Control System: Homework 08 for Units 6A, 6B, 6C: Bode Plot

Assigned: Dec 10, 2021

Due: Dec 16, 2021 (11:59pm)

1. (Frequency response and Bode plot)

2. (a) Calculate the magnitude and phase of

$$G(s) = \frac{1}{s+5}$$

by hand for $\omega = 1,\ 2,\ 5,\ 10,\ 20,\ 50$, and 100 rad/sec.

(b) sketch the asymptotes for $G(s)$ according to the Bode plot rules, and compare these with your computed results from part (a).

For (a), you can use calculator to compute the exact numerical results.

In exam, we will provide the problem which can be easily calculated by pen.

Hence, you cannot use any calculators in exam.

Solution:

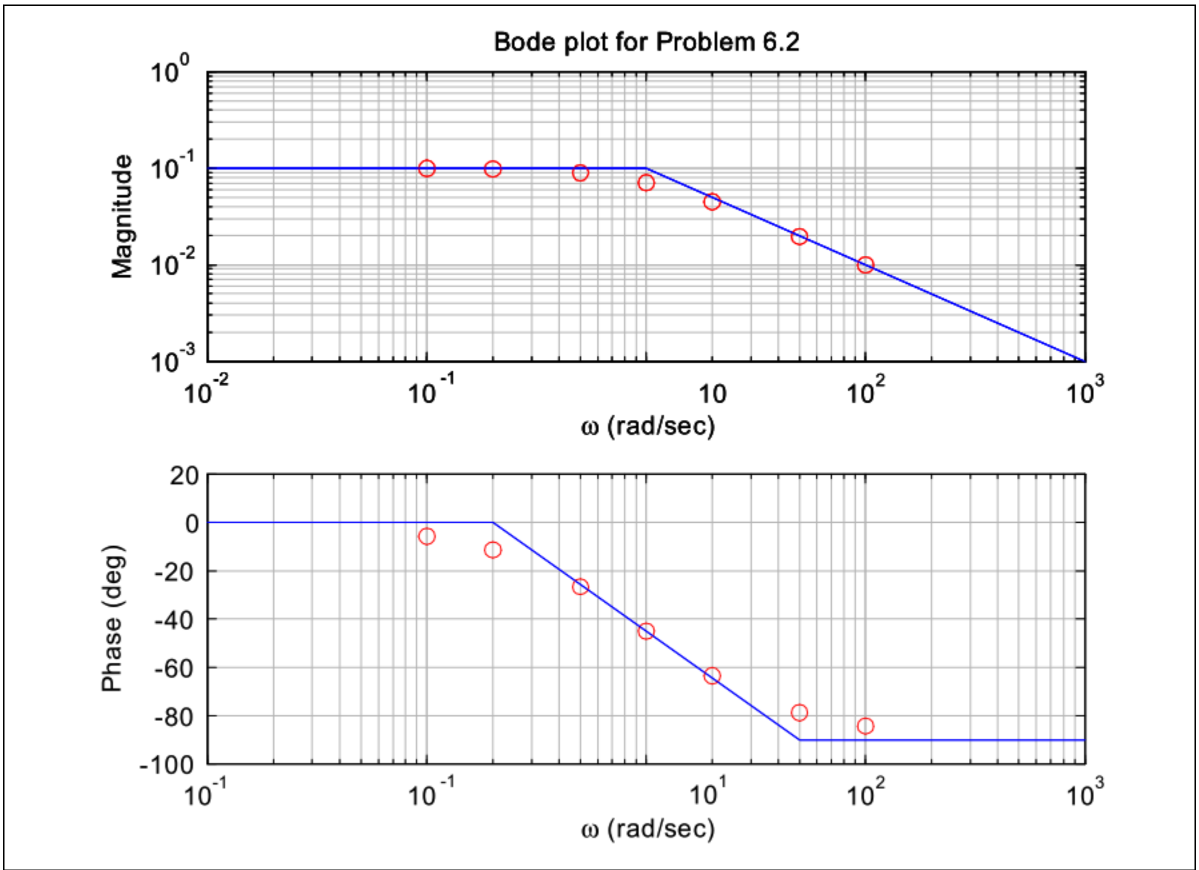
(a)

$$G(s) = \frac{1}{s+5},\ G(j\omega) = \frac{1}{5+j\omega} = \frac{5-\ j\omega}{25+\omega^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{25+\omega^2}},\ \angle G(j\omega) = -\tan^{-1}\frac{\omega}{5}$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
1	0.1961	−11.309
2	0.18569	−21.801
5	0.1414	−45
10	0.08944	−63.43
20	0.0485	−75.96
50	0.0199	−84.3
100	0.00998	−87.137

(b) To plot the asymptotes, you first note that $n = 0$, as defined in Section 6.1.1. That signifies that the leftmost portion of the asymptotes will have zero slope. That portion of the asymptotes will be located at the DC gain of the transfer function, which, in this case it can be seen by inspection to be 0.1. So the asymptote starts with a straight horizontal line at 0.1 and that continues until the breakpoint at $\omega = 5$, at which point the asymptote has a slope of $n = -1$ that continues until forever, at least until the edge of the paper. The values computed above by "hand" (at least we hope you didn' tcheat) are plotted on the graph below and you see they match quite well except very near the breakpoint, a you should have expected. The Bode plot is :



2. (Bode Plot and Frequency Properties)

3. For the open-loop transfer functions of the unity feedback control systems. given below, sketch the Bode magnitude and phase plots. Find their gain margins, gain crossover frequencies, and phase crossover frequencies.

(a) $L(s) = \dfrac{10}{s\,[s+10]}$

(d) $L(s) = \dfrac{50}{s(0.5s+1)^2}$

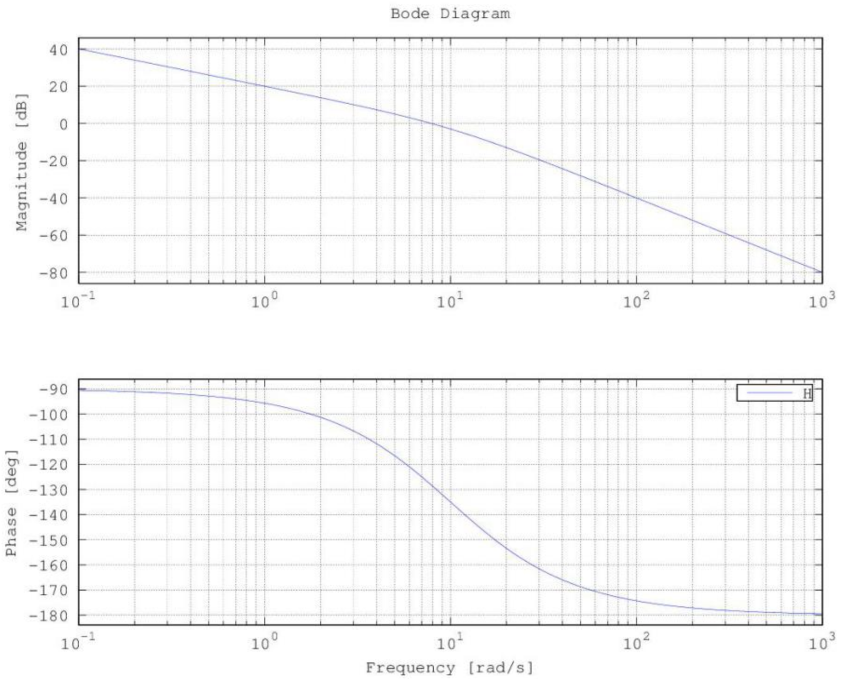
Try to sketch the plots by hand-pen and, then use Matlab code to verify your results.

Solution:

(a) $L(s) = \dfrac{10}{s\,[s+10]}$

In this case, Break frequency is 10.

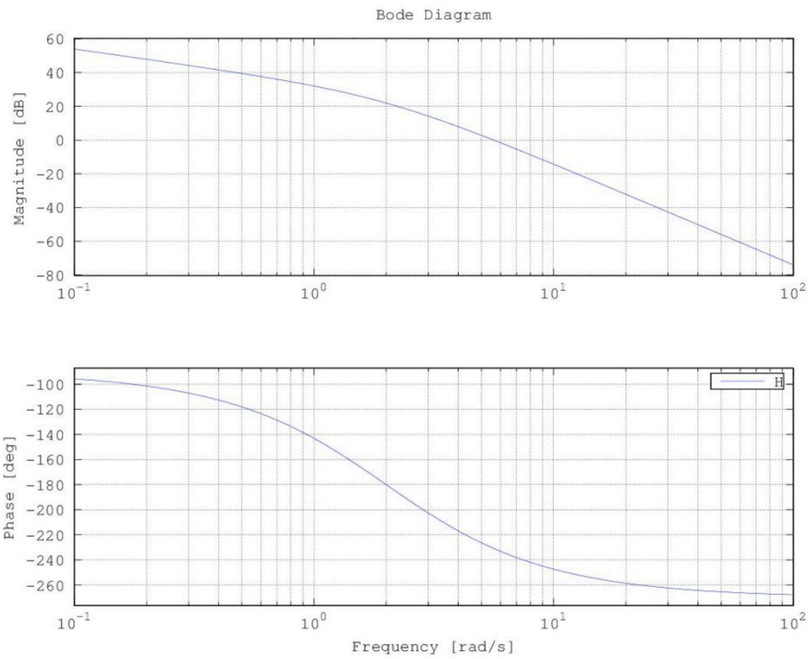
Bode plot for Prob. 6.3 (a)



(d) $L(s) = \dfrac{50}{s(0.5s+1)^2}$

Breaking frequency = 2.

Bode plot for Prob 6.3(d)



3. (Bode Plot and Timing Properties)

11. A normalized second-order system with a damping ratio $\zeta = 0.5$ and an additional zero is given by

$$G(s) = \frac{s/a + 1}{s^2 + s + 1}.$$

Use MATLAB to compare the M_p from the step response of the system for $a = 0.01, 0.1, 1, 10,$ and 100 with the M_r from the frequency response of each case. Is there a correlation between M_r and M_p?

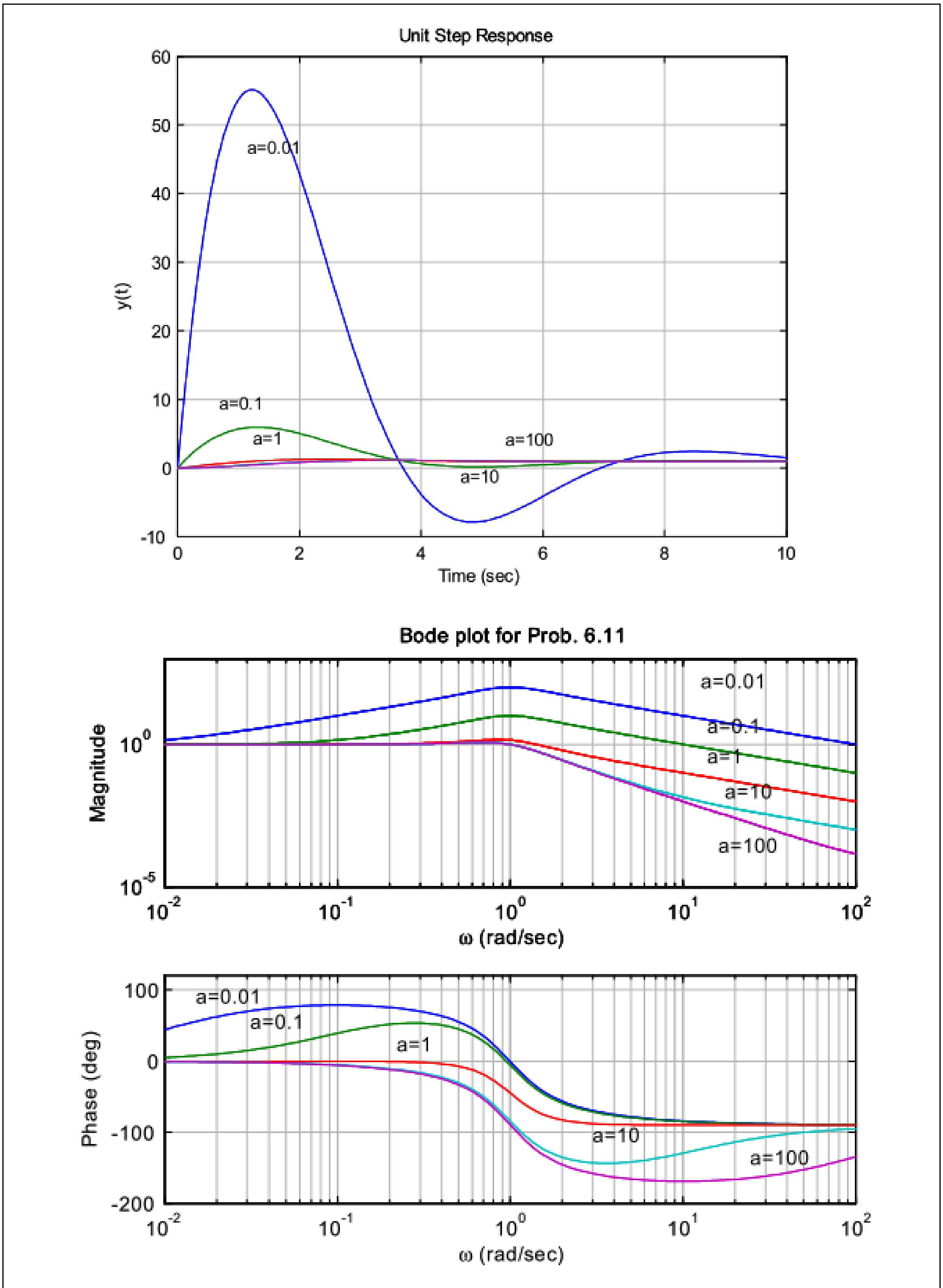
Solution:

α	Resonant peak, M_r	Overshoot, M_p
0.01	98.8	54.1
0.1	9.93	4.94
1	1.46	0.30
10	1.16	0.16
100	1.15	0.16

As α is reduced, the resonant peak in frequency response increases. This leads us to expect extra peak overshoot in transient response. This effect is significant in case of $\alpha = 0.01, 0.1, 1$, while the resonant peak in frequency response is hardly changed in case of $\alpha = 10$. Thus, we do not have considerable change in peak overshoot in transient response for $\alpha \geq 10$.

The response peak in frequency response and the peak overshoot in transient response are correlated.

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.


Unit Step Response
y(t)
Time (sec)
a=0.01
a=0.1
a=1
a=100
a=10
Bode plot for Prob. 6.11
Magnitude
ω (rad/sec)
a=0.01
a=0.1
a=1
a=10
a=100
Phase (deg)
ω (rad/sec)
a=0.01
a=0.1
a=1
a=10
a=100

HW 8: Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 鍾銀香 B08901120	Date: December 2021

1.1 Problem 1

a. Consider a lead compensation $G_{lead}(s)=\frac{s+z}{s+p}$ for $z<p$. Sketch the compensation's magnitude and phase bode plot.

b. Consider a lag compensation $G_{lag}(s)=\frac{s+z}{s+p}$ for $z>p$. Sketch the compensation's magnitude and phase bode plot.

c. Compare the 2 compensation's bode plot.

d. Consider a notch compensation $G_{notch}(s)=\frac{s^2+0.8s+3600}{(s+60)^2}$. Sketch the magnitude and phase bode plot. With the help of the bode plot, explain why notch compensation is capable of filtering specific frequency of oscillation.

HW 8: Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 鍾銀香 B08901120	Date: December 2021

Solution

a. First we rewrite it as

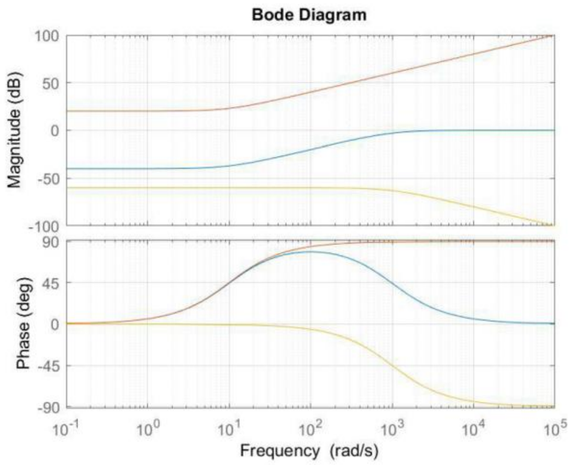
$$G_{lead}(s) = \frac{p}{z}\left(\frac{s/z + 1}{s/p + 1}\right)$$

Since the zero is smaller than pole we will get:

Red : zero

Yellow : pole

Blue : $G_{lead}(s)$



b. First we rewrite it as

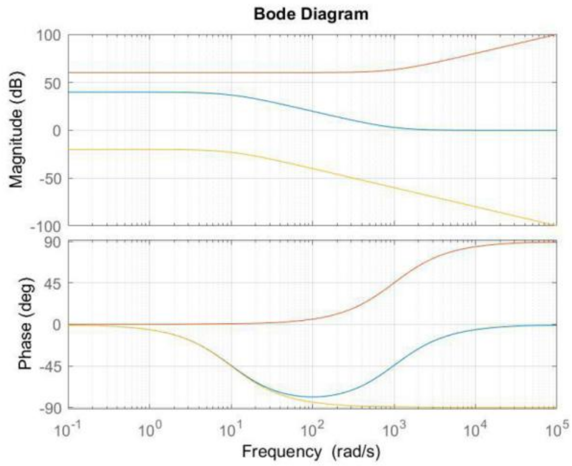
$$G_{lag}(s) = \frac{p}{z}\left(\frac{s/z + 1}{s/p + 1}\right)$$

Since the pole is smaller than zero, we will get:

Red : zero

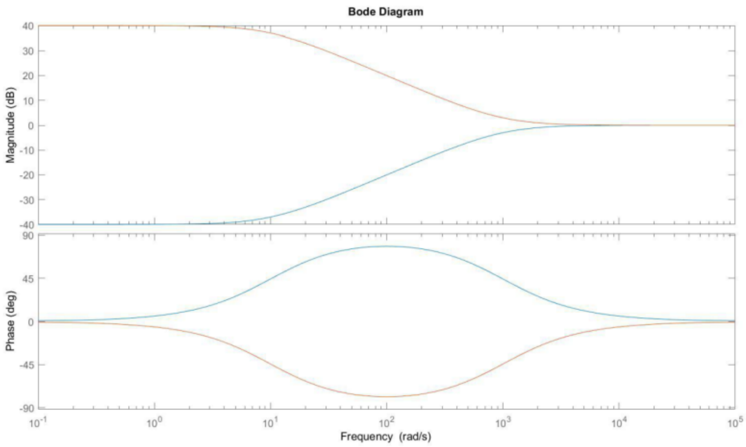
Yellow : pole

Blue : $G_{lag}(s)$



2

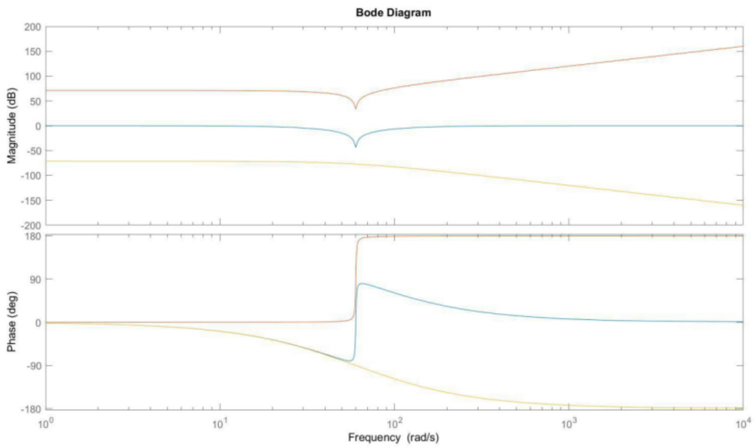
c. We can see that the lead and lag compensations in bode are just mirrors of each other with respect to the frequency axis. Here I have a MATLAB simulation for a lead (blue) and lag(yellow) compensation with the same values.



From the MATLAB graph, we can see that the lead

d. $G_{notch}(s) = \frac{s^2 + 0.8s + 3600}{(s+60)^2}$ has 2 pole break points at $\omega = 60$ and 2 zero break points at the same frequency. However the zeros has an overshoot as it is a second order term. We can predict that at frequency far from the break points, the gain from zero will be canceled by the poles. We rewrite :

$G_{notch}(s) = \frac{\left(s/60\right)^2 + 2 \cdot 1/150 \cdot 1/60\, s + 1}{\left(s/60 + 1\right)^2}$ and obtain : $\omega_n = 60$ and $\zeta = 1/150$.



Red : zeros Yellow : poles Blue : G_{notch}

From the magnitude plot we can analyze that the compensation has unity gain for all frequencies except near the break frequency 60rad/s. Around the 60 rad/s, the compensation has a significant negative gain. This means that this compensation does not change the magnitude of an input that does not contain frequencies near 60 rad/s and mute the inputs at around 60 rad/s.

簡宏哲 B08901111

Control System Homework 08 電機三 B08901111 簡宏哲

Question : For the open-loop transfer functions given below, please

(I) $T_1(s) = \dfrac{s\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)}$ (II) $T_2(s) = \dfrac{1000\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)\left[\left(\frac{s}{\sqrt{10}}\right)^2+\frac{3s}{10}+1\right]}$

(a) Sketch the Bode magnitude and phase plots of both systems.

(b) Use Matlab code to verify the results in (a).

(c) Find the gain crossover frequency and the phase margin of system (I).

(d) Find the phase crossover frequency and the gain margin of system (II).

Solution :

(a) For system (I):

$$T_1(s) = \frac{s\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)}$$

For magnitude :

$\omega \ll 10$, $|T_1(j\omega)| \doteqdot \omega$

$\omega = 10$, $|T_1(j\omega)| \doteqdot 5$

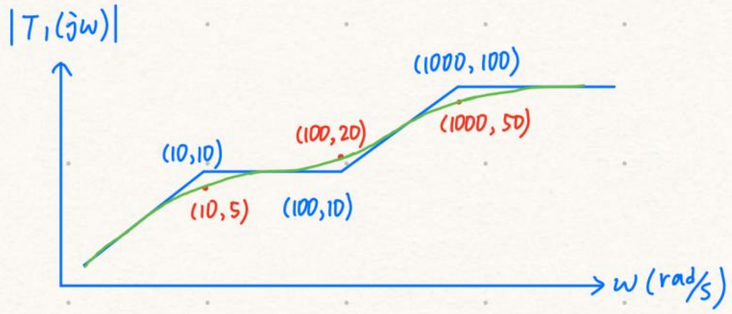
$10 \ll \omega \ll 100$, $|T_1(j\omega)| \doteqdot 10$

$\omega = 100$, $|T_1(j\omega)| \doteqdot 20$

$100 \ll \omega \ll 1000$, $|T_1(j\omega)| \doteqdot \frac{\omega}{10}$

$\omega = 1000$, $|T_1(j\omega)| \doteqdot 50$

$\omega \gg 1000$, $|T_1(j\omega)| \doteqdot 100$



For phase:

$\omega \ll 10$, $\angle T_1(j\omega) \doteq 90^\circ$

$\omega = 10$, $\angle T_1(j\omega) \doteq 45^\circ$

$10 \ll \omega \ll 100$, $\angle T_1(j\omega) \doteq 0^\circ$

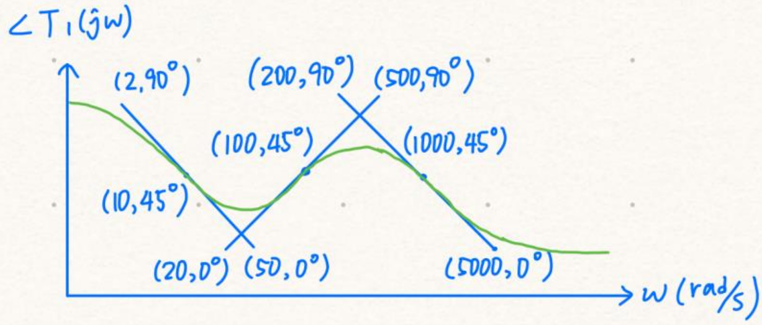
$\omega = 100$, $\angle T_1(j\omega) \doteq 45^\circ$

$100 \ll \omega \ll 1000$, $\angle T_1(j\omega) = 90^\circ$

$\omega = 1000$, $\angle T_1(j\omega) \doteq 45^\circ$

$\omega \gg 1000$, $\angle T_1(j\omega) \doteq 0^\circ$

$$T_1(s) = \frac{s\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)}$$



For system (II):

(II) $T_2(s) = \dfrac{1000\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)\left[\left(\frac{s}{\sqrt{10}}\right)^2+\frac{3s}{10}+1\right]}$

For magnitude:

$\omega \ll \sqrt{10}$, $|T_2(j\omega)| \doteq 1000$

$\omega = \sqrt{10}$, $|T_2(j\omega)| \doteq 333$

$\sqrt{10} \ll \omega \ll 10$, $|T_2(j\omega)| \doteq \dfrac{10000}{\omega^2}$

$\omega = 10$, $|T_2(j\omega)| \doteq 50$

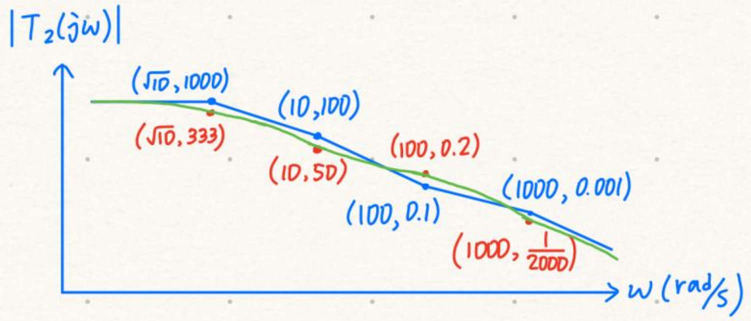
$10 \ll \omega \ll 100$, $|T_2(j\omega)| \doteq \dfrac{100000}{\omega^3}$

$\omega = 100$, $|T_2(j\omega)| \doteq 0.2$

$100 \ll \omega \ll 1000$, $|T_2(j\omega)| \doteq \dfrac{1000}{\omega^2}$

$\omega = 1000$, $|T_2(j\omega)| \doteq \dfrac{1}{2000}$

$\omega \gg 1000$, $|T_2(j\omega)| \doteq \dfrac{10^6}{\omega^3}$



For phase:

$\omega \ll \sqrt{10}$, $\angle T_2(j\omega) \doteq 0^\circ$

$\omega = \sqrt{10}$, $\angle T_2(j\omega) \doteq -90^\circ$

$\sqrt{10} \ll \omega \ll 10$, $\angle T_2(j\omega) \doteq -180^\circ$

$\omega = 10$, $T_2(j\omega) \doteq -225^\circ$

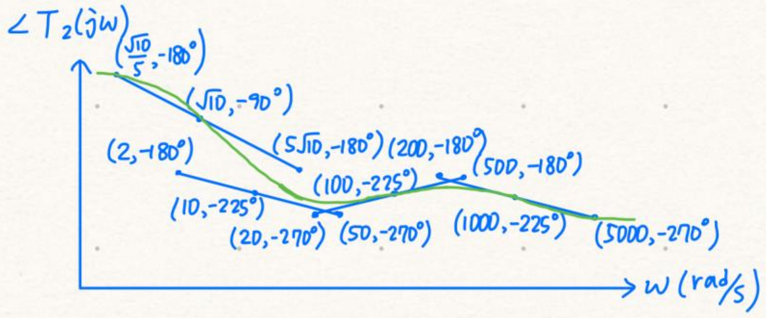
$10 \ll \omega \ll 100$, $\angle T_2(j\omega) \doteq -270^\circ$

$\omega = 100$, $\angle T_2(j\omega) \doteq -225^\circ$

$100 \ll \omega \ll 1000$, $\angle T_2(j\omega) \doteq -180^\circ$

$\omega = 1000$, $\angle T_2(j\omega) \doteq -225^\circ$

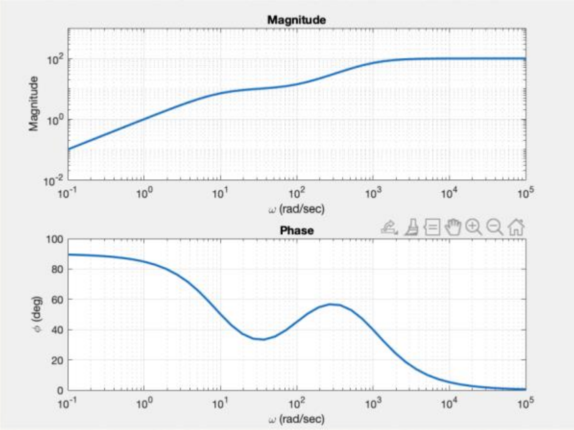
$\omega \gg 1000$, $\angle T_2(j\omega) \doteq -270^\circ$



(II) $$T_2(s) = \frac{1000\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)\left[\left(\frac{s}{\sqrt{10}}\right)^2+\frac{3s}{10}+1\right]}$$

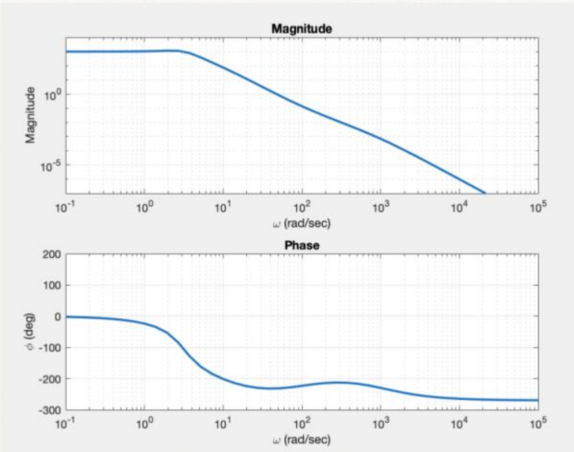
(b) For system (I):

```
clear all;
clf
s=tf('s');

sysD=(s)*(s/100+1)/((s/10+1)*(s/1000+1));
w=logspace(-2,5);
[mag,ph]=bode(sysD,w);

subplot(2,1,1)

loglog(w,squeeze(mag),'LineWidth',2);

grid on;
axis([0.1 100000 0.01 1000])
title('Magnitude');
xlabel('\omega (rad/sec)');
ylabel('Magnitude');

subplot(2,1,2)

semilogx(w,squeeze(ph),'LineWidth',2);

grid on;
axis([0.1 100000 0 100])
title('Phase');
ylabel('\phi (deg)');
xlabel('\omega (rad/sec)')
```



For system (II):

```
clear all;
clf
s=tf('s');

sysD=1000*(s/100+1)/((s/10+1)*(s/1000+1)*(s^2/10+0.3*s+1));
w=logspace(-2,5);
[mag,ph]=bode(sysD,w);

subplot(2,1,1)

loglog(w,squeeze(mag),'LineWidth',2);

grid on;
axis([0.1 100000 0.0000001 10000])
title('Magnitude');
xlabel('\omega (rad/sec)');
ylabel('Magnitude');

subplot(2,1,2)

semilogx(w,squeeze(ph),'LineWidth',2);

grid on;
axis([0.1 100000 -300 200])
title('Phase');
ylabel('\phi (deg)');
xlabel('\omega (rad/sec)')
```



$$T_1(s) = \frac{s\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)} \quad (\text{II})\ T_2(s) = \frac{1000\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{1000}+1\right)\left[\left(\frac{s}{\sqrt{10}}\right)^2+\frac{3s}{10}+1\right]}$$

(c) gain crossover frequency w_c is where the magnitude gain is 1

$\therefore w_c \doteqdot 1\ \text{rad}/s$

When $w = w_c \doteqdot 1\ \text{rad}/s$, $\angle T_1(jw) \doteqdot 84.8^\circ$

$\Rightarrow$ phase margin PM : $\angle T_1(jw) + 180^\circ \doteqdot 264.8^\circ$

(d) phase crossover frequency w_{cp} is where the phase is -180°

$\therefore w_{cp} \doteqdot 6\ \text{rad}/s$

When $w = w_{cp} \doteqdot 6\ \text{rad}/s$, $|T_2(jw)| \doteqdot 270$

$\Rightarrow$ gain margin GM : $\frac{1}{|T_2(jw_{cp})|} \doteqdot 3.68 \cdot 10^{-3}$

HW 8: Frequency Response and Bode Plot	Control Systems NTU-EE
Name: 賴哲緯 B07502079	Date: 12/14, 2021

自行編輯的題目：

For the open-loop transfer functions of the unity feedback control systems given below, sketch the Bode magnitude and phase plots. Find their gain margins, gain crossover frequencies, and phase crossover frequencies.

(a) $KL(s) = K\frac{5}{s(s+5)}$

(b) $KL(s) = K\frac{5}{s(2s+1)^2}$

< sol > :

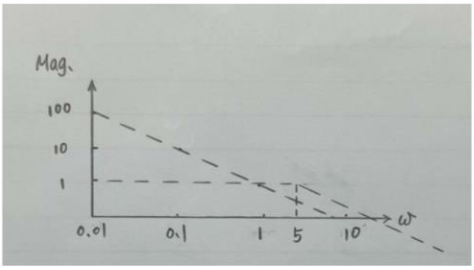
(a) 此系統的 relative order 為 2 階，代表無論 K 值調整多少，根軌跡的漸進線會位於±90°，系統的 pole 不會跑到右半平面。

將系統轉移函數化為頻率響應的標準式：

$$KL(j\omega) = K(j\omega)^{-1}\frac{1}{(\frac{1}{5}j\omega + 1)} \ldots (1)$$

根據上述頻率響應函數可見系統屬於 Type 1，$K_v = \left.\frac{K}{\omega}\right|_{\omega=1}$。

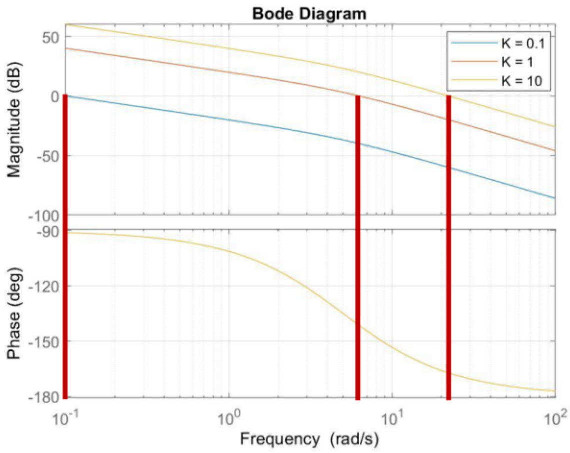
繪製此系統的波德圖，可以觀察出 error constant、crossover frequency 等等資訊。根據式(1)的標準形式，頻率分界點分別位於$\omega = 0$、5 [rad/s]。手繪出漸進線如下圖一所示：



圖一，手繪波德圖漸進線。

其轉折頻率位在 5 rad/s，頻率小於 5 rad/s 以前的斜率為 -1，大於 5 rad/s 則斜率變為-2。利用 MATLAB 來驗證的結果如下圖二所示。設定不同的 K 值並不會影響頻率響應的相位圖形，由圖二的結果可見，無論 K 值為多少，對應的 magnitude 為 1 的相位皆小於 180 度，因此無論 K 值為多少，gain margin 永遠大於零，系統永遠保持穩定，與根軌跡分析結果相同。

1

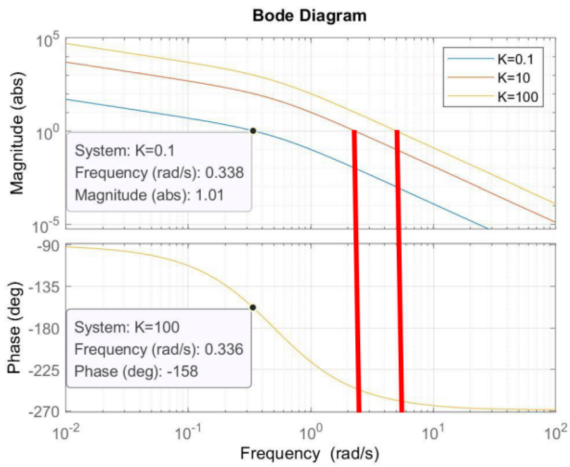


圖二，不同 K 值系統 Bode plot。

(b) 將系統轉移函數寫為標準形式，可以得知轉折頻率位於ω = 0、0.5 [rad/s]。

$$KL(j\omega) = K\frac{50}{(j\omega)\left(\frac{1}{0.5}(j\omega)+1\right)^2} \dots (2)$$

同理於(a)分別設定 K 為 0.1、1 及 10 來觀察頻率響應情形，如下圖三所示。可見只有當 K=0.1 時系統存在 gain margin。K=0.1 時，在輸入頻率為 0.338 [rad/s] 處，增益大小為 1、相位為-158 度，與 critically stable 的相位-180 度還有 22 的裕度，因此此系統仍為穩定的情形。然而 K=1 及 K=10 的情形，增益為 1 的相位已經跑到-180 度以下，gain margin 小於零，對應到根軌跡位置上已經在 s domain 的右半平面。



圖二，Bode diagram。

2

《附件—MATLAB code》

```
clear all;
clc;
s = tf('s');
% set different K values
set_K = [0.1 10 100];
L = 5/(s*(2*s+1)^2);

for i=1:1:3
    K = set_K(i);
    sys(i) = K*L;
    figure(1)
    bode(sys(i));
    grid on;
    grid minor;
    hold on;
end;
```

HW08 電機三 B08502048 陳杰伸

(Revised from problem 2, adding zeros and second order terms)

2. Bode Plot

(2-1) For the following open-loop transfer functions $L(s)$, sketch the magnitude and phase Bode plots. Verify your results using MATLAB.

(a) $L_1(s) = \frac{10(s+2)}{s(s+10)}$

(b) $L_2(s) = \frac{50}{s(s^2+s+25)}$

(2-2) Please use the Bode plot to find the velocity-error constant K_v of the unity feedback system using $L_2(s)$. Verify your result using direct calculation.

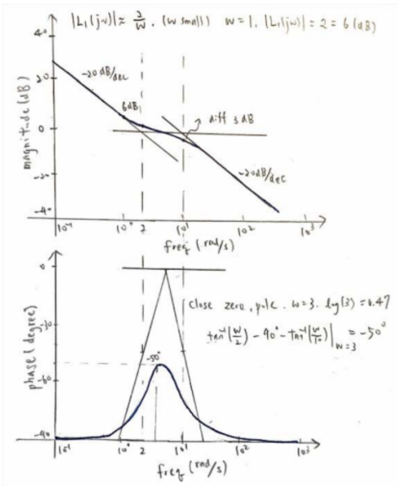
(2-3) Please use the Bode plot to find the value of K such that the unity feedback system using $KL_2(s)$ is neutral stable. Verify your result using root locus.

Sol.

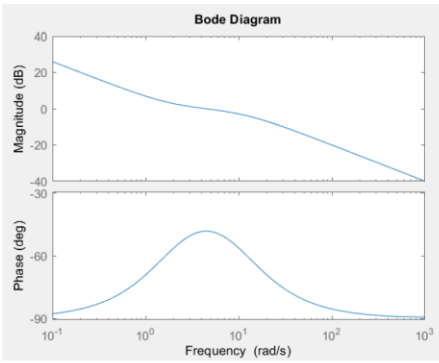
(2-1)

(a) $L_1(s) = \frac{10(s+2)}{s(s+10)}, L_1(j\omega) = \frac{2(\frac{j\omega}{2}+1)}{j\omega(\frac{j\omega}{10}+1)}$

Break points: 2, 10

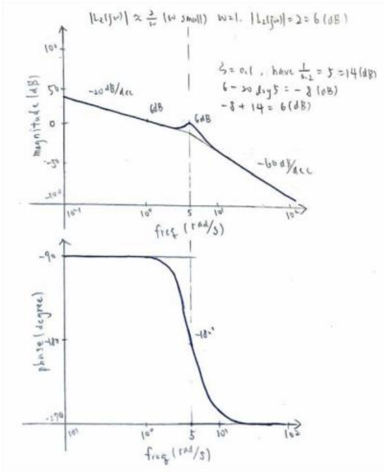


Verification: Using MATLAB.

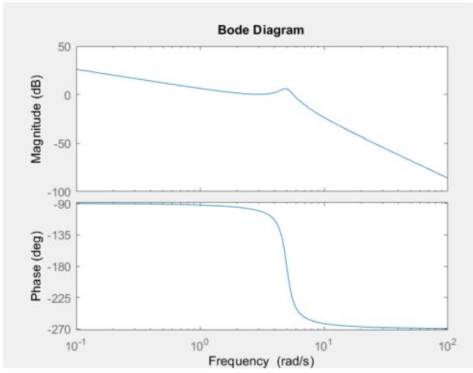


(b) $L_2(s) = \frac{50}{s(s^2+s+25)}, L_2(j\omega) = \frac{2}{j\omega(\frac{(j\omega)^2}{25}+2(0.1)\frac{j\omega}{5}+1)}$

Break points: 5



Verification: Using MATLAB.



(2-2)

Using magnitude Bode plot of $L_2(s)$.

The low frequency asymptote is $\frac{2}{\omega}$, with value 2 at $\omega = 1$. Therefore, $\mathbf{K_v = 2}$.

Verification: $K_v = \lim_{s\to 0} sL_2 = \lim_{s\to 0} s\frac{50}{s(s^2+s+25)} = 2$

(2-3)

Using magnitude and phase Bode plot of $L_2(s)$.

For neutral stable, we want Bode plot of $KL_2(s)$ have magnitude 0 (dB) phase -180° at the same time. Since K does not affect the phase, and

$$20log|KL_2(j\omega)| = 20logK + 20log|L_2(j\omega)| \text{ (dB)}$$

We want ω_1, the frequency in Bode plot of $L_2(s)$ with phase -180°, to have

$$0 = 20logK + 20log|L_2(j\omega_1)| \text{ (dB)}$$

From Bode plot of $L_2(s)$, we have

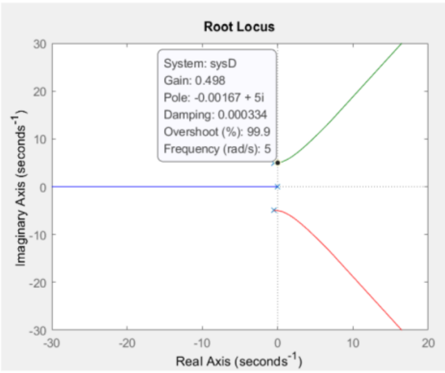
$$20log|L_2(j\omega_1)| = 6 \text{ (dB)}$$

Therefore,

$$\mathbf{K = 10^{\frac{-6}{20}} = 0.501}$$

2/3

Verification: K = 0.498 using root locus.



Appendix: MATLAB code for the whole problem.

```
s=tf('s');
sysD = (10*(s+2))/(s*(s+10));
figure
bode(sysD);

sysD = 50/(s*(s^2+s+25));
figure
bode(sysD);

figure
rlocus(sysD);
```

3/3

Homework 08 for Units 6A, 6B, 6C: Bode Plot

Digital Control System, Fall 2021, NTU-EE

Name: B07901004 電機四 陳恩庭 Date: 12/15, 2021

Problem 3 (revised)

A normalized second-order system with a damping ratio $\boldsymbol{\zeta = 0.7}$ and an additional zero is given by

$$\mathbf{G(s)} = \frac{\frac{s}{\alpha} + 1}{s^2 + 1.4s + 1}$$

(1) Use MATLAB to compare the M_p from the step response of the system for $\alpha = 0.01, 0.1, 1, 10, 100$, and ∞ with the M_r from the frequency response of each case. Is there a correlation between M_r and M_P?

(2) If the transfer function of the system is given by adding **an additional pole** instead of an additional zero, such as

$$\mathrm{G(s)} = \frac{1}{(\frac{s}{\alpha} + 1)(s^2 + 1.4s + 1)}$$

Then, what would the step response and the frequency response be like?

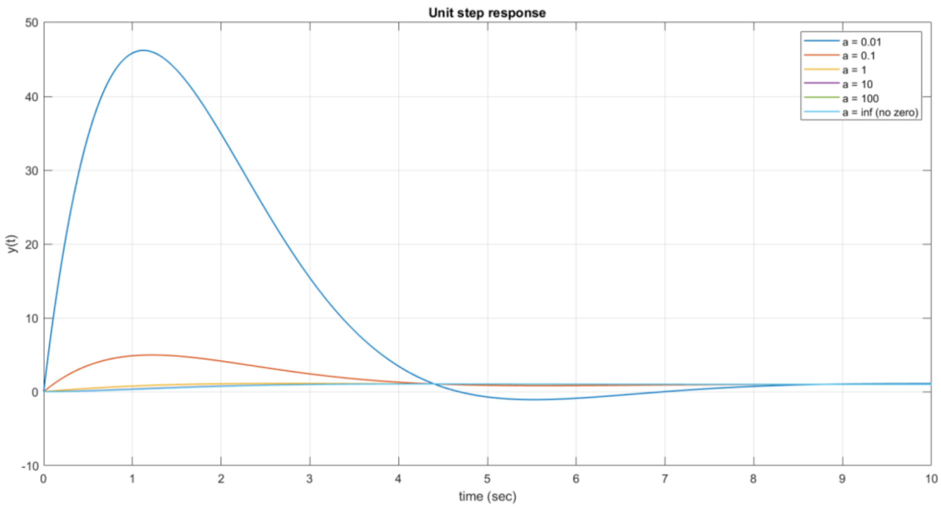
(3) Observe the answer in (2), find the **bandwidth** from the frequency response of each case, and describe your findings.

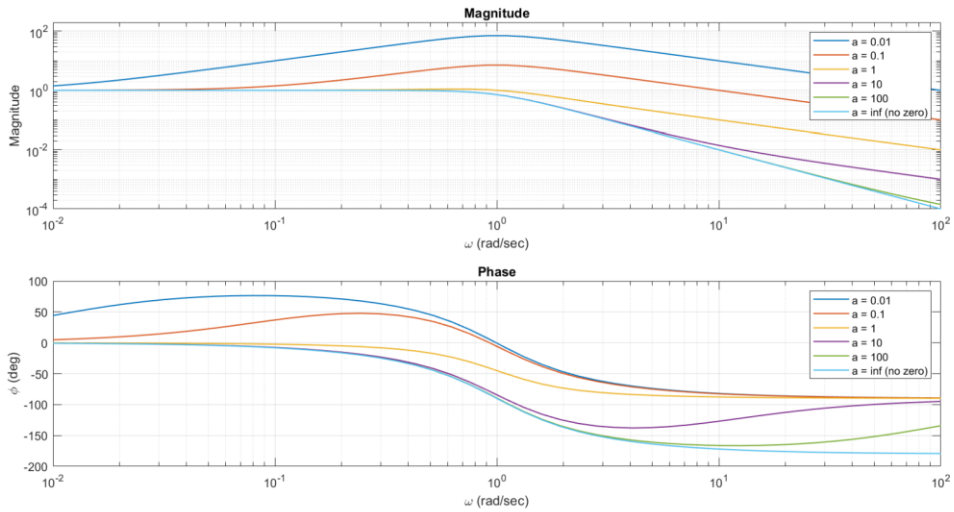
Solutions:

(1) Get the values below by using Data Tips in MATLAB.

α	Resonant peak, M_r	Overshoot, M_p
0.01	70.8	45.21
0.1	7.122	3.968
1	1.106	0.112
10	1	0.046
100	1	0.046
∞	1	0.046

As α increases, both the resonant peak M_r and the overshoot M_p reduce. Besides, in the cases of $\alpha = 0.01, 0.1, 1$, we can find that the resonant peak has significant reduction as α increases, with the overshoot of step response also dropping significantly. While in the case of $\alpha = 10, 100, \infty$, the resonant peak becomes hardly changed, at the same time, the overshoot turns to be hardly changed as well. Thus, the resonant peak in frequency response and the overshoot in transient response are correlated.


Unit step response
y(t)
time (sec)
a = 0.01
a = 0.1
a = 1
a = 10
a = 100
a = inf (no zero)


Magnitude
Magnitude
ω (rad/sec)
a = 0.01
a = 0.1
a = 1
a = 10
a = 100
a = inf (no zero)
Phase
φ (deg)
ω (rad/sec)
a = 0.01
a = 0.1
a = 1
a = 10
a = 100
a = inf (no zero)

Matlab command:

```
s = tf('s');
w = logspace(-2,2);

a = 0.01;
sys1 = (s/a+1)/(s^2+1.4*s+1);
[m1,p1] = bode(sys1,w);
a = 0.1;
sys2 = (s/a+1)/(s^2+1.4*s+1);
[m2,p2] = bode(sys2,w);
a = 1;
sys3 = (s/a+1)/(s^2+1.4*s+1);
[m3,p3] = bode(sys3,w);
a = 10;
sys4 = (s/a+1)/(s^2+1.4*s+1);
[m4,p4] = bode(sys4,w);
a = 100;
sys5 = (s/a+1)/(s^2+1.4*s+1);
[m5,p5] = bode(sys5,w);
sys6 = 1/(s^2+1.4*s+1);
[m6,p6] = bode(sys6,w);

figure()
t = 0:.01:10;
y1 = step(sys1,t);
y2 = step(sys2,t);
y3 = step(sys3,t);
y4 = step(sys4,t);
y5 = step(sys5,t);
y6 = step(sys6,t);
plot(t,y1,t,y2,t,y3,t,y4,t,y5,t,y6,'LineWidth',1);

grid on;
title('Unit step response');
xlabel('time (sec)');
ylabel('y(t)');
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no zero)');
```

```
figure()
subplot(2,1,1)
loglog(w,squeeze(m1),w,squeeze(m2),w,squeeze(m3),w,squeeze(m4),w,squeez
e(m5),w,squeeze(m6),'LineWidth',1);

grid on;
axis([.01 100 .0001 200])
title('Magnitude');
xlabel('\omega (rad/sec)');
ylabel('Magnitude');
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no zero)');

subplot(2,1,2)
semilogx(w,squeeze(p1),w,squeeze(p2),w,squeeze(p3),w,squeeze(p4),w,squeez
e(p5),w,squeeze(p6),'LineWidth',1);

grid on;
title('Phase');
ylabel('\phi (deg)');
xlabel('\omega (rad/sec)')
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no zero)');
```

(2) Get the values below by using Data Tips in MATLAB.

α	Resonant peak, M_r	Overshoot, M_p
0.01	1	0
0.1	1	0
1	1	0.015
10	1	0.046
100	1	0.046
∞	1	0.046

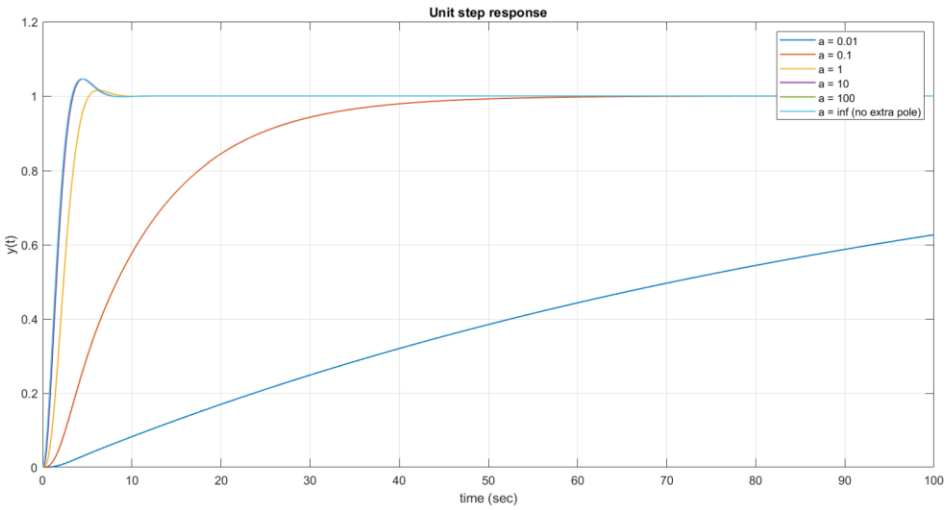
In this case, unlike the case of adding an additional zero as (1), we observe that adding an additional pole doesn't lead to changes in resonant peak ($M_r = 1$ in every case.) And in the case of $\alpha \geq 1$, there is an overshoot in the transient response. The value of the peak overshoot is very small and changes slightly as α increases.

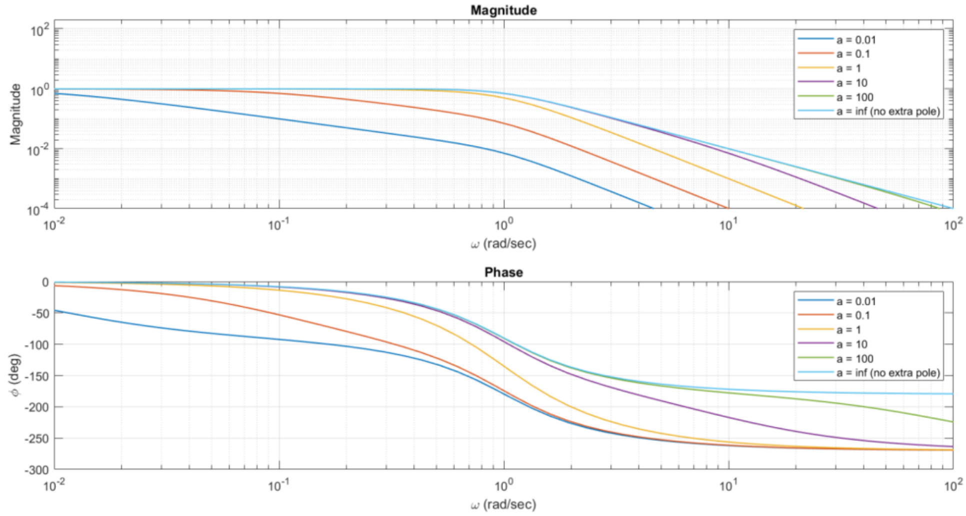
(3) With our discussion in (2), we find that considering the relationship between the resonant peak and the overshoot is quite useless in the case of an additional pole.

However, by observing the plot we have sketched, we can easily find that as α decreases, the rising time of the unit step response increases significantly, and the bandwidth of the frequency response decreases simultaneously. This result is consistent with what we have taught in class that bandwidth is a measure of speed of response.

We can get the values below by using Data Tips in MATLAB.

α	Rising time, t_r (sec)	Bandwidth, ω_{BW} (rad/sec)
0.01	> 100	$<$ 0.010
0.1	24.48	0.100
1	3.99	0.743
10	2.73	0.996
100	2.63	1.002
∞	2.63	1.002





Matlab command:

```
s = tf('s');
w = logspace(-2,2);

a = 0.01;
sys1 = 1/(s/a+1)/(s^2+1.4*s+1);
[m1,p1] = bode(sys1,w);
a = 0.1;
sys2 = 1/(s/a+1)/(s^2+1.4*s+1);
[m2,p2] = bode(sys2,w);
a = 1;
sys3 = 1/(s/a+1)/(s^2+1.4*s+1);
[m3,p3] = bode(sys3,w);
a = 10;
sys4 = 1/(s/a+1)/(s^2+1.4*s+1);
[m4,p4] = bode(sys4,w);
a = 100;
sys5 = 1/(s/a+1)/(s^2+1.4*s+1);
[m5,p5] = bode(sys5,w);
sys6 = 1/(s^2+1.4*s+1);
[m6,p6] = bode(sys6,w);
```

```
figure()
t = 0:.01:100;
y1 = step(sys1,t);
y2 = step(sys2,t);
y3 = step(sys3,t);
y4 = step(sys4,t);
y5 = step(sys5,t);
y6 = step(sys6,t);
plot(t,y1,t,y2,t,y3,t,y4,t,y5,t,y6,'LineWidth',1);

grid on;
title('Unit step response');
xlabel('time (sec)');
ylabel('y(t)');
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no extra pole)');

figure()
subplot(2,1,1)
loglog(w,squeeze(m1),w,squeeze(m2),w,squeeze(m3),w,squeeze(m4),w,squeez
e(m5),w,squeeze(m6),'LineWidth',1);

grid on;
axis([.01 100 .0001 200])
title('Magnitude');
xlabel('\omega (rad/sec)');
ylabel('Magnitude');
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no extra pole)');

subplot(2,1,2)
semilogx(w,squeeze(p1),w,squeeze(p2),w,squeeze(p3),w,squeeze(p4),w,squeez
e(p5),w,squeeze(p6),'LineWidth',1);

grid on;
title('Phase');
ylabel('\phi (deg)');
xlabel('\omega (rad/sec)')
legend('a = 0.01','a = 0.1','a = 1','a = 10','a = 100','a = inf (no extra pole)');
```

邱泓翔 B08901095

HW 08: Unit 6A, 6B, 6C Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 12/16, 2021

Problem

A second-order system with a damping ratio of $\zeta = 0.3$ and an additional pole is given by

$$G(s) = \frac{1}{\left(\frac{s}{\zeta\alpha}+1\right)(s^2+2\zeta s+1)} = \frac{1}{\left(\frac{s}{0.3\alpha}+1\right)(s^2+0.6s+1)}.$$

(a) Sketch the asymptotes when $\alpha = 2$ according to the Bode plot rules.
(b) Use MATLAB to draw the Bode plots.
(c) Find the gain margin, phase margin, gain crossover frequency, and phase crossover frequency for G(s) when $\alpha = 2$ by the MATLAB plots in (b). Then verify your results using MATLAB commands.
(d) Use MATLAB to draw the step responses of the system when $\alpha = 100, 4, 2, 1$.
(e) Use MATLAB to draw the Bode plots of the system when $\alpha = 100, 4, 2, 1$.
(f) Is there a correlation between resonant peak M_r and overshoot M_p?

Solution

(a) When $\alpha = 2$,

$$\mathrm{G(s)} = \frac{1}{\left(\frac{s}{0.6}+1\right)(s^2+0.6s+1)}.$$

Note that the break frequencies are 0.6 and 1 (rad/s). The magnitude asymptotes are as shown in Figure 1 and the phase asymptotes are as shown in Figure 2.

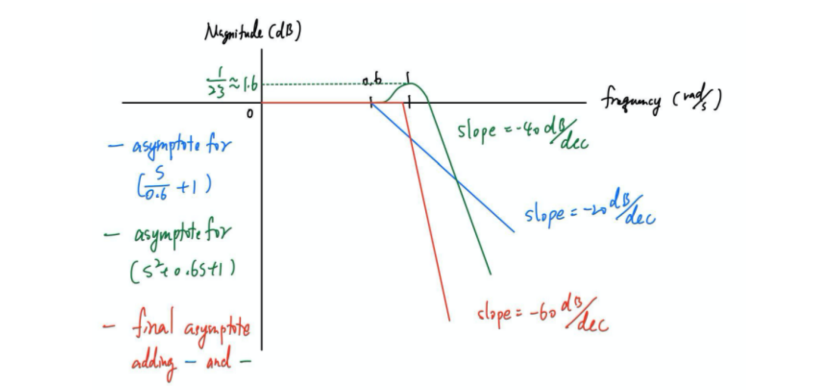


Figure 1. Magnitude asymptotes for G(s).

1

HW 08: Unit 6A, 6B, 6C Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 12/16, 2021

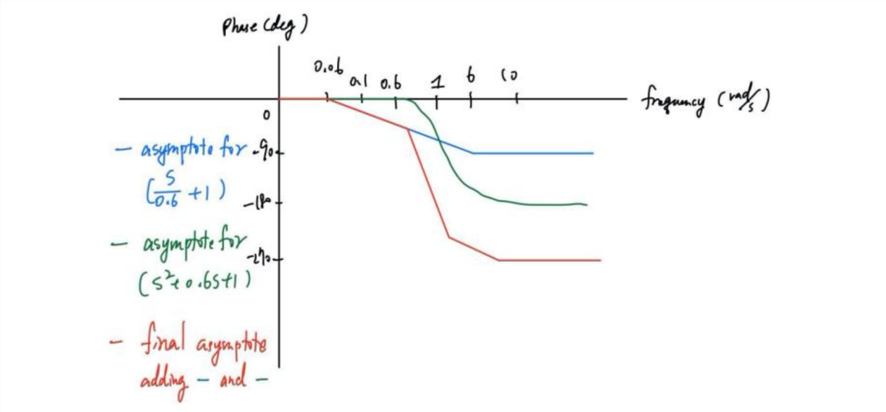


Figure 2. Phase asymptotes for G(s).

(b) The Bode plots are shown in Figure 3.

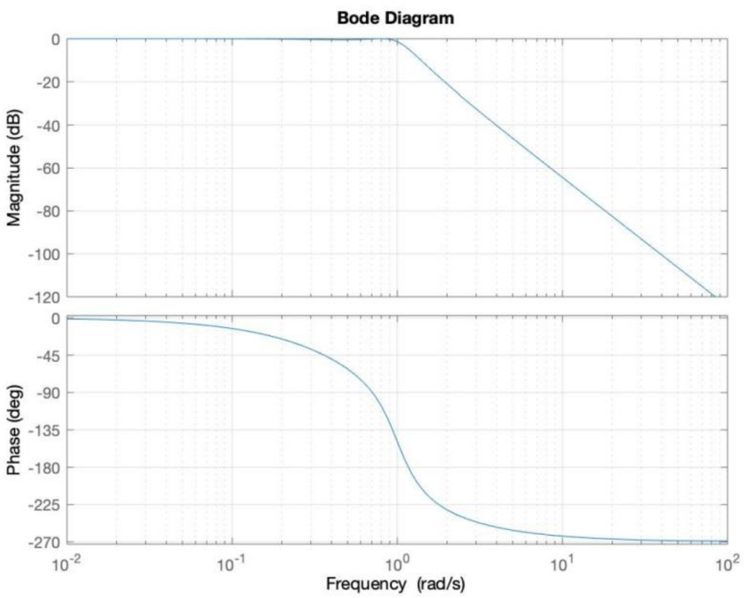


Figure 3. Bode plots of G(s) by MATLAB.

(c) By figure 3, when the phase reaches -180°, the absolute value of the magnitude is about 4.5, so gain margin $\approx 10^{\frac{4.5}{20}} \approx 1.68$. Phase margin is obviously -180°. Gain crossover frequency = 0. Phase crossover frequency ≈ 1.15.

2

HW 08: Unit 6A, 6B, 6C Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 12/16, 2021

The results using MATLAB commands:
Gain margin = 1.72, phase margin = -180°,
gain crossover frequency = 0, phase crossover frequency = 1.1662.

MATLAB code for (b) and (c):

```
zeta = 0.3;
k = 1/zeta;
alpha = 2;
s = tf('s');
G = 1/((k*s/alpha+1)*(s^2+2*zeta*s+1));
t = 0:0.01:20;
[Gm, Pm, Wcg, Wcp] = margin(G)

figure;
bode(G)
grid on;
```

(d) The step responses are shown in Figure 4.

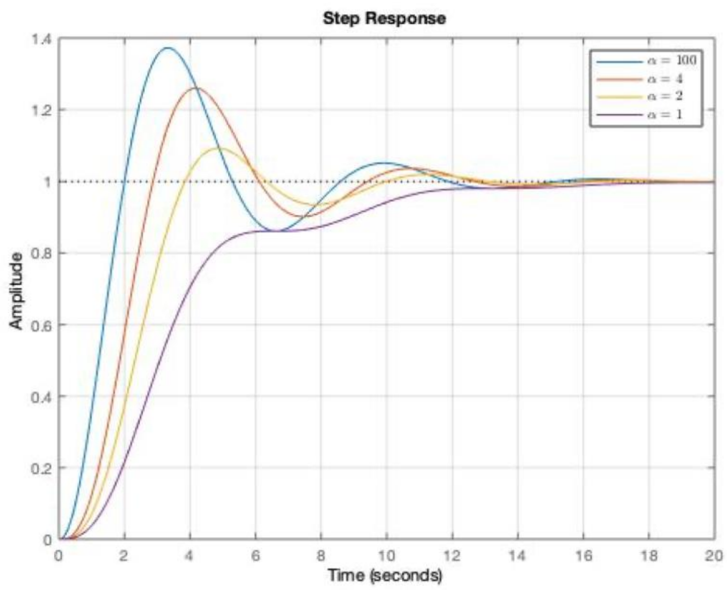


Figure 4. Step responses for different α

3

HW 08: Unit 6A, 6B, 6C Bode Plot	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 12/16, 2021

(e) The Bode plots are shown in Figure 5.

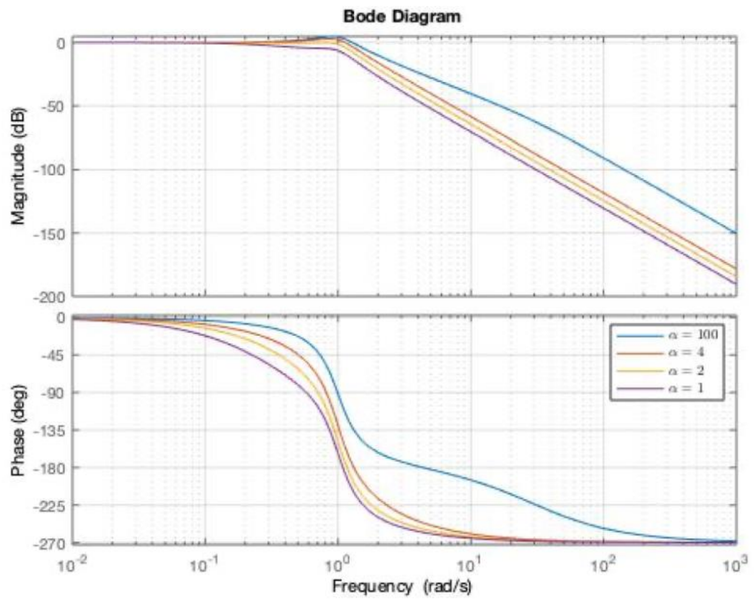


Figure 5. Bode plots for different α

MATLAB code for (d) and (e):

```
zeta = 0.3;
k = 1/zeta;
alpha = [100 4 2 1];
s = tf('s');
G_1 = 1/((k*s/alpha(1)+1)*(s^2+2*zeta*s+1));
G_2 = 1/((k*s/alpha(2)+1)*(s^2+2*zeta*s+1));
G_3 = 1/((k*s/alpha(3)+1)*(s^2+2*zeta*s+1));
G_4 = 1/((k*s/alpha(4)+1)*(s^2+2*zeta*s+1));

t = 0:0.01:20;

figure;
step(G_1, G_2, G_3, G_4, t)
leg = legend( ...
    '$\alpha = 100$', ...
    '$\alpha = 4$', ...
    '$\alpha = 2$', ...
```

```
    '$\alpha = 1$');
set(leg, 'interpreter', 'latex')
grid on;

figure;
bode(G_1, G_2, G_3, G_4)
leg = legend( ...
    '$\alpha = 100$', ...
    '$\alpha = 4$', ...
    '$\alpha = 2$', ...
    '$\alpha = 1$');
set(leg, 'interpreter', 'latex')
grid on;
```

(f) From figure 4 and figure 5, we can observe that when α is reduced, both the resonant peak and overshoot decreases. This means that the resonant peak in frequency response and the peak overshoot in transient response are correlated.