Control System: Homework 07 for Unit 5D, 5E, 5F: Root Locus

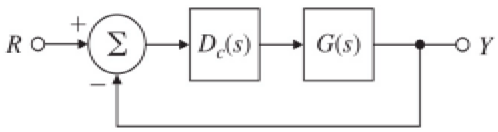
Assigned: Nov 11, 2022

Due: Nov 17, 2022 (23:59)

Please read the following problems and their solutions. Then, choose one of them and edit your solution for the selected problem. Submit your homework file in PDF format to the NTU-Cool website.

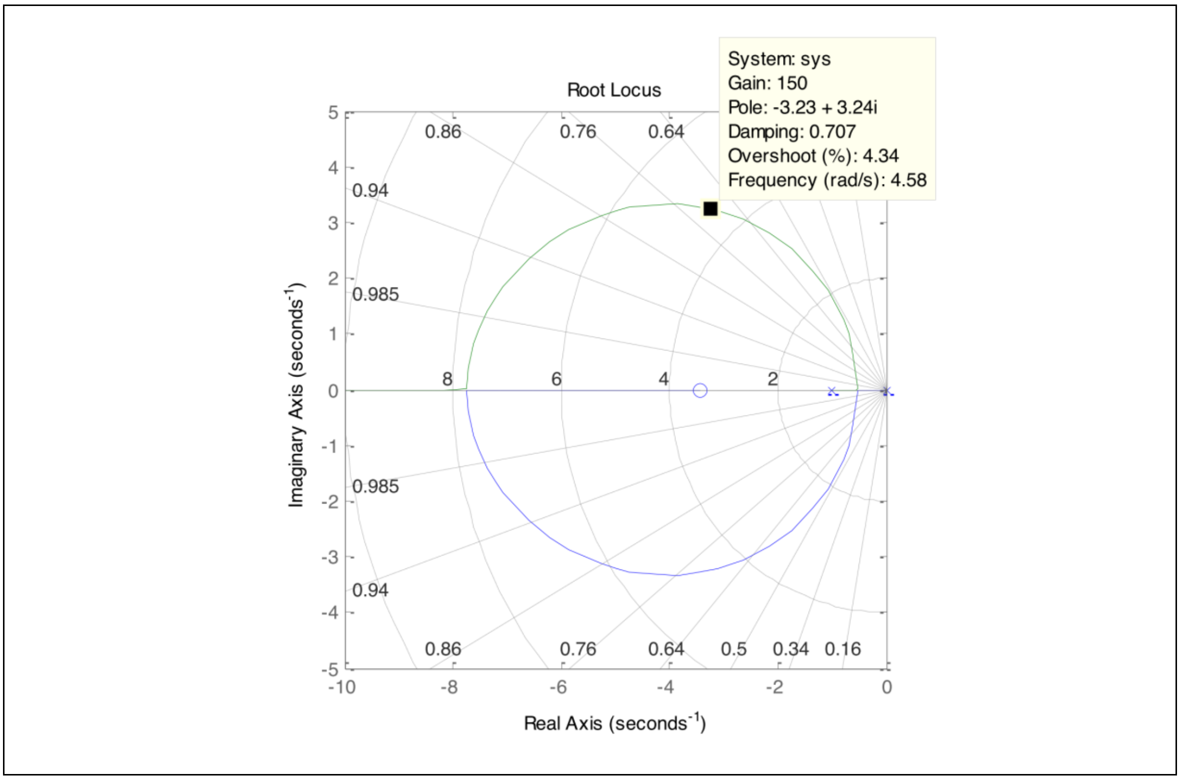
1. (U5D: Lead-Lag Compensator)

23. Suppose the unity feedback system of Fig. 5.59 has an open-loop plant given by $G(s) = \frac{1}{s(s+1)}$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.



Solution:

Setting the pole of the lead to be at $p = -30$, and the zero is at $z = -3$ produces a locus with a circle that goes a bit too high and misses the desired $-3.2 + 3.2j$. So move the zero a bit to the West, i.e. let $z = -3.44$. It does the job, so put your cursor on the spot and find that with a gain of $K = 150$ gives the desired roots. The locus is plotted below.



Root Locus
System: sys
Gain: 150
Pole: -3.23 + 3.24i
Damping: 0.707
Overshoot (%): 4.34
Frequency (rad/s): 4.58
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.86
0.76
0.64
0.94
0.985
0.5
0.34
0.16
8
6
4
2
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-8
-6
-4
-2
0

2. (U5D: Lead-Lag Compensator)

26. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D_c(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.
- The steady-state error to a unit ramp at the reference input must be less than 0.05.

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications, ignoring the error requirement.

(b) What is the velocity constant K_v for your design? Does it meet the error specification?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the Matlab plot of the root locus of your final design.

(e) Give the Matlab response of your final design to a reference step.

Solution:

(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245. With the lead compensator, the overshoot is reduced to 3.64% and the rise time is 0.35 sec.

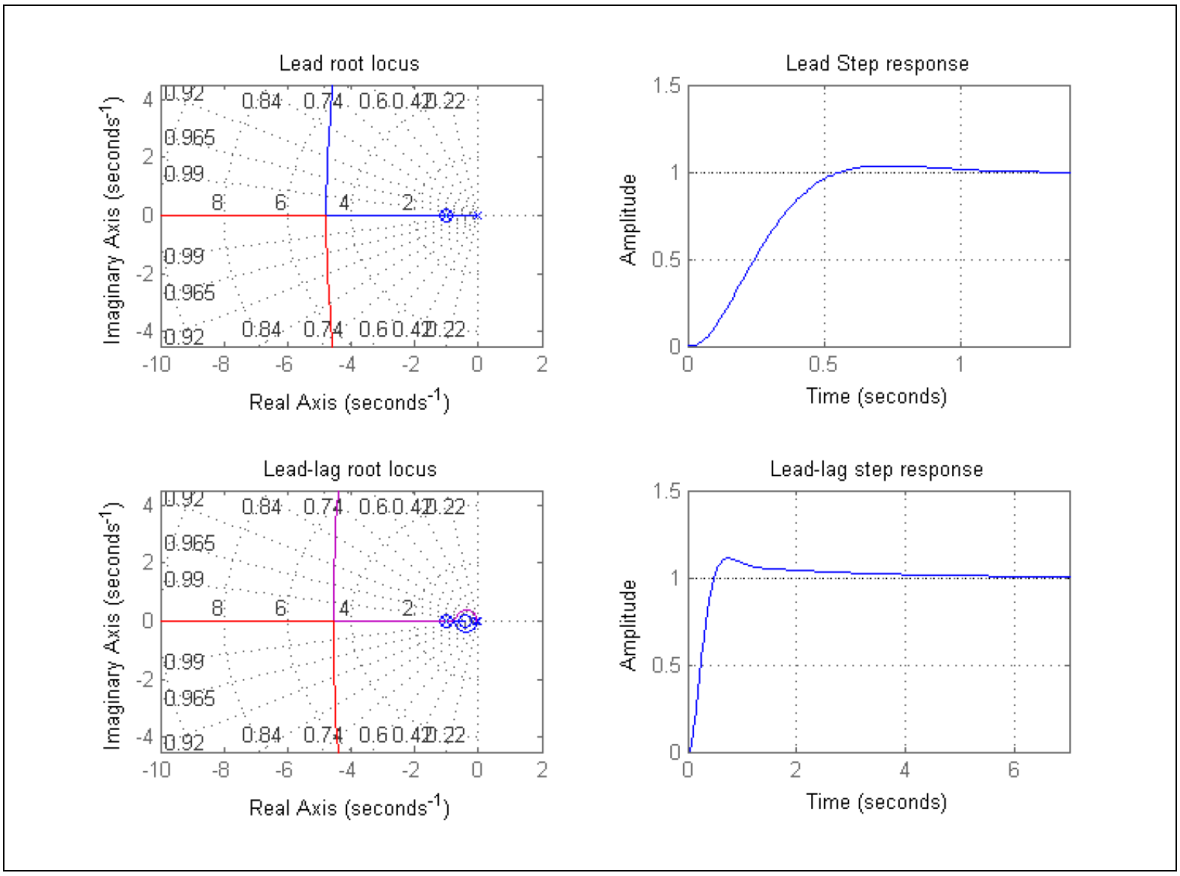
(b)

$$K_v = \lim_{s \to 0} sGD_c = \lim_{s \to 0} s \frac{10}{s(s+1)(s+10)} \frac{245(s+1)}{(s+6)} = 4.083$$

(c) To meet the steady-state requirement, we need a new $K_v = 20$, which is an increase of a factor of 5. If we set the lag zero at $z = -0.4$, the pole needs to be at $p = -0.08$.

(d) The root locus is plotted below.

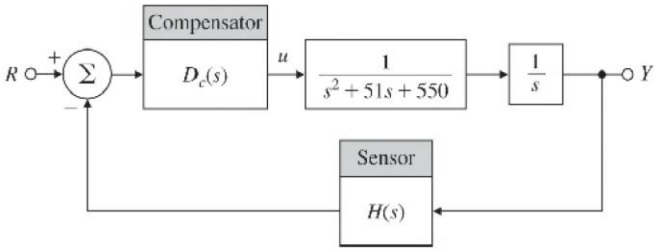
(e) The step response is plotted below.



Lead root locus
Lead Step response
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Amplitude
Time (seconds)
Lead-lag root locus
Lead-lag step response
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Amplitude
Time (seconds)

3. (U5E: Design using the Root Locus)

40. Consider the instrument servomechanism with the parameters given in Fig.5.68. For each of the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your final design.



(a) *Lead network* : Let

$$H(s) = 1, \quad D_c(s) = K\frac{s+z}{s+p}, \quad \frac{p}{z} = 6.$$

Select z and K so that the roots nearest the origin (the dominant roots) yield

$$\zeta \geq 0.4, \quad -\sigma \leq -7, \quad K_v \geq 16\frac{2}{3}\text{sec}^{-1}.$$

(b) *Output-velocity (tachometer) feedback*: Let

$$H(s) = 1 + K_T s \quad \text{and} \quad D_c(s) = K.$$

Select K_T and K so that the dominant roots are in the same location as those of part (a). Compute K_v. If you can, give a physical reason explaining the reduction in K_v when output derivative feedback is used.

(c) *Lag network* : Let

$$H(s) = 1 \quad \text{and} \quad D(s) = K\frac{s+1}{s+p}.$$

Using proportional control, is it possible to obtain a $K_v = 12$ at $\zeta = 0.4$? Select K and p so that the dominant roots correspond to the proportional-control case but with $K_v = 100$ rather than $K_v = 12$.

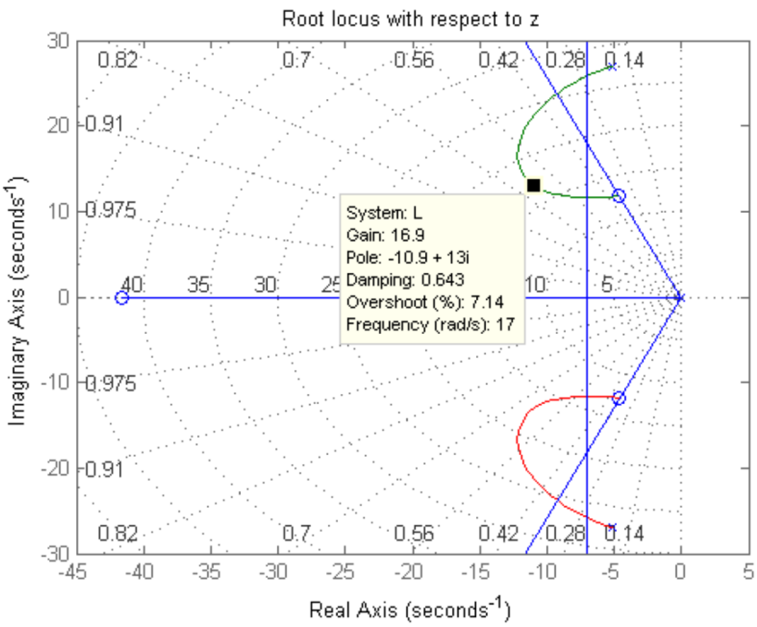
Solution:

(a) Setting $p = 6z$, the velocity constant is

$$K_v = \lim_{s \to 0} s \frac{K(s+z)}{s+6z} \frac{1}{s(s^2+51s+550)} = \frac{K}{3300}$$

Thus the K_v requirement leads to $K \geq 35200$. With $K = 40000$, a root locus can be drawn with respect to z.

$$1 + z\frac{6s(s^2+51s+550)+40000}{s^2(s^2+51s+550)+40000s} = 0$$



Root locus for Problem 5.40(a)

At the point of maximum damping, the values are $z = 16.8$ and the dominant roots are at $s = -11 \pm 13j$. So the compensator is $D_c(s) = 40000\dfrac{s+16.8}{s+100.8}$.

(b) With $H(s) = 1 + K_T s$ and $D_c(s) = K$, the closed-loop transfer function is

$$\frac{Y}{R} = \frac{K}{s^3 + 51s^2 + (550 + KK_T)s + K}$$

For this system to have poles at $s = -11 \pm 13j.$, the characteristic polynomial should be in the form of

$$(s+p)(s^2 + 22s + 290) = s^3 + (p+22)s^2 + (22p + 290)s + 290p$$

Equating the coefficients leads to $p = 29$, $K = 8410$, and $K_T = 0.045$. With these value, the velocity constant is

$$\frac{1}{K_v} = \lim_{s \to 0} s\left(1 - \frac{Y}{R}\right)\frac{1}{s^2} = \frac{550 + KK_T}{K} \quad \Rightarrow \quad K_v = 9.058$$

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.

(c) Using proportional control ($D_c(s) = K$), the velocity constant is

$$K_v = \lim_{s \to 0} sK\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550}$$

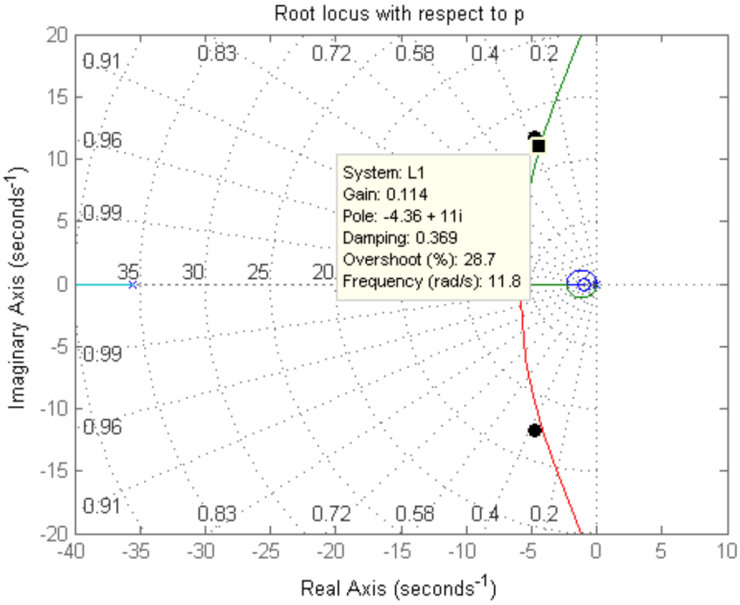
Therefore $K_v = 12$ can be obtained by setting $K = 6600$. With this value, the dominant roots are at $s = -4.7 \pm 11.69j$, and $\zeta = 0.37$.

With $D_c(s) = K\frac{s+1}{s+p}$, the velocity constant is

$$K_v = \lim_{s \to 0} s\frac{K(s+1)}{s+p}\frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550p}$$

So $K_v = 100$ can be obtained by setting $\frac{K}{p} = 55000$. Setting $K = 55000p$, a root locus can be drawn with the parameter p

$$1 + p\frac{s(s^2 + 51s + 550) + 55000(s+1)}{s^2(s^2 + 51s + 550)} = 0$$



Root locus for Problem 5.40(c)

In the plot, the desired pole locations are marked with a dot (•). Thus we can choose $p = 0.11$ to place the poles near the desired locations. Thus the compensator is $D_c(s) = 6050\dfrac{s+1}{s+0.11}$.