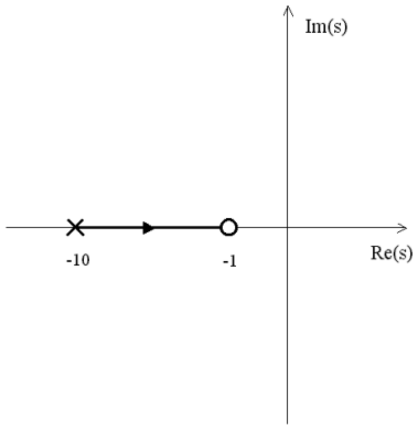
$$G_3(j\omega) = \frac{s+1}{s-10}\Big|_{s=j\omega}$$



(d) Minimum phase system $G_1(j\omega)$:

i. For any $K > 0$, $N = 0$, $P = 0 \Longrightarrow Z = 0 \Longrightarrow$ The system is stable, as verified by the root locus being entirely in the LHP.

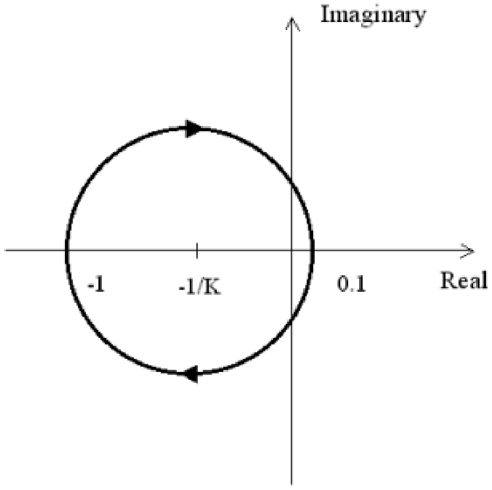


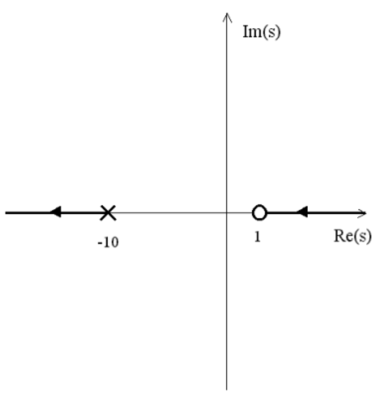


Non-minimum phase system $G_2(j\omega)$: the $-1/K$ point will not be encircled if $K < 1$.

$$\begin{array}{ll} 0 < K < 1 & N = 0,\ P = 0 \Longrightarrow Z = 0 \Longrightarrow \text{Stable} \\ 1 < K & N = 1,\ P = 0 \Longrightarrow Z = 1 \Longrightarrow \text{Unstable} \end{array}$$

This is verified by the Root Locus shown below where the branch of the locus to the left of the pole is from $K < 1$.

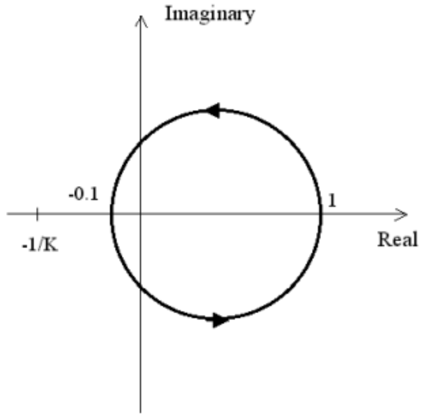


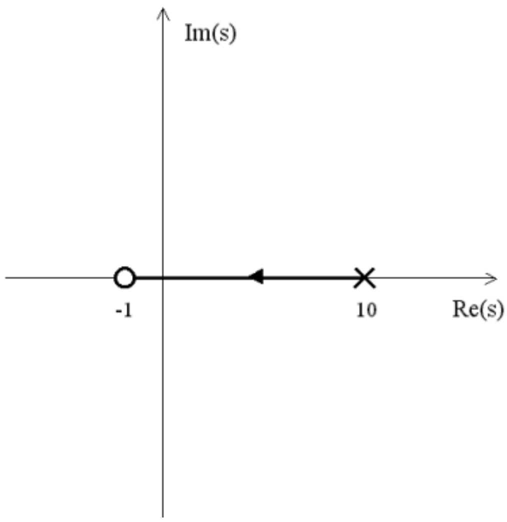


Unstable system $G_3(j\omega)$: The $-1/K$ point will be encircled if $K > 10$, however, $P = 1$, so

$$\begin{aligned} 0 < K < 10: &\quad N = 0,\ P = 1 \Longrightarrow Z = 1 \Longrightarrow \text{Unstable} \\ 10 < K\ : &\quad N = -1,\ P = 1 \Longrightarrow Z = 0 \Longrightarrow \text{Stable} \end{aligned}$$

This is verified by the Root Locus shown below right, where the locus crosses the imaginary axis when $K = 10$, and stays in the LHP for $K > 10$.





2. (Stability margin)

27. Consider the system given in Fig. 6.91.

(a) Use MATLAB to obtain Bode plots for $K = 1$ and use the plots to estimate the range of K for which the system will be stable.

(b) Verify the stable range of K by using margin to determine PM for selected values of K.

(c) Use rlocus and rlocfind to determine the values of K at the stability boundaries.

(d) Sketch the Nyquist plot of the system, and use it to verify the number of unstable roots for the unstable ranges of K.

(e) Using Routh's criterion, determine the ranges of K for closed-loop stability of this system.

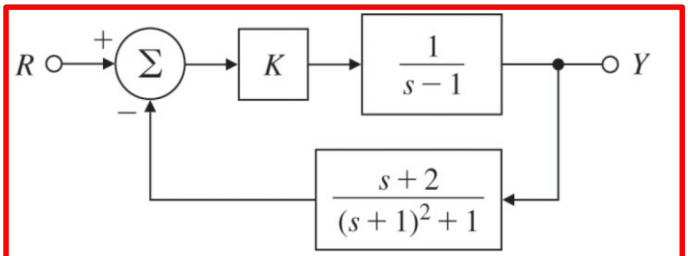
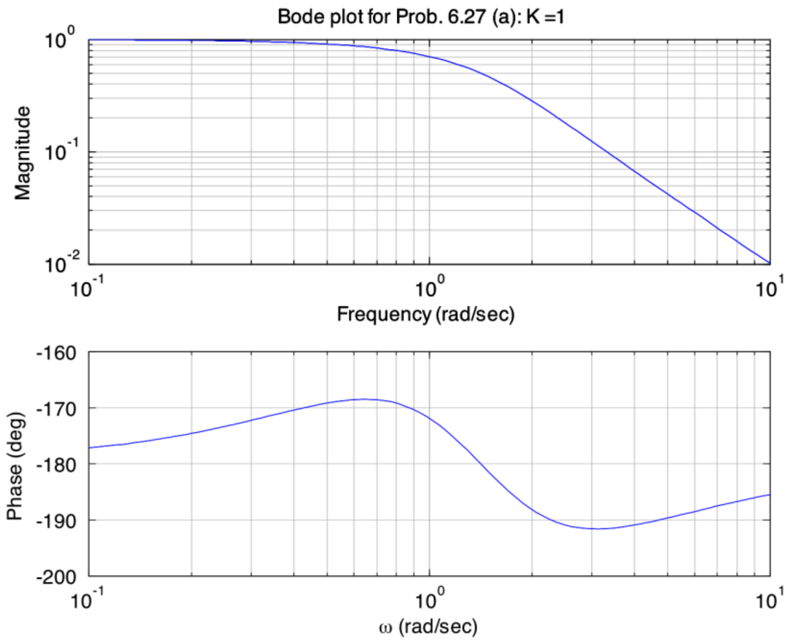


Fig. 6.91 Control system for Problem 6.27

Solution :

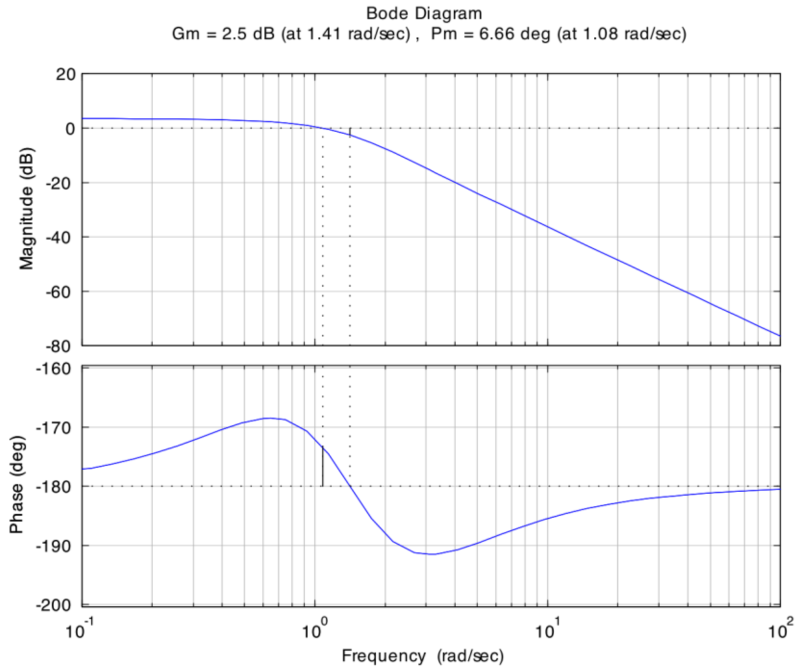
(a) The Bode plot for $K = 1$ is :



From the Bode plot, the closed-loop system is unstable for $K = 1$. But we can make the closed-system stable with positive GM by increasing the gain K up to where the crossover frequencyis at $\omega = 1.414$ rad/sec ($K = 2$), where the phase plot crosses the $-180°$ line. Therefore :

$$1 < K < 2 \implies \text{The closed-loop system is stable.}$$

(b) For example, $PM = 6.66\,\text{deg}$ for $K = 1.5$.



Margin determination for K=1.5

(c) Root locus is :
and it shows that the $j\omega$-crossing information is $K = 2$ and $\omega = \pm\sqrt{2}$, or $K = 1$ at the origin. It can also be calculated by:

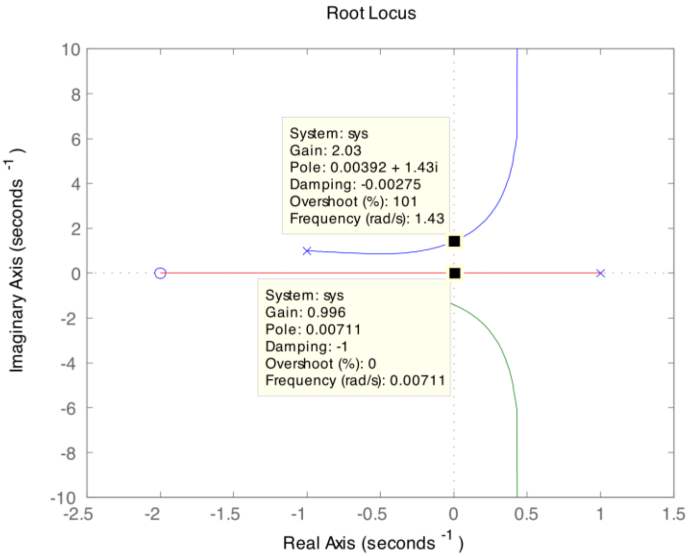
$$1 + K\frac{j\omega + 2}{(j\omega)^3 + (j\omega)^2 - 2} = 0$$

$$\begin{aligned} \omega^2 - 2K + 2 &= 0 \\ \omega(\omega^2 - K) &= 0 \end{aligned}$$

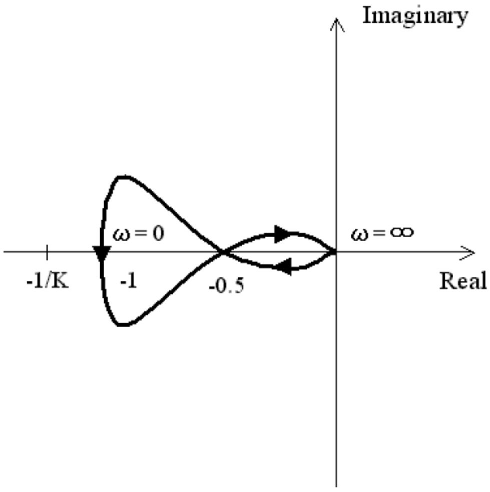
$$K = 2,\ \omega = \pm\sqrt{2},\ \text{or}\ K = 1,\ \omega = 0$$

Therefore,

$$1 < K < 2 \implies \text{The closed-loop system is stable.}$$



(d)



i. $0 < K < 1$
$N = 0,\ P = 1 \implies Z = 1$
One unstable closed-loop root.

ii. $1 < K < 2$
$N = -1,\ P = 1 \implies Z = 0$
Stable.

iii. $2 < K$
$N = 1,\ P = 1 \implies Z = 2$
Two unstable closed-loop roots.

(e) The closed-loop transfer function of this system is :

$$\frac{y(s)}{r(s)} = \frac{k\dfrac{1}{s-1}}{1+k\dfrac{1}{s-1}\times\dfrac{s+2}{(s+1)^2+1}} = \frac{K(s^2+2s+2)}{s^3+s^2+Ks+2K-2}$$

So the characteristic equation is :

$$\Longrightarrow s^3+s^2+Ks+2K-2=0$$

Using the Routh's criterion,

$$\begin{array}{lcc} s^3: & 1 & K \\ s^2: & 1 & 2K-2 \\ s^1: & 2-K & 0 \\ s^0: & 2k=2 & \end{array}$$

For stability,

$$\begin{aligned} 2-K &> 0 \\ 2K-2 &> 0 \end{aligned}$$

$$\Longrightarrow 2 > K > 1$$

$$\begin{array}{cc} 0<K<1 & \text{Unstable} \\ 1<K<2 & \text{Stable} \\ 2<K & \text{Unstable} \end{array}$$

3. (Gain margin and phase margin)

36. For the system shown in Fig. 6.96, determine the Nyquist plot and apply the Nyquist criterion.

(a) to determine the range of values of K (positive and negative) for which the system will be stable, and

(b) to determine the number of roots in the RHP for those values of K for which the system is unstable. Check your answer using a rough root-locus sketch.

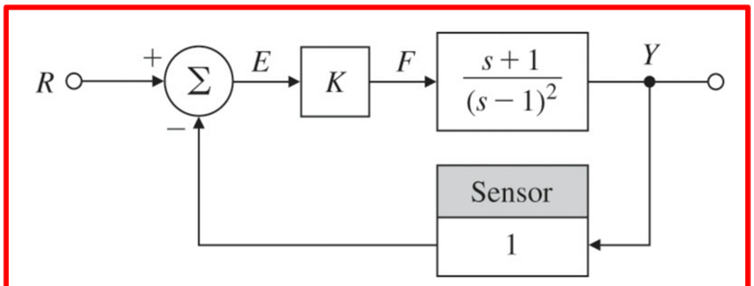


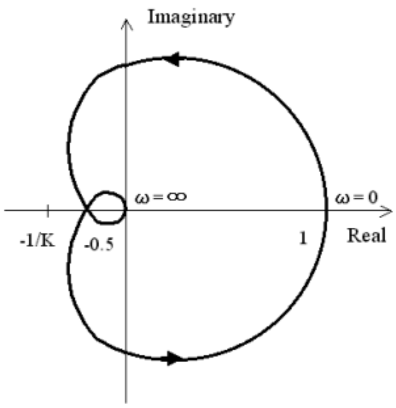
Fig. 6.96 Control system for Problem 6.36

Solution :

(a) & b.

$$KG(s) = K\frac{s+1}{(s-1)^2}$$

Use of the nyquist routine in Matlab yields the plot below, where the contour crosses the real axis at -0.5 and +1.



From the Nyquist plot we see that:

i.

$$-\infty < -\frac{1}{K} < -\frac{1}{2} \implies 0 < K < 2$$

$$N = 0,\ P = 2 \implies Z = 2$$

Two closed-loop roots in RHP.

ii.

$$-\frac{1}{2} < -\frac{1}{K} < 0 \implies 2 < K$$

$$N = -2,\ P = 2 \implies Z = 0$$

The closed-loop system is stable.

iii.

$$0 < -\frac{1}{K} < 1 \implies K < -1$$

$$N = -1,\ P = 2 \implies Z = 1$$

One closed-loop root in RHP.

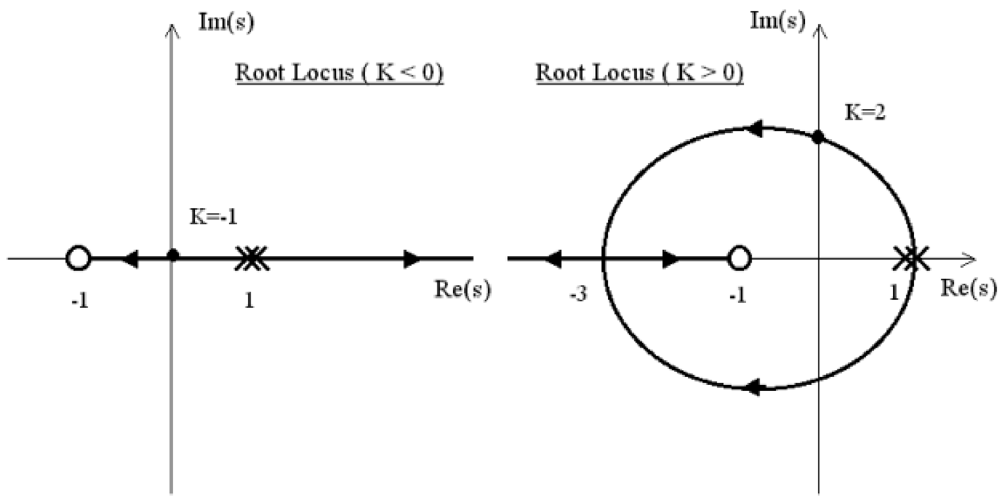
iv.

$$1 < -\frac{1}{K} < \infty \implies -1 < K < 0$$

$$N = 0,\ P = 2 \implies Z = 2$$

Two closed-loop roots in RHP.

These results are confirmed by looking at the root loci below:



4. (Gain-Phase relation)

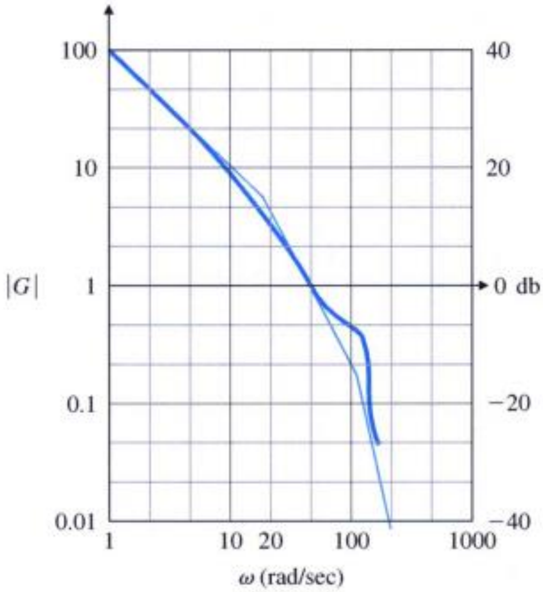
41. The frequency response of a plant in a unity feedback configuration is sketched in Fig. 6.99. Assume the plant is open-loop stable and minimum phase.

(a) What is the velocity constant K_v for the system as drawn?

(b) What is the damping ratio of the complex poles at $\omega = 100$?

(c) What is the PM of the system as drawn? (Estimate to within $\pm 10^o$.)

Figure 6.99: Magnitude frequency response for Prc



Solution :

(a) From Fig. 6.99,

$$K_v = \lim_{s \to 0} sG = |\text{Low frequency asymptote of } G(j\omega)|_{\omega=1} = 100)$$

(b) Let

$$G_1(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$$

For the second order system $G_1(s)$,

$$|G_1(j\omega)|_{\omega=1} = \frac{1}{2\zeta} \quad (1)$$

From Fig. 6.99 :

$$|G_1(j\omega)|_{\omega=100} = \frac{|G(j\omega)|_{\omega=100}}{|\text{Asymptote of } G(j\omega)|_{\omega=100}} \cong \frac{0.4}{0.2} = 2 \qquad (2)$$

From (1) and (2) we have :

$$\frac{1}{2\zeta} = 2 \implies \zeta = 0.25$$

(c) Since the plant is a minimum phase system, we can apply the Bode's approximate gain-phase relationship.

When $|G| = 1$, the slope of $|G|$ curve is $\cong$ -2.

$$\implies \angle G(j\omega) \cong -2 \times 90^\circ \ = -180^\circ$$

$$PM \cong \angle G(j\omega) + 180^\circ \ = 0^\circ$$

Note : Actual PM by Matlab calculation is 6.4°, so this approximation is within the desired accuracy.

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Problem

Consider

$$G_1(s) = \frac{s+1}{(s-1)^2}, G_2(s) = \frac{s-1}{(s+1)^2}.$$

(a) Draw the magnitude and phase Bode plots of the two systems using MATLAB.

(b) Use Nyquist plot and Nyquist criterion to determine the range of values of K for which the systems $KG_1(\mathrm{s})$ and $KG_2(\mathrm{s})$ are stable, respectively.

(c) For those values of K for which the systems $\mathrm{K}G_1(\mathrm{s})$ and $KG_2(\mathrm{s})$ are unstable, use Nyquist plot and Nyquist criterion to determine the number of roots in the RHP, respectively.

(d) Check the results of (b) and (c) using root-locus.

(e) Check the results of (b) using Routh's criterion.

Solution

We first do (a)-(e) to $G_1(s)$.

(a) MATLAB code:

```
G_1 = tf([1, 1], [1, -2, 1]);
figure;
bode(G_1)
```

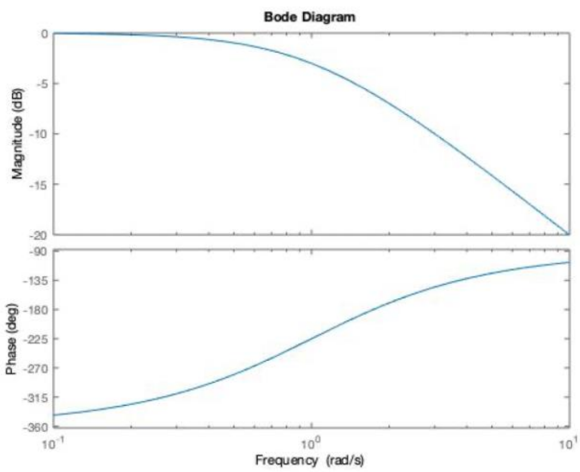

Figure 1. Bode plots of $G_1(s)$.

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(b) Note that $KG_1(s) = K\frac{s+1}{(s-1)^2}$, and the Nyquist plot is as shown in Figure 2.

MATLAB code:

```
G_1 = tf([1, 1], [1, -2, 1]);
figure;
nyquist(G_1)
```

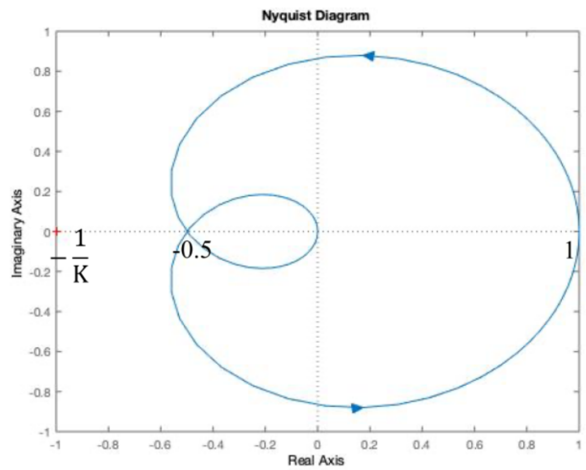

Figure 2. Nyquist plot of $KG_1(s)$.

The curve is the case when $K = 1$ and it crosses the real axis at -0.5 and 1.

From the Nyquist plot we can observe that

(1) $-\frac{1}{K} < -\frac{1}{2}$

In this case, we have $0 < K < 2, N = 0, P = 2$, so $Z = 2$.
That is, when $0 < K < 2$, the system is unstable and there are two closed-loop roots in RHP.

(2) $-\frac{1}{2} < -\frac{1}{K} < 0$

In this case, we have $K > 2, N = -2, P = 2$, so $Z = 0$.
That is, when $K > 2$, the system is stable and there are no closed-loop roots in RHP.

(3) $0 < -\frac{1}{K} < 1$

In this case, we have $K < -1, N = -1, P = 2$, so $Z = 1$.
That is, when $K < -1$, the system is unstable and there is one closed-loop root in RHP.

(4) $1 < -\frac{1}{K}$

In this case, we have $-1 < \mathrm{K} < 0, \mathrm{N} = 0, \mathrm{P} = 2$, so $\mathrm{Z} = 2$.
That is, when $-1 < \mathrm{K} < 0$, the system is unstable and there are two closed-loop roots in RHP.

(c) The solution is shown in (b).

(d) The root-loci when $\mathrm{K} < 0$ and $\mathrm{K} > 0$ are shown in Figure 3 and Figure 4, respectively.

MATLAB code:

```
G_1 = tf([-1, -1], [1, -2, 1]);
figure;
rlocus(G_1)
```

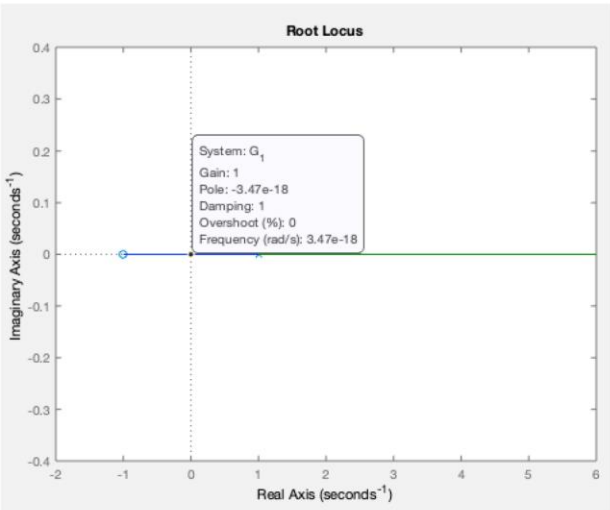

Figure 3. Root-locus of $KG_1(s)$ when $\mathrm{K} < 0$.

MATLAB code:

```
G_1 = tf([1, 1], [1, -2, 1]);
figure;
rlocus(G_1)
```

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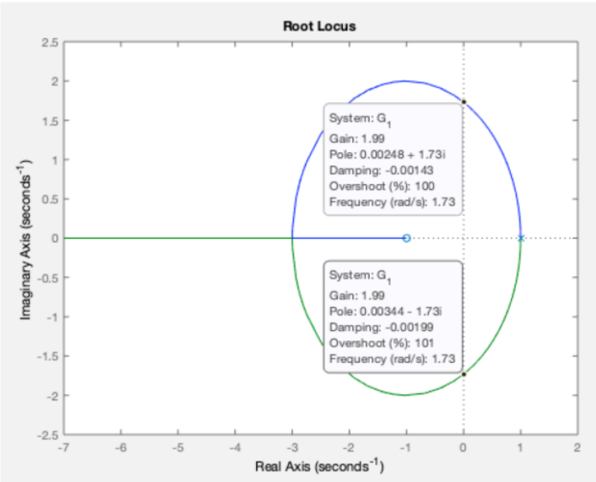

Figure 4. Root-locus of $KG_1(s)$ when $K > 0$.

From the two root-loci, we can see that the only situation that there are no closed-loop poles in the RHP is when $K > 2$, same as the result shown in part (c).

(e) We can use Routh's criterion to verify that the closed-loop system of KG_1 is stable if $K > 2$. The steps are shown in Figure 5.

Closed-loop transfer function:

$$\frac{KG_1(s)}{1+KG_1(s)} = \frac{K\frac{s+1}{(s-1)^2}}{1+K\frac{s+1}{(s-1)^2}} = \frac{K(s+1)}{s^2-(2-K)s+(1+K)}$$

Characteristic equation: $s^2-(2-K)s+(1+K)=0$

Using Routh's criterion:

s^2:	1	1+K
s^1:	−2+K	0
s^0:	1+K	

For stability, $-2+K>0$ and $1+K>0$.

That is, $K>2$ and $K>-1$.

$\therefore$ $K>2 \Rightarrow$ stable, otherwise $\Rightarrow$ unstable.

Figure 5. The steps of using Routh's criterion to verify the condition for stability of $KG_1(s)$.

We now do (a)-(e) to $G_1(s)$.

(a) MATLAB code:

```
G_2 = tf([1, -1], [1, 2, 1])
figure;
bode(G_2)
```

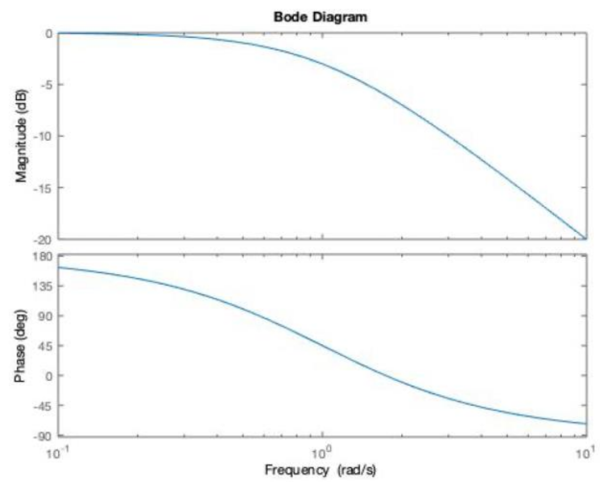

Figure 6. Bode plots of $G_2(s)$.

(b) Note that $KG_2(s) = \mathrm{K}\frac{s-1}{(s+1)^2}$, and the Nyquist plot is as shown in Figure 7.

MATLAB code:

```
G_2 = tf([1, -1], [1, 2, 1]);
figure;
nyquist(G_2)
```

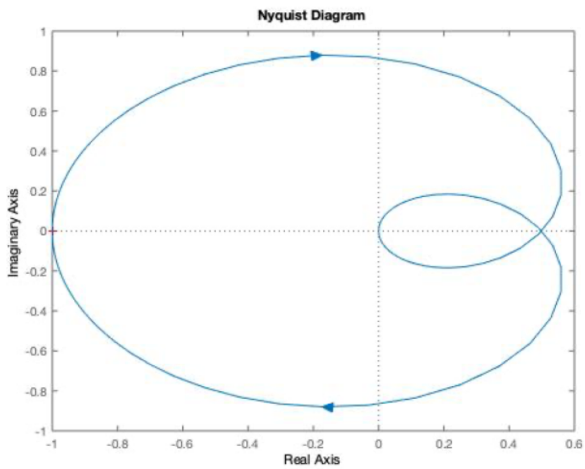

Figure 7. Nyquist plot of $KG_2(s)$.

The curve is the case when $K = 1$ and it crosses the real axis at 0.5 and -1.

From the Nyquist plot we can observe that

(1) $0 < K < 1$

In this case, we have $N = 0, P = 0$, so $Z = 0$.

That is, the system is stable and there are no closed-loop roots in RHP.

(2) $K > 1$

In this case, we have $N = 1, P = 0$, so $Z = 1$.

That is, the system is unstable and there is one closed-loop root in RHP.

(3) $K < -2$

In this case, $N = 2, P = 0$, so $Z = 2$.

That is, the system is unstable and there are two closed-loop roots in RHP.

(4) $-2 < K < 0$

In this case, we have $N = 0, P = 0$, so $Z = 0$.

That is, the system is stable and there are no closed-loop roots in RHP.

(c) The solution is shown in (b).

(d) The root-loci when $K < 0$ and $K > 0$ are shown in Figure 8 and Figure 9, respectively.

MATLAB code:

```
G_2 = tf([-1, 1], [1, 2, 1]);
figure;
rlocus(G_2)
```

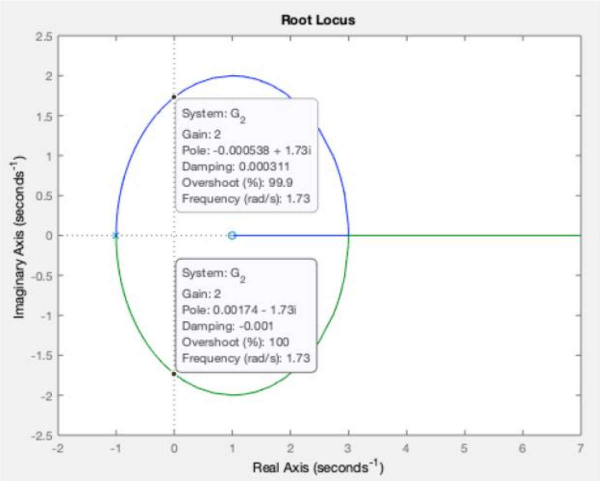

Figure 8. Root-locus of $KG_2(s)$ when $K < 0$.

MATLAB code:

```
G_2 = tf([1, -1], [1, 2, 1]);
figure;
rlocus(G_2)
```

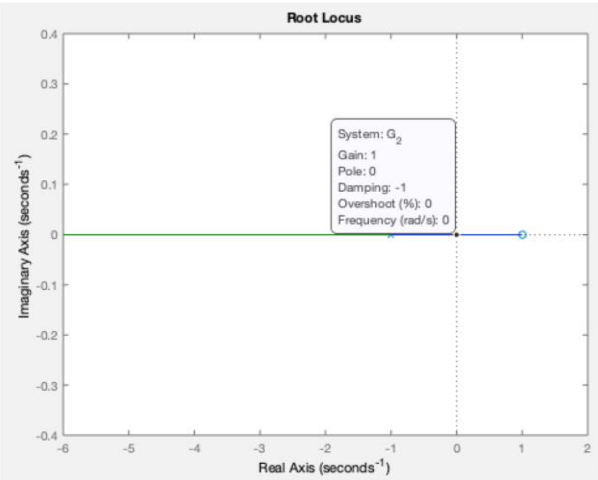

Figure 9. Root-locus of $KG_2(s)$ when $K > 0$.

From the two root-loci, we can see that the only situation that there are no closed-loop poles in the RHP is when $-2 < K < 1$, same as the result shown in part (c).

(e) We can use Routh's criterion to verify that the closed-loop system of KG_2 is stable if $-2 < K < 1$. The steps are shown in Figure 10.

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Closed-loop transfer function :

$$\frac{KG_2(s)}{1+KG_2(s)} = \frac{K\frac{s-1}{(s+1)^2}}{1+K\frac{s-1}{(s+1)^2}} = \frac{K(s-1)}{s^2+(2+K)s+(1-K)}$$

Characteristic equation : $s^2+(2+K)s+(1-K)=0$

Using Routh's criterion:

s^2:	1	1-K
s^1:	2+K	0
s^0:	1-K	

For stability, $2+K>0$ and $1-K>0$.

That is, $-2<K<1$.

$\therefore$ $-2<K<1 \Rightarrow$ stable , otherwise $\Rightarrow$ unstable

Figure 10. The steps of using Routh's criterion to verify the condition for stability of $KG_2(s)$.

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HW 9: Bode Plot - Analysis	Control Systems, Fall 2021, NTU-EE
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1.1 Problem 1

Given a system $G(s)=\dfrac{s+10}{s(s^2+3s+50)}$.

a. Draw the Nyquist plot for the the system by noting the important points. Compare the plot you drew to MATLAB plot.

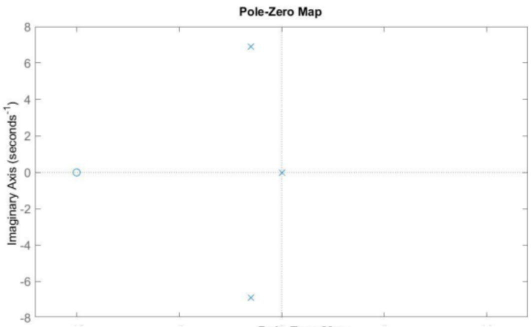
b. Based on the Nyquist plot, define if the system in stable for different values of K.

c. With the help of MATLAB, graph the root locus and see for which K will the system be stable. Match with (b)

d. Use Routh's stability theorem to check the value of K for system stability and compare with (b).

SOLUTION

a. Let us plot the poles and zeros:

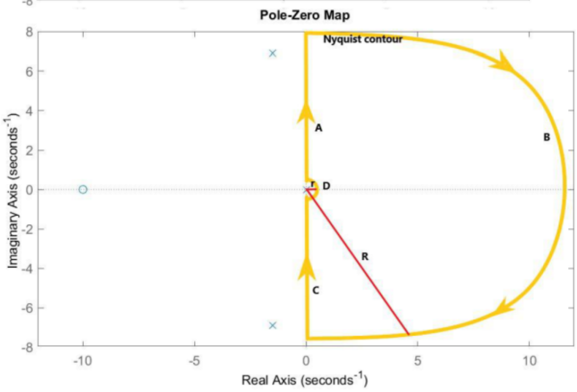


Zero : s = -10

Pole : s = -1.5 ± j 6.9, s = 0

(No RHS pole)

Realize that we have a pole at 0. Thus we need to set our Nyquist contour as such.



Where $r \to 0$ and $R \to \infty$

The four parts are:

$A: s = j\omega, \omega = 0,\ldots,\infty$

$B: s = \mathrm{R}e^{j\theta}, \theta = \pi/2,\ldots,-\pi/2$

$C: s = -j\omega, \omega = -\infty,\ldots,0$

$D: s = re^{j\theta}, \theta = -\pi/2,\ldots,\pi/2$

Then we rewrite G(jw) for different parts:

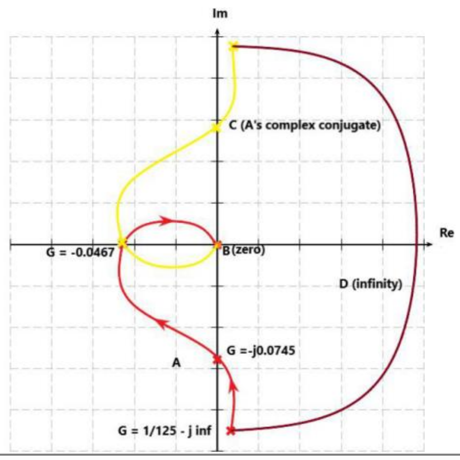
Part A : $G(j\omega)=\frac{j\omega+10}{j\omega(-\omega^2+3j\omega+50)}$ $=\frac{j\omega+10}{-j\omega^3-3\omega^2+50j\omega}\cdot\frac{-3\omega^2-j(50\omega-\omega^3)}{-3\omega^2-j(50\omega-\omega^3)}$ $=\frac{20-\omega^2+j(7\omega^2-500)/\omega}{\omega^4-91\omega^2+2500}$ $=\frac{20-\omega^2}{\omega^4-91\omega^2+2500}+j\frac{7\omega^2-500}{\omega^5-91\omega^3+2500\omega}$	(1)
Part B : $G(\lim_{R\to\infty}\mathrm{R}e^{j\theta})=\lim_{R\to\infty}\frac{\mathrm{R}e^{j\theta}+10}{\mathrm{R}e^{j\theta}(\mathrm{R}e^{2j\theta}+3s+50)}$ $=\lim_{R\to\infty}\frac{e^{j\theta}/R+10/R^2}{e^{j\theta}(e^{2j\theta}+3s/R+50/R)}=0$	(2)
Part C : *Complex conjugate of A* $G(-j\omega)=\frac{20-\omega^2}{\omega^4-91\omega^2+2500}-j\frac{7\omega^2-500}{\omega^5-91\omega^3+2500\omega}$	(3)
Part D : $G(\lim_{r\to0}re^{j\theta})=\lim_{r\to0}\frac{re^{j\theta}+10}{re^{j\theta}(re^{2j\theta}+3s+50)}=\infty$	(4)

Check the important points in A: $\omega = 0$, $\omega = \infty$, $\text{Re}[G(j\omega)] = 0$, $\text{Im}[G(j\omega)] = 0$

(part C is just A's mirror since it is the complex conjugate)

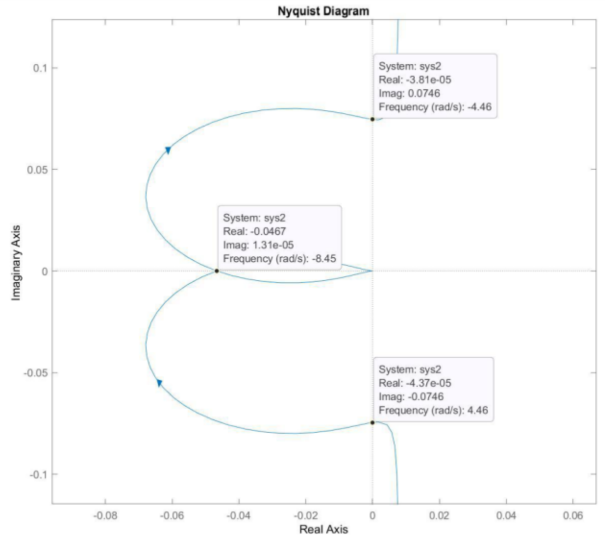
$G(0) = \frac{20}{2500} + j\frac{-500}{0} = \frac{1}{125} - \infty j$ $\rightarrow \text{Re}[G(0)] = \frac{1}{125}\ \&\ \text{Im}[G(0)] = -\infty$ $G(\infty) = \frac{(20-\omega^2)/\omega^4}{(\omega^4 - 91\omega^2 + 2500)/\omega^4} + j\frac{(7\omega^2 - 500)/\omega^5}{(\omega^5 - 91\omega^3 + 2500\omega)/\omega^5} = \frac{0}{1} + j\frac{0}{1}$ $\rightarrow \text{Re}[G(\infty)] = 0\ \&\ \text{Im}[G(\infty)] = 0$ $\text{Re}[G(j\omega)] = 0 \rightarrow \omega = 2\sqrt{5}$ $\rightarrow \text{Im}[G(j2\sqrt{5})] = \frac{7\cdot 20 - 500}{20^{5/2} - 91\cdot 20^{3/2} + 2500\cdot 2\sqrt{5}} = -0.0745$ $\text{Im}[G(j\omega)] = 0 \rightarrow \omega = \sqrt{500/7}$ $\rightarrow \text{Re}[G(j\sqrt{500/7})] = \frac{(20 - 500/7)}{((500/7)^2 - 91\cdot 500/7 + 2500)} = -0.0467$	(5)

Thus we can draw and compare to MATLAB's generated graph.



(a) hand-drawn nyquist plot based on the calculation

3



(b) MATLAB plot

(note: MATLAB does not handle infinity values nicely, thus the cropped plot)

Thus, we see that the hand-drawn graph depicts the real Nyquist plot roughly.

b. We analyze the plot and calculate for 3 different regions. Remember that we have no RHS pole thus P = 0.

a) For $-\infty < -\dfrac{1}{K} < -0.0467 \Leftrightarrow 0 < K < 21.4$:

N = 0, P = 0 so Z = 0 [There is no RHS root. **Stable**]

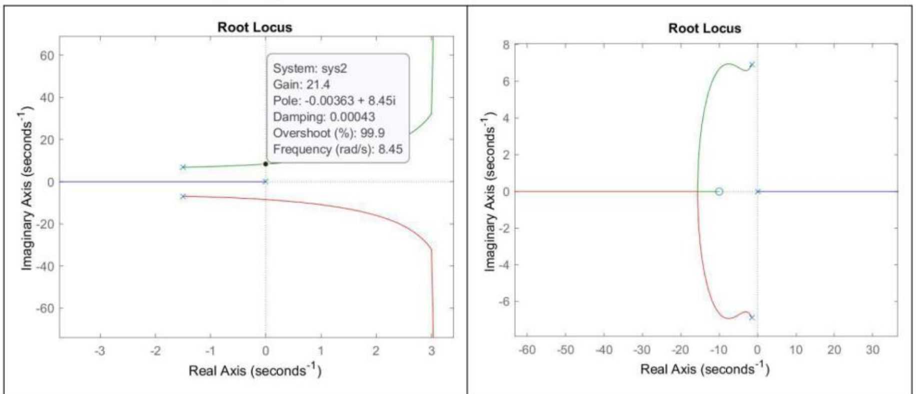
b) For $-0.0467 < -\dfrac{1}{K} < 0 \Leftrightarrow 21.4 < K < \infty$:

N = 2, P = 0 so Z = 1 [There are 2 RHS roots. **Unstable**]

c) For $0 < -\dfrac{1}{K} < \infty \Leftrightarrow -\infty < K < 0$:

N = 1, P = 0 so Z = 1 [There is 1 RHS root. **Unstable**]

c. With MATLAB, we get:



(a) for K>0	(b) for K<0
[see that for K>21.4 the root goes to RHS]	[pole at 0 makes system unstable for any value]

The two root locus graph correspond to the right K value as (b)

d. Rewrite to acquire the characteristics equation

$s(s^2+3s+50)+K(s+10)=0$ $\rightarrow s^3+3s^2+(50+K)s+10K=0$	(6)

Routh's matrix:

s^3	1	50+K
s^2	3	10K
s	$\dfrac{3(50+K)-10K}{3}$	0
1	10K	0

To reach stability:

$$\frac{3(50+K)-10K}{3}>0$$

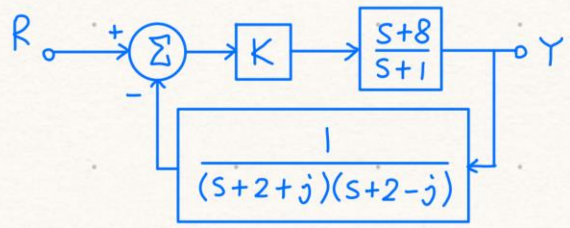
$3(50+K)>10K$ and $10K>0 \Leftrightarrow K>0$. Thus if we combine both $0<K<21.43$.

$K<150/7=21.43$

This solution also match (b).

Control System Homework 09 電機三 B08901111 簡宏哲

Question : Consider the system below ,



(a) Please use MATLAB to obtain the Bode plots (magtitude and phase) for K = 1 , and use "margin" to obtain the gain margin (GM) and the phase margin (PM).

(b) Please use "rlocus" to obtain the Root Locus diagram and find the stable range of positive K.

(c) Please obtain the Nyquist Plot of the system, and use it to find the stable range of positive K.

(d) Please use Routh's criterion to find the stable range of positive K.

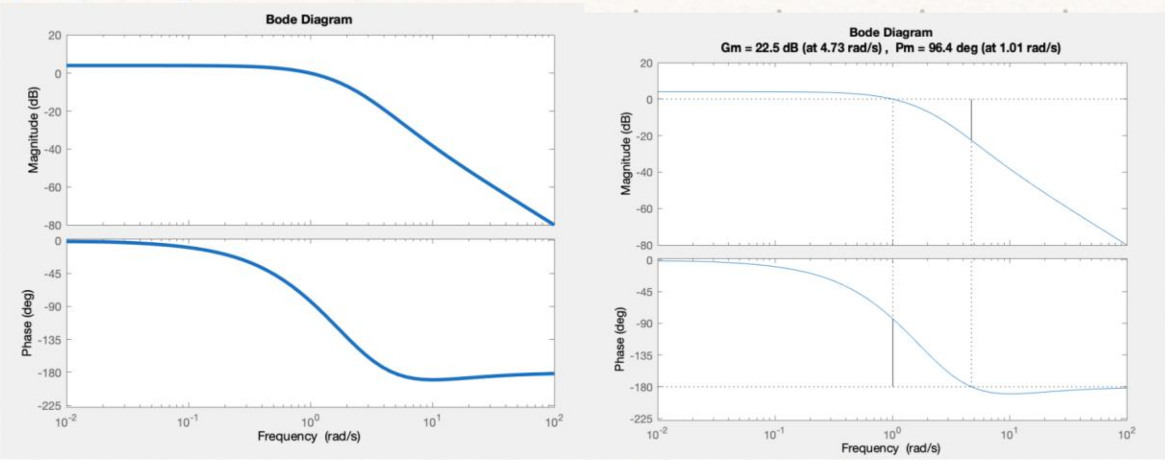
Solution :

(a) Matlab Code:

```
clear all; close all
s = tf( 's' )
K = 2;
sysG = (s+8)/((s+1));
sysH = (1)/((s+2)^2+1);
figure(2)
bode( sysG*sysH );
grid
%axis('equal')
set(findall(gcf,'type','line'),'linewidth',3);
grid

figure(4)
margin(sysG*sysH );
```

Bode Plots :

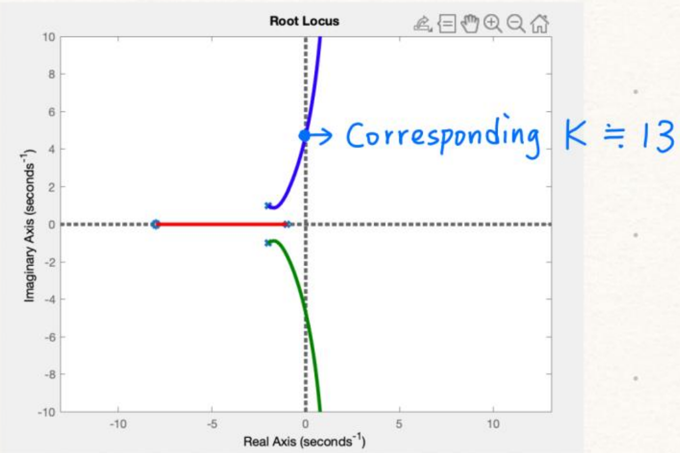


We obtain that GM = 22.5 dB and PM = 96.4°

(b) Matlab Code:

```
figure(1)
rlocus( sysG*sysH );
axis( [ -5 5, -10, 10 ])
axis('equal')
set(findall(gcf,'type','line'),'linewidth',3);
```

Root Locus:



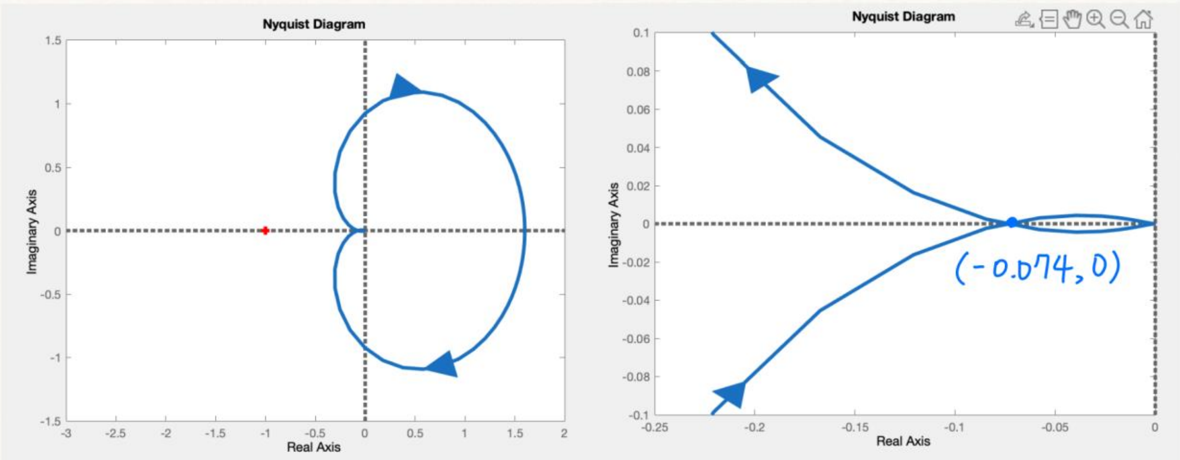
∴ We obtain that the stable range of positive K is $K < 13$

(c) Matlab Code:

```
figure(3)
nyquist( sysG*sysH );
axis('equal')
set(findall(gcf,'type','line'),'linewidth',3);
axis( [ -3 ,2, -1.5, 1.5 ])

figure(5)
nyquist( sysG*sysH );
axis('equal')
set(findall(gcf,'type','line'),'linewidth',3);
axis( [ -0.25 ,0, -0.1, 0.1 ])
```

Nyquist Plot:



∴ We obtain that the stable range of positive K is $-\frac{1}{K} < -0.074$

$\Rightarrow K < 13.5$

(d) Closed-loop transfer function:

$$\frac{1+K\cdot\frac{s+8}{s+1}}{1+K\cdot\frac{s+8}{s+1}\cdot\frac{1}{(s+2+j)(s+2-j)}} = \frac{N(s,K)}{s^3+5s^2+(9+K)s+(5+8K)}$$

⇒ Characteristic equation: $s^3+5s^2+(9+K)s+(5+8K)=0$

Using the Routh's criterion

1	$9+K$
5	$5+8K$
$\frac{3K-40}{-5}$	0
$5+8K$	

∴ The stable range of positive K is $K<\frac{40}{3} \fallingdotseq 13.3$

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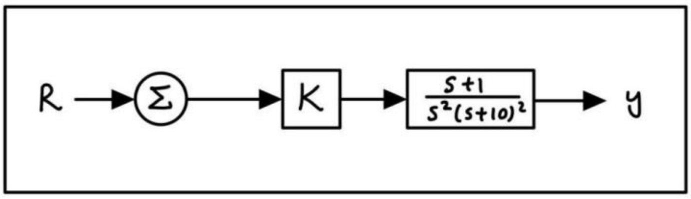
HW9

改編自第 2 題：

Consider the system given below



(a) Use Routh's criterion, determine the ranges of K for this open-loop stability of this system.

(b) Use MATLAB to obtain the bode plot of the system, choose K inside of your answer in (a). Check if the outcome is stable or not.

(c) Use MATLAB to find the root locus and determine the values of K at the stability boundaries. Check if the outcome is match with (a).

(d) Determine the K where PM = 30 degree. Calculate it by hand.

(e) Confirm your answer in (d) using MATLAB.

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Solution:

(a)

$1 + G(s) = 0$

$\Rightarrow \ 1 + \dfrac{K(s+1)}{s^2(s+10)^2} = 0$

$\Rightarrow \ s^2(s+10)^2 + k(s+1) = 0$

$\Rightarrow \ s^4 + 20s^3 + 100s^2 + ks + k = 0$

By Routh Criterian

s^4	1	100	k
s^3	20	K	0
s^2	$\frac{2000-k}{20}$	k	
s^1	①	0	
s^0	k		

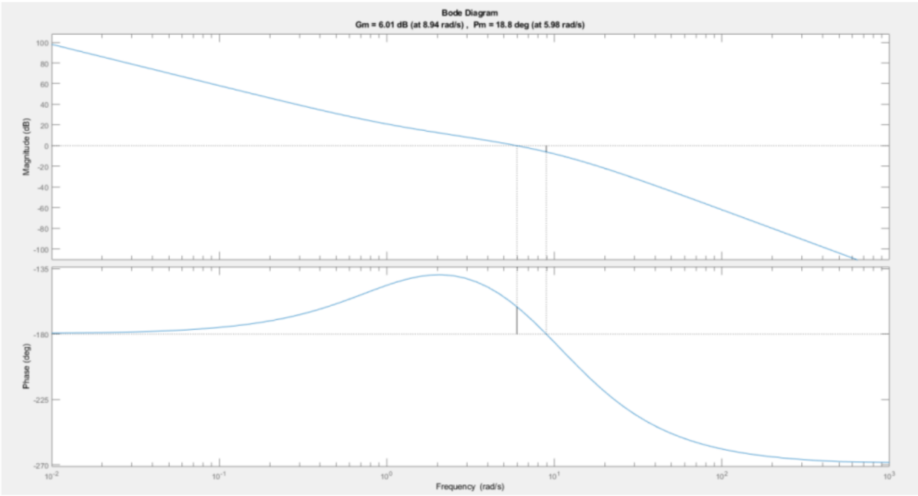
$$① = -\dfrac{20k - \left(\frac{2000-k}{20}\right)k}{\left(\frac{2000-k}{20}\right)} = \dfrac{1600k - k^2}{2000-k}$$

- first collumn >0 . no sign change

$$\begin{cases} \dfrac{2000-k}{20} > 0 \ \Rightarrow \ K > 2000 \\ ① > 0 \ \Rightarrow \ \dfrac{1600k-k^2}{2000-k} > 0 \ \Rightarrow \ 1600 > k \\ k > 0 \end{cases}$$

$\Rightarrow \ 0 < k < 1600$

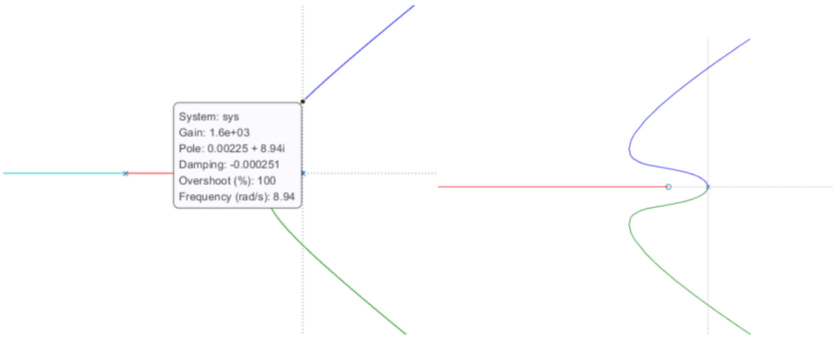
(b) Choose K = 800



PM = 18.8 degree

It is stable.

(c)



We can see that the boundary of the root locus is

0 < K < 1600

The result is same as the result we get from Routh criterian.

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(d)

$$PM = 30^\circ = 180^\circ + \angle G(jw)\big|_{\text{zero dB}}$$

$$\Rightarrow \angle G(jw)\big|_{\text{zero dB}} = -150^\circ$$

$$\Rightarrow -150^\circ = \tan^{-1}\left(\frac{1}{w}\right) - 180^\circ - 2\tan^{-1}\left(\frac{10}{w}\right)$$

$$= \tan^{-1}\left(\frac{1}{w}\right) - 180^\circ - \tan^{-1}\left(\frac{2\left(\frac{10}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)$$

$$\Rightarrow 30^\circ = \tan^{-1}\left(\frac{1}{w}\right) - \tan^{-1}\left(\frac{2\left(\frac{10}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)$$

$$\Rightarrow 30^\circ = \tan^{-1}\left(\frac{\frac{1}{w} - \left(\frac{2\left(\frac{10}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)}{1+\left(\frac{1}{w}\right)\left(\frac{2\left(\frac{10}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)}\right)$$

$$\Rightarrow \tan 30^\circ = \left(\frac{\frac{1}{w} - \left(\frac{2\left(\frac{}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)}{1+\left(\frac{1}{w}\right)\left(\frac{2\left(\frac{10}{w}\right)}{1-\left(\frac{10}{w}\right)^2}\right)}\right)$$

$$\Rightarrow 0.577 = \frac{-19w^2-100}{w^3-100w+20w}$$

$$\Rightarrow -0.577w^3 - 19w^2 + 46.18w - 100 = 0$$

$\Rightarrow$ By calculator : $w = -35.31$

$$|G(j\cdot w)| = \frac{\sqrt{1+w^2}}{\sqrt{w^2}\sqrt{(20w)^2+(100-w^2)^2}}\Bigg|_{w=-35.31}$$

$$\approx \frac{35.3}{35.3 \times 705.387} \approx 0.001418$$

$$k = \frac{1}{|G(jw)|} \approx \frac{1}{0.001418} \approx 705 \quad \#$$

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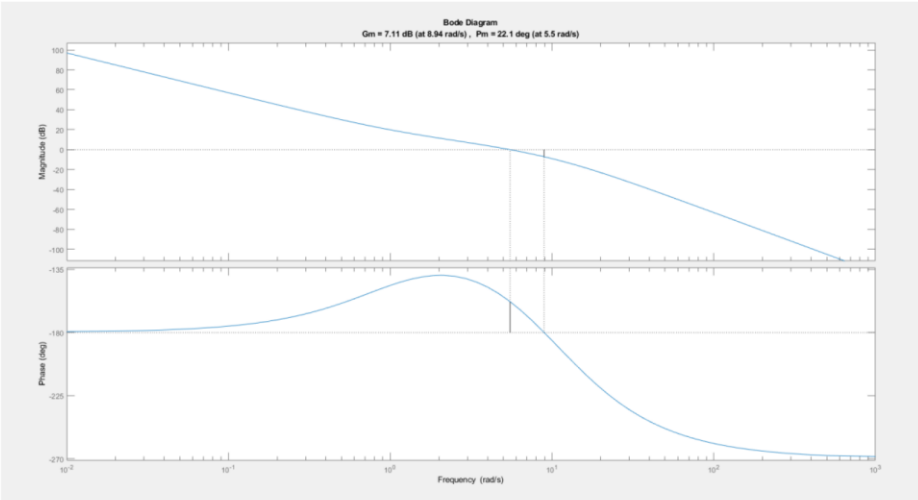
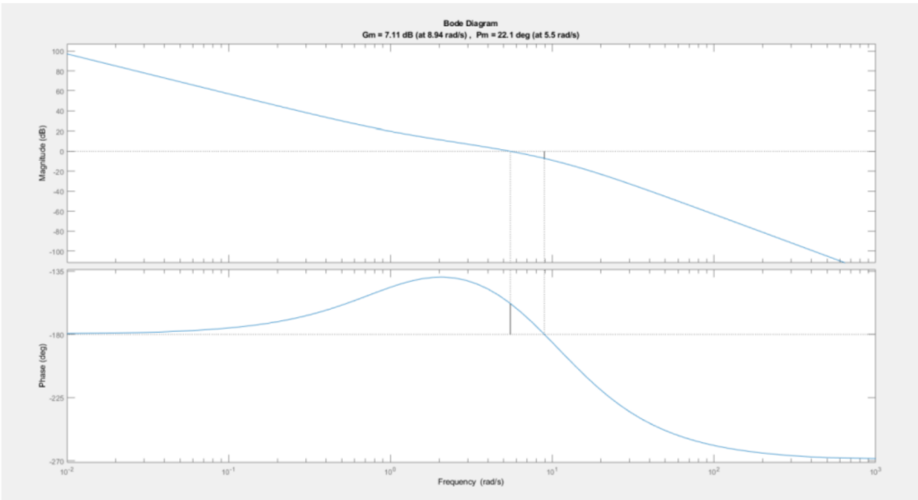
(e)



The PM = 22.1, there is 7.9 degree of error.



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MATLAB:

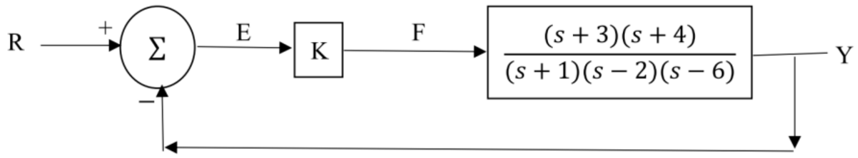
hw9.m

```
s = tf('s');
sys = 705 * (s + 1)/((s ^ 2)*(s + 10)^ 2);
bode(sys);
margin(sys);
```

HW 9: Nyquist plot	Control Systems, Fall 2021, NTU-EE
Name: 楊昇曄 B08502059	Date: 12/23, 2021

改編 Problem 3.

給定如圖所示的系統:



請求出:

(a)找出 K = 1 , s = jω 時 , ω = 0、ω = ∞、ω分別與虛軸和實軸的交點 ，畫出 Nyquist plot 並用 MATLAB 驗證。

(b)用 Nyquist plot 決定 K 的值在那些範圍裡會使系統 stable，那些範圍會使系統 unstable

(c)在不同的 K 值下各有幾個 closed loop roots 在 RHP

(d)用 Root locus 來驗證 K 的範圍

(e)從(b)推出使系統 stable 的範圍內選擇一個 K 值來找出 GM 和 PM，觀察並說明此系統具有何種性質

HW 9: Nyquist plot	Control Systems, Fall 2021, NTU-EE
Name: 楊昇曄 B08502059	Date: 12/23, 2021

Solution.

(a)

$G = \frac{(s+3)(s+4)}{(s+1)(s-2)(s-6)}$ → 2 個 pole 在 RHP → P = 2

$KG = \frac{(j\omega+3)(j\omega+4)}{(j\omega+1)(j\omega-2)(j\omega-6)} = \frac{(12+7\omega^2)(12-\omega^2)+7\omega^2(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2} + j\frac{7\omega(12+7\omega^2)-\omega(12-\omega^2)(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2}$

$\omega = 0$ 時，KG = 1+0j，$\omega = \infty$時，KG = 0+0j

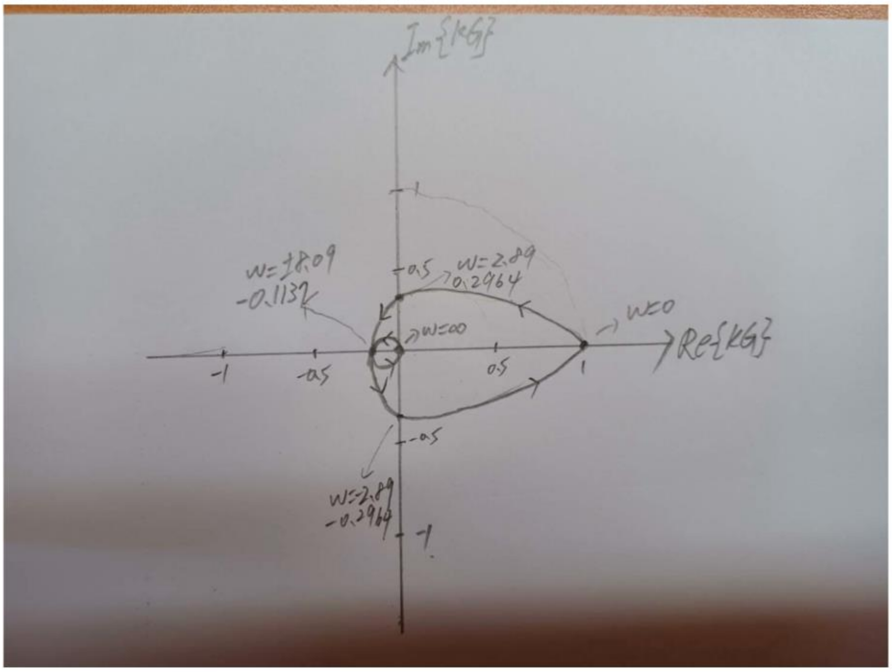
與實軸相交 → $\frac{7\omega(12+7\omega^2)-\omega(12-\omega^2)(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2} = 0$ → $\omega = \pm 8.09$

→$\frac{(12+7\omega^2)(12-\omega^2)+7\omega^2(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2}\Big|_{\omega=\pm 8.09}$ = -0.1137 → KG = -0.1137±0j

與虛軸相交 → $\frac{(12+7\omega^2)(12-\omega^2)+7\omega^2(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2} = 0$ → $\omega = \pm 2.89$

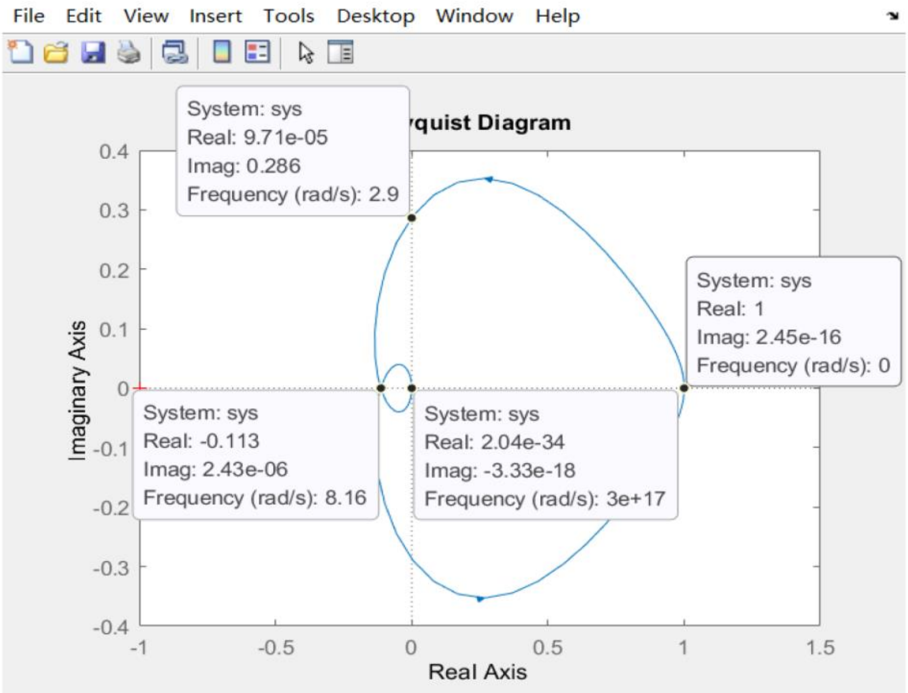
→$\frac{7\omega(12+7\omega^2)-\omega(12-\omega^2)(4-\omega^2)}{(12+7\omega^2)^2+\omega^2(4-\omega^2)^2}\Big|_{\omega=\pm 2.89}$ = 0.2964 → KG = 0±0.2964j

用這些交點畫出 Nyquist plot 大概的樣子:



HW 9: Nyquist plot	Control Systems, Fall 2021, NTU-EE
Name: 楊昇曄 B08502059	Date: 12/23, 2021

MATLAB 模擬的結果:



(b)(c)

根據 G , P = 2

K = 1 時，看 encirclement of s = -1 → K 為變數時，看 encirclement of s = $\frac{-1}{K}$

根據(a)所畫出來的 Nyquist plot:

1.$-\infty < \frac{-1}{K} < -0.113$ →0 < K < 8.795 , N = 0 → Z = N + P = 2 → 2 CL roots in RHP → unstable

2.$-0.113 < \frac{-1}{K} < 0$ → K > 8.795 , N = -2 → Z = N + P = 0 → 0 CL roots in RHP → stable

3.$0 < \frac{-1}{K} < 1$ → K < -1 , N = -1 → Z = N + P = 1 → 1 CL roots in RHP → unstable

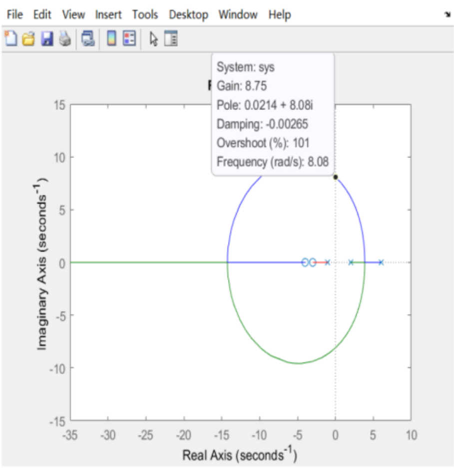
4.$1 < \frac{-1}{K} < \infty$ → -1 < K < 0 , N = 0 → Z = N + P = 2 → 2 CL roots in RHP → unstable

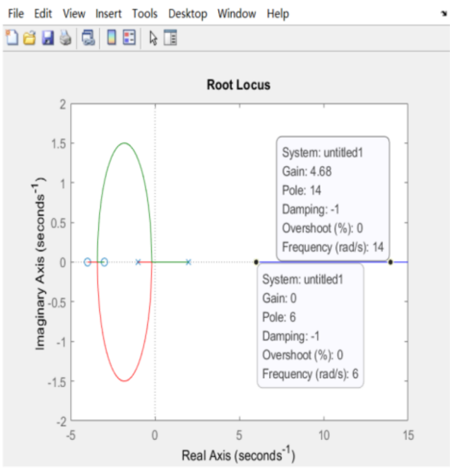
Stable: K > 8.795 , Unstable: otherwise

(d)

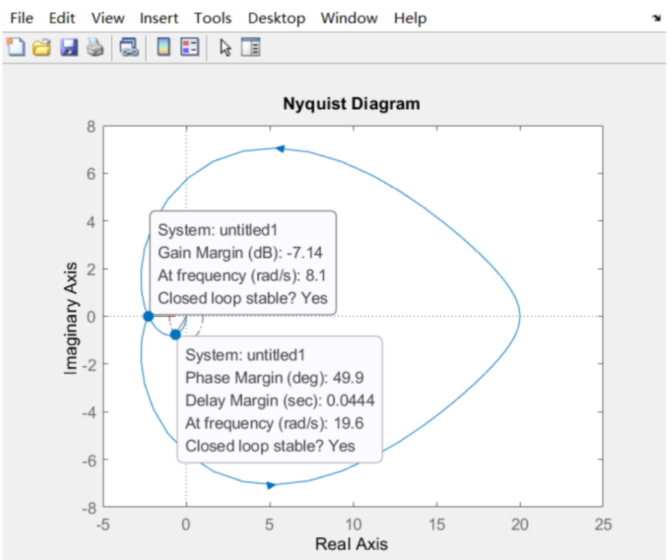
從圖可以看出當 K<8.75 時系統為 stable → Nyquist plot 的 K 的範圍正確

(1)K > 0 (2)K < 0





(e)根據(b)的結果，選擇 K = 20，畫出 Nyquist plot:



從圖得到 GM = -7.14(dB) = 0.44 < 1 (unstable) , PM = 49.9(deg) > 0 (stable)

→conflict →重新看 Nyquist encirclements →在 s = -1 處逆時針轉了兩次 → N = -2

→ Z = N+P = 0 →確定系統在 K = 20 是 stable →根據觀察得出此系統為

Conditionally Stable System

HW 9: Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 12/23, 2021

1.1 Question

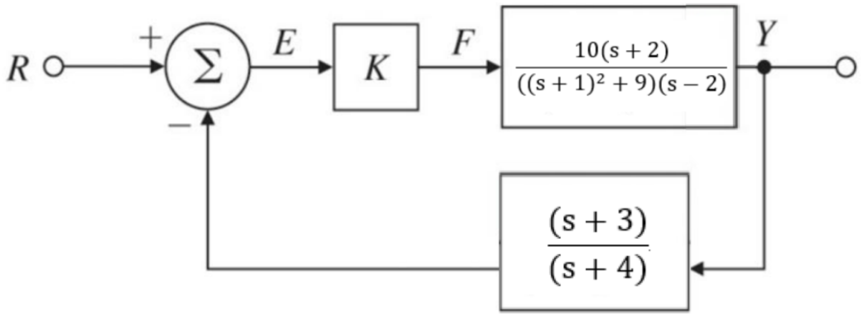
For the system shown in below, determine the Nyquist plot and apply the Nyquist criterion.

(a) To determine the range of values of K (positive and negative) for which the system will be stable

(b) To determine the number of roots in the RHP for those values of K for which the system is unstable. Check your answer using a root-locus plot.

(c) Obtain Bode plots for K=1 and use the plots to verify the result for K=1 that you get in (a) and (b).

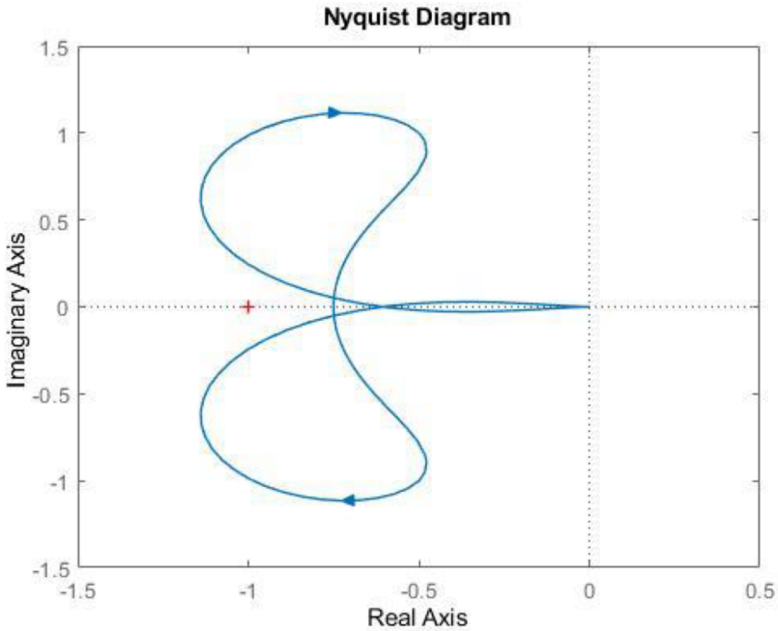
(d) Verify the stable range of K by using margin to determine PM for selected values of K.



1.2 Answer(a)(b)

$$KG(s)H(s) = K\frac{10(s+2)}{((s+1)^2+9)(s-2)}\frac{(s+3)}{(s+4)}$$

Nyquist plot:



Contour crosses the real axis at -0.75, -0.6 and 0

1. $-\infty < -\frac{1}{K} < -0.75 \rightarrow 0 < K < \frac{4}{3}$

   沒圈到$\rightarrow N = 0, P = 1 \rightarrow Z = 1$

   One closed-loop roots in RHP.

2. $-0.75 < -\frac{1}{K} < -0.6 \rightarrow \frac{4}{3} < K < \frac{5}{3}$

   逆時針圈到一次$\rightarrow N = -1, P = 1 \rightarrow Z = 0$

   The closed-loop system is stable.

3. $-0.6 < -\frac{1}{K} < 0 \rightarrow \frac{5}{3} < K$

   順時針圈到一次$\rightarrow N = 1, P = 1 \rightarrow Z = 2$

   Two closed-loop roots in RHP.

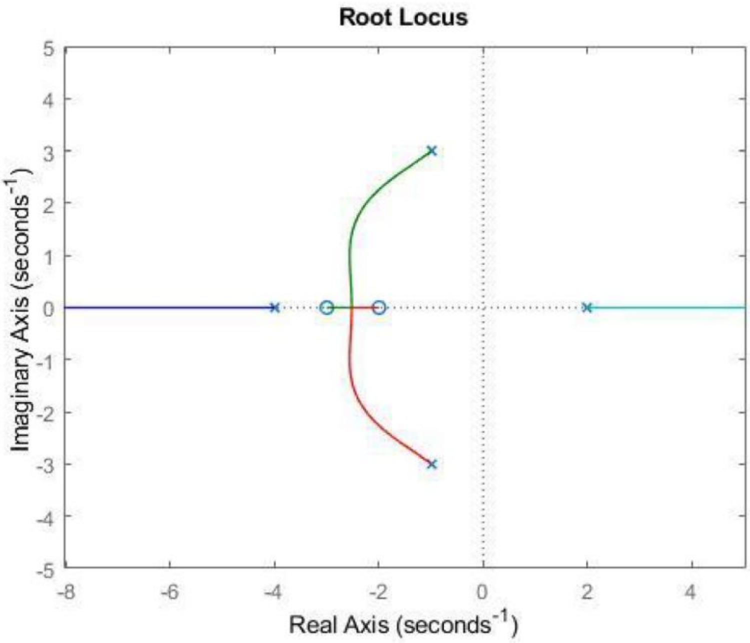
4. $0 < -\frac{1}{K} < \infty \rightarrow K < 0$

   沒圈到$\rightarrow N = 0, P = 1 \rightarrow Z = 1$

   One closed-loop roots in RHP.

These results are confirmed by looking at the root loci below:

Root Locus (K < 0)



HW 9: Bode Plot	Control Systems, Fall 2021, NTU-EE
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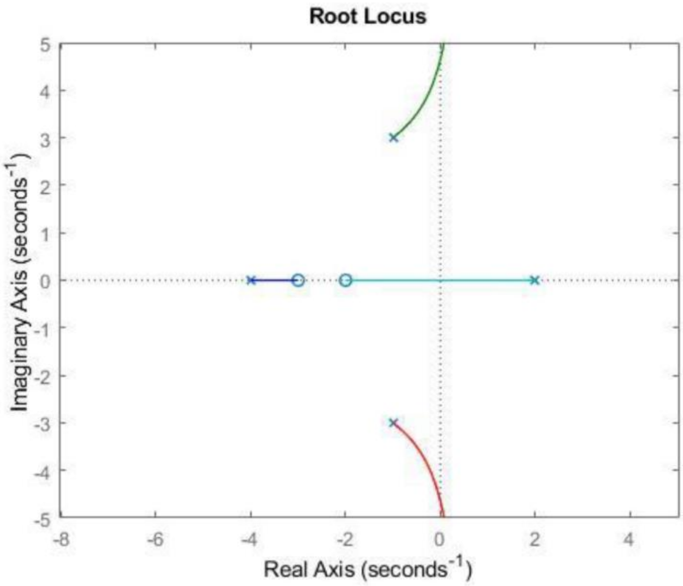
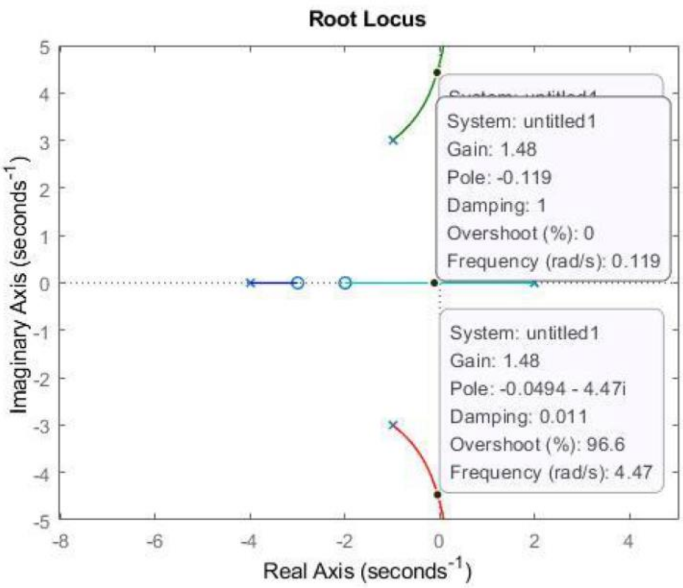
Root Locus (K > 0)



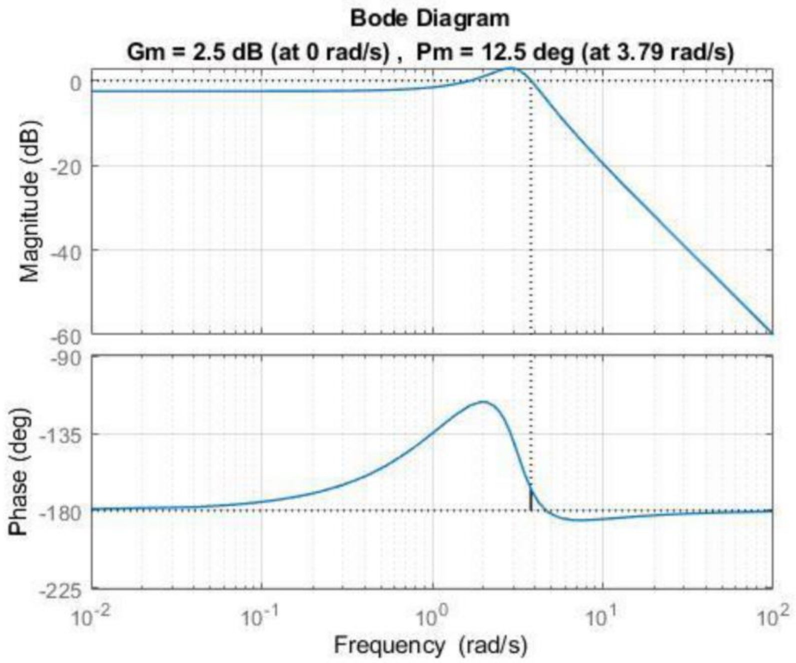
To show that if $\frac{4}{3} < K < \frac{5}{3}$, the closed-loop system is stable. $(\frac{4}{3} < 1.48 < \frac{5}{3})$



When K=1.48, all the poles are in LHP. → The closed-loop system is stable.

1.3 Answer(c)

Bode plots for K=1

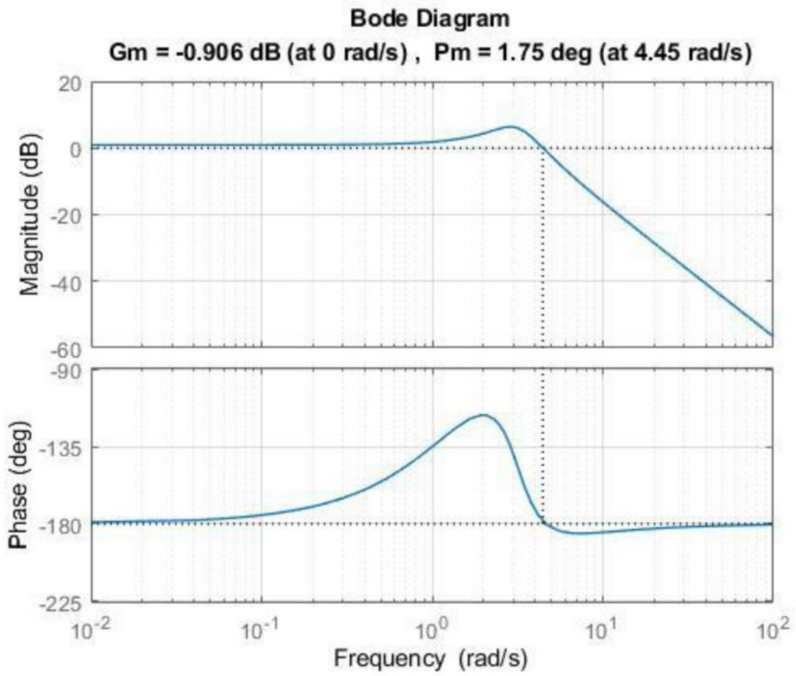


Gm=2.5dB > 0dB = 1 → The closed-loop system is unstable.

1.4 Answer(d)

For example, for K = 1.48, PM = 1.75deg and GM = -0.906dB < 0dB = 1

→ The closed-loop system is stable.



1.5 MATLAB code

```
clear all; close all

s = tf( 's' )

K = 1;

%sysG = (s+1)/((s-1)^2);
%sysH = 1;
sysG = 10*(s+2)/(((s+1)^2+9)*(s+4));
sysH = (s+3)/(s-2);

figure(1)
nyquist( sysG*sysH );
%grid
axis('equal')
set(findall(gcf,'type','line'),'linewidth',1);
axis( [ -1.5 0.5, -1.5, 1.5 ])

figure(2)
%K>0
rlocus( 1*sysG*sysH );
axis( [ -5 2, -5, 5 ])
axis('equal')
set(findall(gcf,'type','line'),'linewidth',1);

figure(3)
%K<0
rlocus( -1*sysG*sysH );
axis( [ -5 2, -5, 5 ])
axis('equal')
set(findall(gcf,'type','line'),'linewidth',1);

figure(4)
margin( sysG*sysH );
grid
```

```
%axis('equal')
set(findall(gcf,'type','line'),'linewidth',1);

figure(5)
margin( 1.48*sysG*sysH );
grid
%axis('equal')
set(findall(gcf,'type','line'),'linewidth',1);
```

HW9

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Problem :

Consider the unity-feedback system with the open-loop transfer function :

$$G(s) = \frac{K}{s(s+1)(s+2)^2}$$

a. Use Matlab to draw the Bode plots for $G(jw)$, assuming K = 1

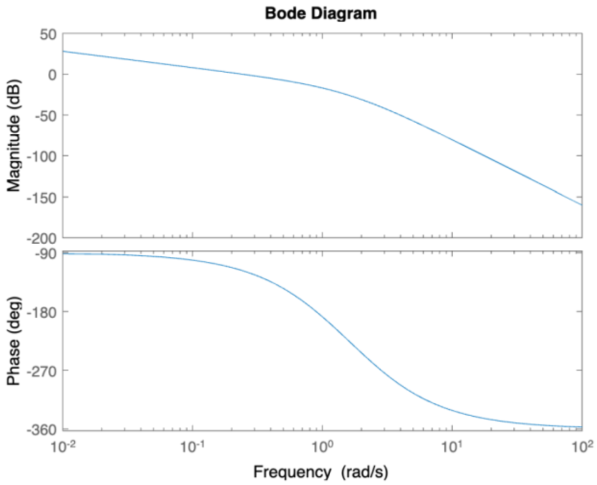
b. What gain K is required for a PM of 45 deg? What is the GM for this value of K ?

c. What is K_v when the gain K is set for PM = 45 deg?

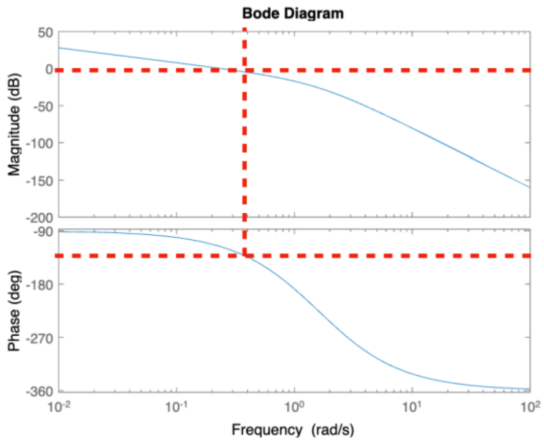
d. Create a root locus with respect to K, and indicate the roots for a PM is 45 deg.

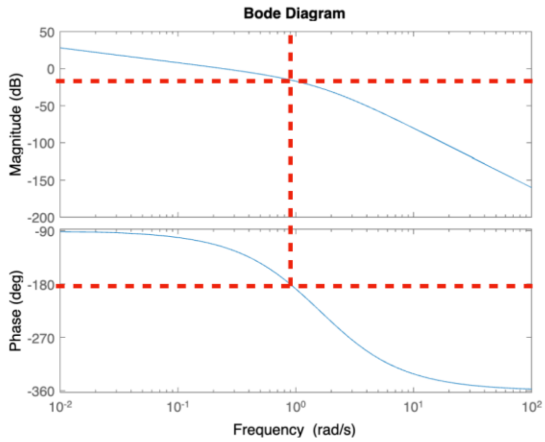
Solution :

a. Use bode() function in Matlab, the plot is as follows:



b. From the Bode plot, when magnitude is 0 dB, PM is 45 deg (left plot). As a result, K = 1. GM = -20 for K = 1 (right plot).





c. For PM = 45 deg, K = 1, and $G(s) = \dfrac{1}{s(s+1)(s+2)^2}$,

$\lim_{s\to 0} s\dfrac{1}{s(s+1)(s+2)^2} = \dfrac{1}{4} = \dfrac{1}{K_v}$, $K_v = 4$. We can also get K_v from the bode plot.

d. The root locus plot is as follows, and root are indicated in right plot: