Control System: Homework 08 for Units 6A, 6B, 6C: Bode Plot

Assigned: Nov 25, 2022

Due: Dec 1, 2022 (11:59pm)

1. (Frequency response and Bode plot)

2. (a) Calculate the magnitude and phase of

$$G(s) = \frac{1}{s+5}$$

by hand for $\omega = 1,\ 2,\ 5,\ 10,\ 20,\ 50$, and 100 rad/sec.

(b) sketch the asymptotes for $G(s)$ according to the Bode plot rules, and compare these with your computed results from part (a).

For (a), you can use calculator to compute the exact numerical results.

In exam, we will provide the problem which can be easily calculated by pen.

Hence, you cannot use any calculators in exam.

Solution:

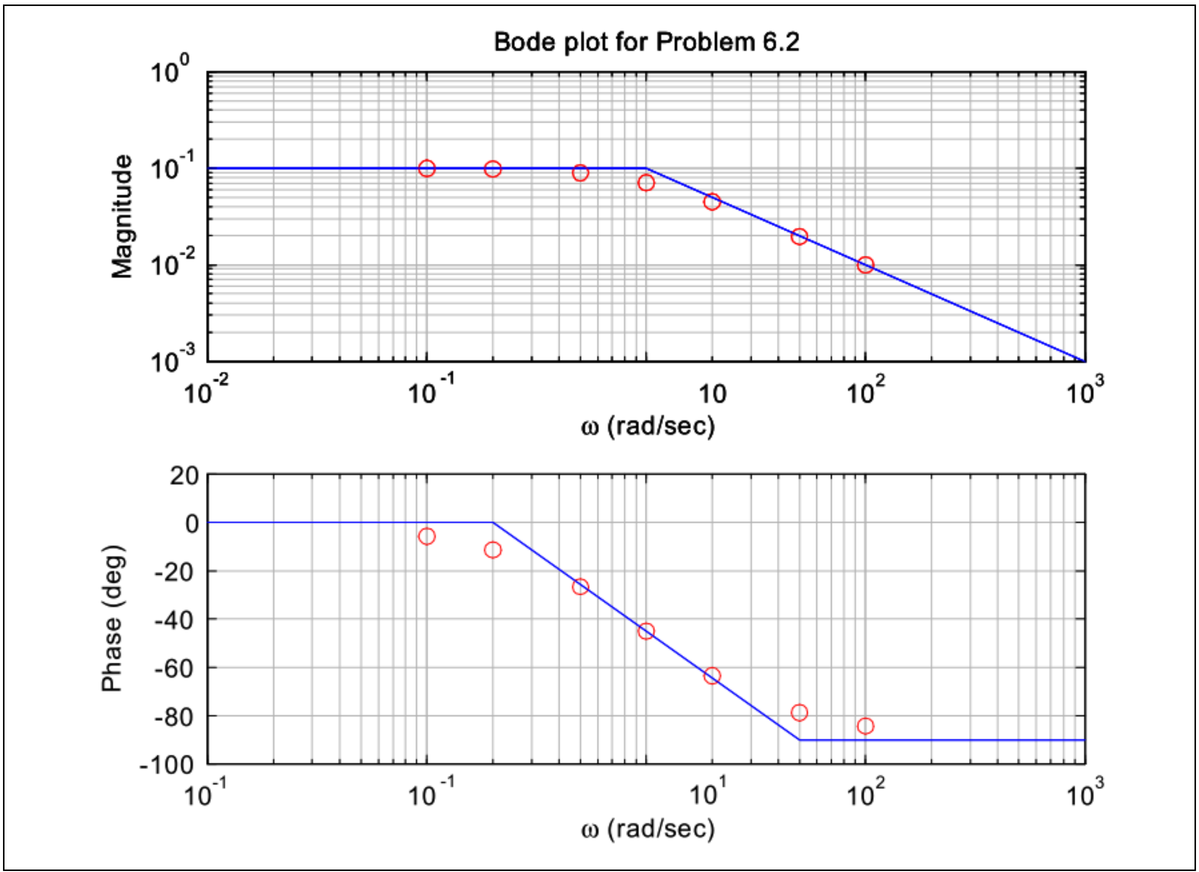
(a)

$$G(s) = \frac{1}{s+5},\ G(j\omega) = \frac{1}{5+j\omega} = \frac{5-\ j\omega}{25+\omega^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{25+\omega^2}},\ \angle G(j\omega) = -\tan^{-1}\frac{\omega}{5}$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
1	0.1961	-11.309
2	0.18569	-21.801
5	0.1414	-45
10	0.08944	-63.43
20	0.0485	-75.96
50	0.0199	-84.3
100	0.00998	-87.137

(b) To plot the asymptotes, you first note that $n = 0$, as defined in Section 6.1.1. That signifies that the leftmost portion of the asymptotes will have zero slope. That portion of the asymptotes will be located at the DC gain of the transfer function, which, in this case it can be seen by inspection to be 0.1. So the asymptote starts with a straight horizontal line at 0.1 and that continues until the breakpoint at $\omega = 5$, at which point the asymptote has a slope of $n = -1$ that continues until forever, at least until the edge of the paper. The values computed above by "hand" (at least we hope you didn' tcheat) are plotted on the graph below and you see they match quite well except very near the breakpoint, a you should have expected. The Bode plot is :



2. (Bode Plot and Frequency Properties)

3. For the open-loop transfer functions of the unity feedback control systems. given below, sketch the Bode magnitude and phase plots. Find their gain margins, gain crossover frequencies, and phase crossover frequencies.

(a) $L(s) = \dfrac{10}{s\,[s+10]}$

(d) $L(s) = \dfrac{50}{s(0.5s+1)^2}$

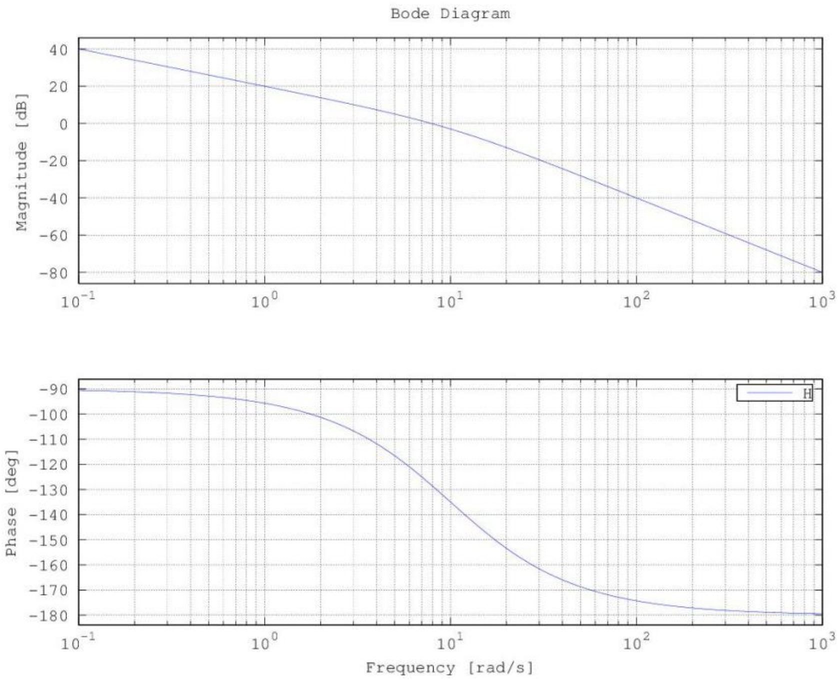
Try to sketch the plots by hand-pen and, then use Matlab code to verify your results.

Solution:

(a) $L(s) = \dfrac{10}{s\,[s+10]}$

In this case, Break frequency is 10.

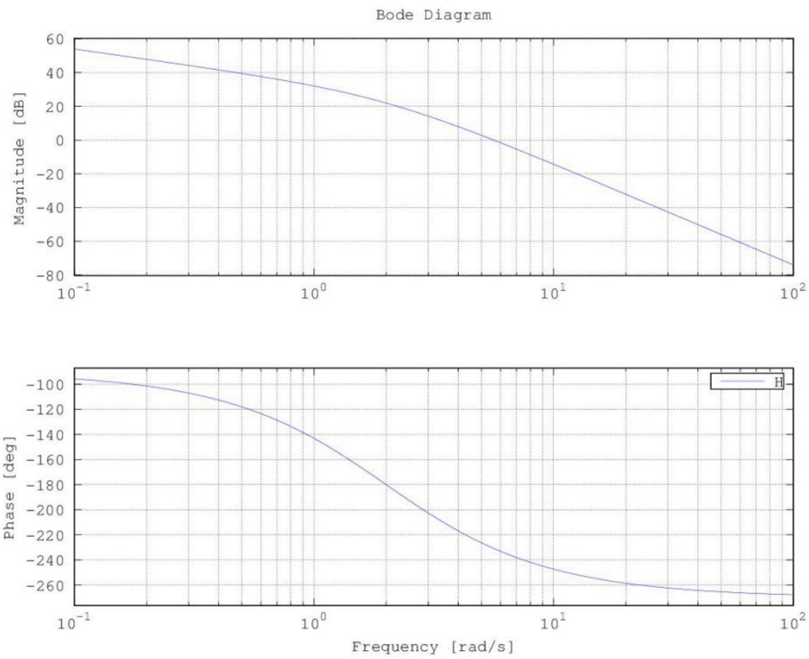
Bode plot for Prob. 6.3 (a)



(d) $L(s) = \dfrac{50}{s(0.5s+1)^2}$

Breaking frequency $= 2$.

Bode plot for Prob 6.3(d)



3. (Bode Plot and Timing Properties)

11. A normalized second-order system with a damping ratio $\zeta = 0.5$ and an additional zero is given by

$$G(s) = \frac{s/a + 1}{s^2 + s + 1}.$$

Use MATLAB to compare the M_p from the step response of the system for $a = 0.01, 0.1$, 1, 10, and 100 with the M_r from the frequency response of each case. Is there a correlation between M_r and M_p?

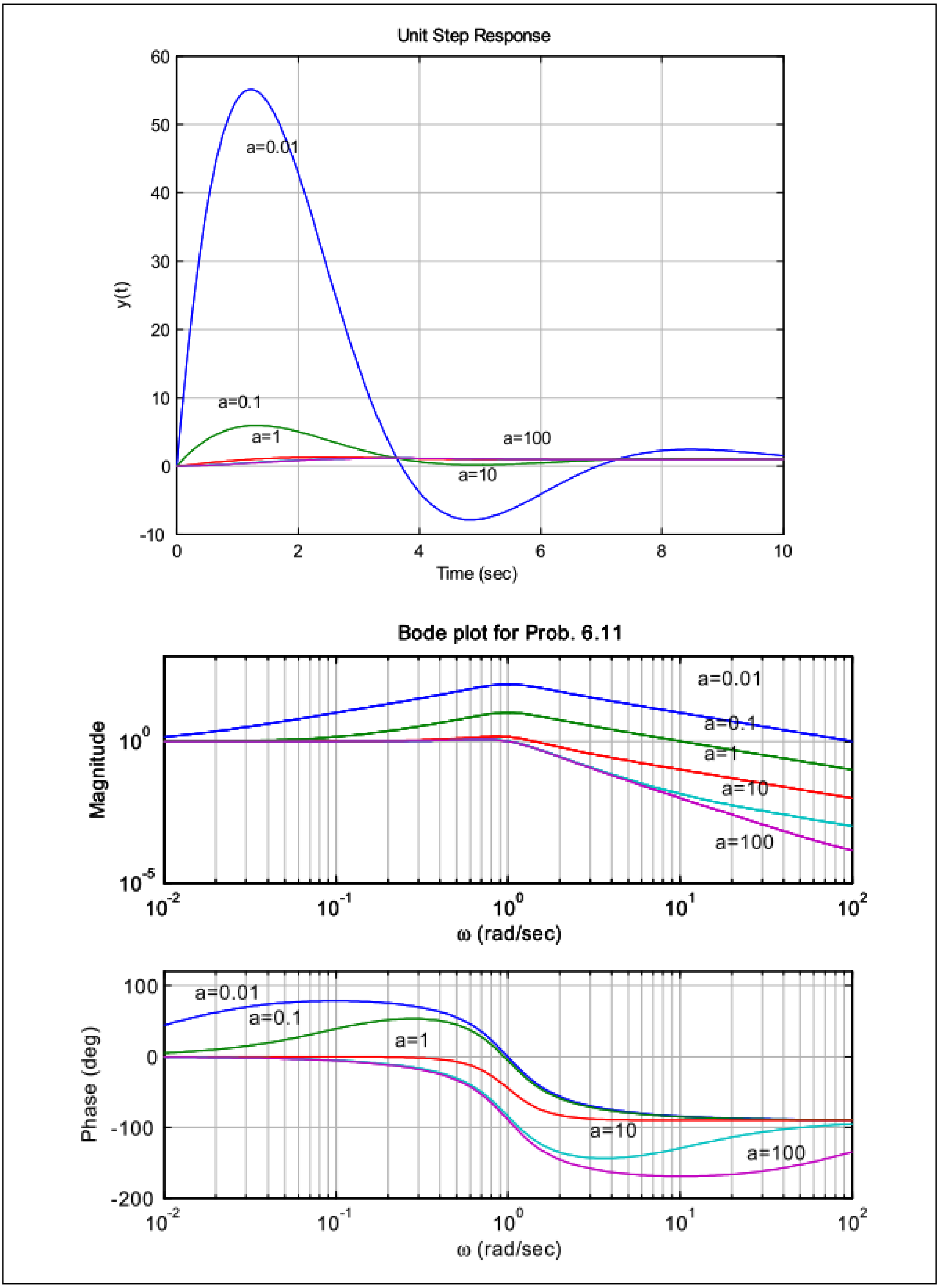
Solution:

α	Resonant peak, M_r	Overshoot, M_p
0.01	98.8	54.1
0.1	9.93	4.94
1	1.46	0.30
10	1.16	0.16
100	1.15	0.16

As α is reduced, the resonant peak in frequency response increases. This leads us to expect extra peak overshoot in transient response. This effect is significant in case of $\alpha = 0.01, 0.1, 1$, while the resonant peak in frequency response is hardly changed in case of $\alpha = 10$. Thus, we do not have considerable change in peak overshoot in transient response for $\alpha \geq 10$.

The response peak in frequency response and the peak overshoot in transient response are correlated.

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.


Unit Step Response
y(t)
Time (sec)
a=0.01
a=0.1
a=1
a=10
a=100
Bode plot for Prob. 6.11
Magnitude
ω (rad/sec)
a=0.01
a=0.1
a=1
a=10
a=100
Phase (deg)
ω (rad/sec)
a=0.01
a=0.1
a=1
a=10
a=100