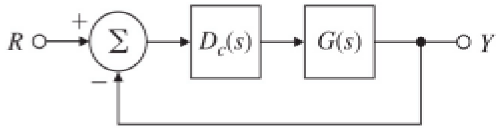
Control System: Homework 07 for Units 5C, 5D, 5E, 5F: Root Locus

Assigned: November 19, 2021

Due: December 2, 2021 (23:59)

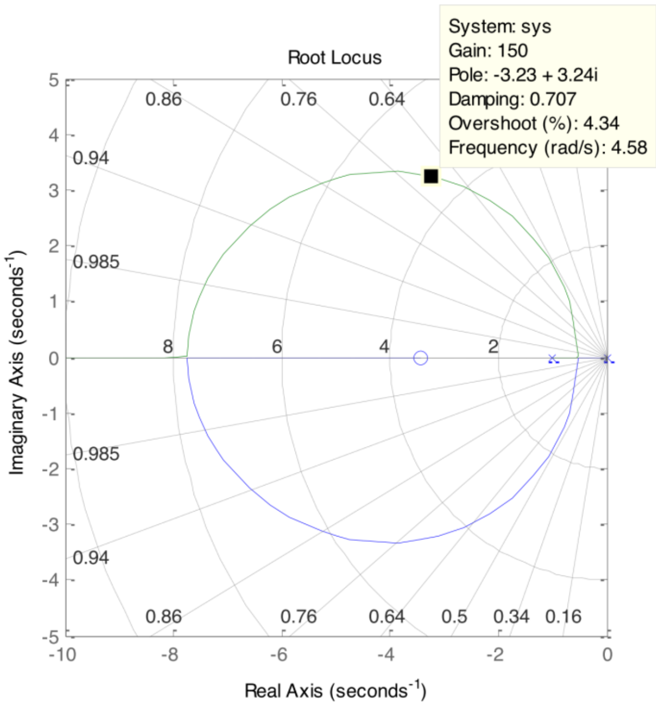
1. (U5D: Lead-Lag Compensator)

23. Suppose the unity feedback system of Fig. 5.59 has an open-loop plant given by $G(s) = \frac{1}{s(s+1)}$. Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.



Solution:

Setting the pole of the lead to be at $p = -30$, and the zero is at $z = -3$ produces a locus with a circle that goes a bit too high and misses the desired $-3.2 + 3.2j$. So move the zero a bit to the West, i.e. let $z = -3.44$. It does the job, so put your cursor on the spot and find that with a gain of $K = 150$ gives the desired roots. The locus is plotted below.



2. (U5D: Lead-Lag Compensator)

26. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}.$$

You are to design a series compensation transfer function $D_c(s)$ in the unity feedback configuration to meet the following closed-loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4 sec.
- The steady-state error to a unit ramp at the reference input must be less than 0.05.

(a) Design a lead compensation that will cause the system to meet the dynamic response specifications, ignoring the error requirement.

(b) What is the velocity constant K_v for your design? Does it meet the error specification?

(c) Design a lag compensation to be used in series with the lead you have designed to cause the system to meet the steady-state error specification.

(d) Give the Matlab plot of the root locus of your final design.

(e) Give the Matlab response of your final design to a reference step.

Solution:

(a) Setting the lead pole at $p = -60$ and the zero at $z = -1$, the dynamic specifications are met with a gain of 245. With the lead compensator, the overshoot is reduced to 3.64% and the rise time is 0.35 sec.

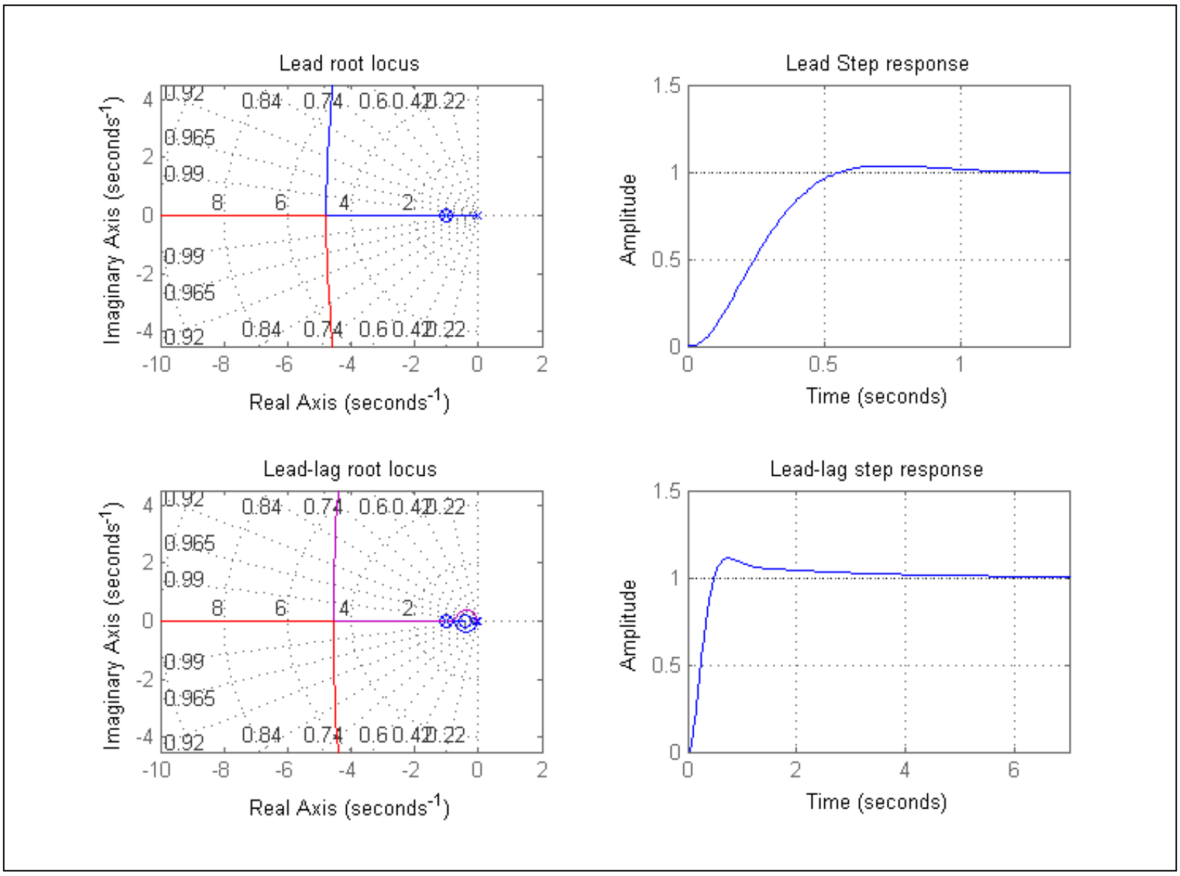
(b)

$$K_v = \lim_{s \to 0} sGD_c = \lim_{s \to 0} s \frac{10}{s(s+1)(s+10)} \frac{245(s+1)}{(s+6)} = 4.083$$

(c) To meet the steady-state requirement, we need a new $K_v = 20$, which is an increase of a factor of 5. If we set the lag zero at $z = -0.4$, the pole needs to be at $p = -0.08$.

(d) The root locus is plotted below.

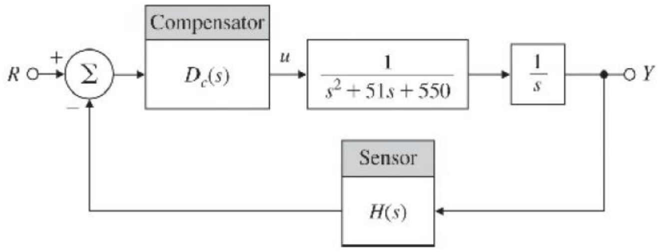
(e) The step response is plotted below.



Lead root locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
0.92
0.84
0.74
0.6
0.42
0.22
0.965
0.99
8
6
4
2
Lead Step response
Amplitude
Time (seconds)
Lead-lag root locus
Lead-lag step response

3. (U5E: Design using the Root Locus)

40. Consider the instrument servomechanism with the parameters given in Fig.5.68. For each of the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your final design.



(a) *Lead network* : Let

$$H(s) = 1, \quad D_c(s) = K\frac{s+z}{s+p}, \quad \frac{p}{z} = 6.$$

Select z and K so that the roots nearest the origin (the dominant roots) yield

$$\zeta \geq 0.4, \quad -\sigma \leq -7, \quad K_v \geq 16\frac{2}{3}\text{sec}^{-1}.$$

(b) *Output-velocity (tachometer) feedback*: Let

$$H(s) = 1 + K_T s \quad \text{and} \quad D_c(s) = K.$$

Select K_T and K so that the dominant roots are in the same location as those of part (a). Compute K_v. If you can, give a physical reason explaining the reduction in K_v when output derivative feedback is used.

(c) *Lag network* : Let

$$H(s) = 1 \quad \text{and} \quad D(s) = K\frac{s+1}{s+p}.$$

Using proportional control, is it possible to obtain a $K_v = 12$ at $\zeta = 0.4$? Select K and p so that the dominant roots correspond to the proportional-control case but with $K_v = 100$ rather than $K_v = 12$.

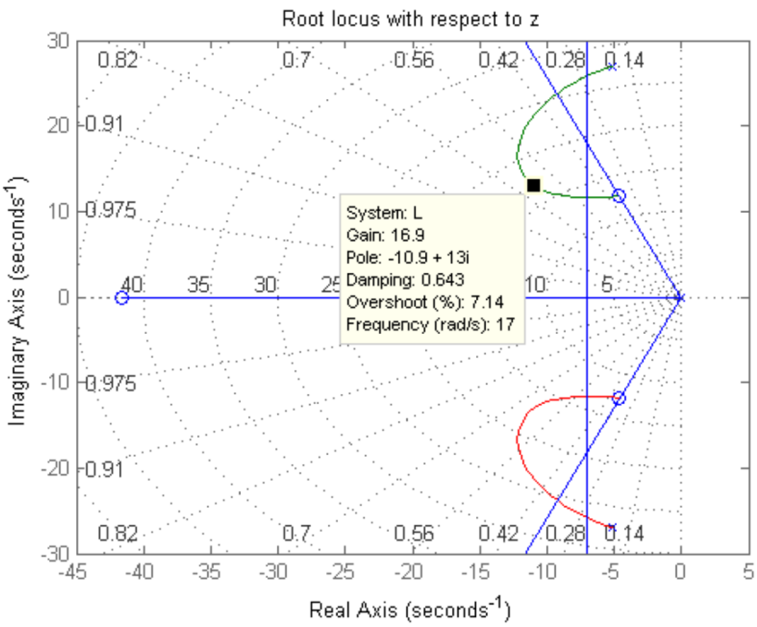
Solution:

(a) Setting $p = 6z$, the velocity constant is

$$K_v = \lim_{s\to 0} s\frac{K(s+z)}{s+6z}\frac{1}{s(s^2+51s+550)} = \frac{K}{3300}$$

Thus the K_v requirement leads to $K \geq 35200$. With $K = 40000$, a root locus can be drawn with respect to z.

$$1 + z\frac{6s(s^2+51s+550)+40000}{s^2(s^2+51s+550)+40000s} = 0$$



Root locus for Problem 5.40(a)

At the point of maximum damping, the values are $z = 16.8$ and the dominant roots are at $s = -11 \pm 13j$. So the compensator is $D_c(s) = 40000\frac{s+16.8}{s+100.8}$.

(b) With $H(s) = 1 + K_T s$ and $D_c(s) = K$, the closed-loop transfer function is

$$\frac{Y}{R} = \frac{K}{s^3 + 51s^2 + (550 + KK_T)s + K}$$

For this system to have poles at $s = -11 \pm 13j$., the characteristic polynomial should be in the form of

$$(s+p)(s^2 + 22s + 290) = s^3 + (p+22)s^2 + (22p + 290)s + 290p$$

Equating the coefficients leads to $p = 29$, $K = 8410$, and $K_T = 0.045$. With these value, the velocity constant is

$$\frac{1}{K_v} = \lim_{s \to 0} s \left(1 - \frac{Y}{R}\right) \frac{1}{s^2} = \frac{550 + KK_T}{K} \quad \Rightarrow \quad K_v = 9.058$$

The output derivative feedback is acting only when there is a change in the output. Therefore, for a ramp input, the derivative action will minimize the deviation from the reference because the input signal is continuously increasing.

(c) Using proportional control ($D_c(s) = K$), the velocity constant is

$$K_v = \lim_{s \to 0} sK \frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550}$$

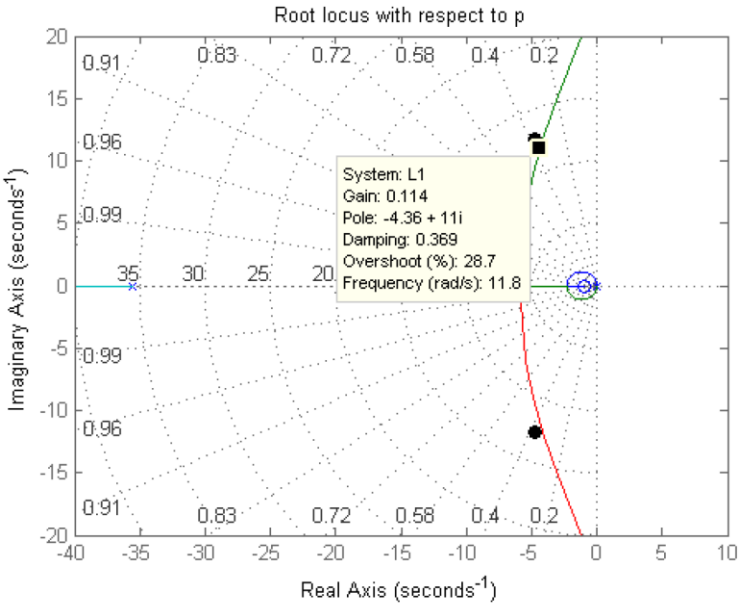
Therefore $K_v = 12$ can be obtained by setting $K = 6600$. With this value, the dominant roots are at $s = -4.7 \pm 11.69j$, and $\zeta = 0.37$.

With $D_c(s) = K\frac{s+1}{s+p}$, the velocity constant is

$$K_v = \lim_{s \to 0} s \frac{K(s+1)}{s+p} \frac{1}{s(s^2 + 51s + 550)} = \frac{K}{550p}$$

So $K_v = 100$ can be obtained by setting $\frac{K}{p} = 55000$. Setting $K = 55000p$, a root locus can be drawn with the parameter p

$$1 + p\frac{s(s^2 + 51s + 550) + 55000(s+1)}{s^2(s^2 + 51s + 550)} = 0$$



Root locus for Problem 5.40(c)

In the plot, the desired pole locations are marked with a dot (•). Thus we can choose $p = 0.11$ to place the poles near the desired locations. Thus the compensator is $D_c(s) = 6050\dfrac{s+1}{s+0.11}$.

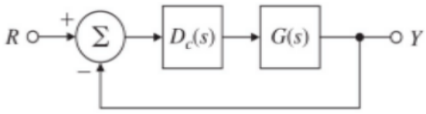
Homework 07 for Units 5C, 5D, 5E, 5F: Root Locus

Digital Control System, Fall 2021, NTU-EE

Name: B07901004 電機四 陳恩庭 Date: 12/1, 2021

Problem 1 (revised)

Suppose the unity feedback system of the figure below has an open-loop plant given by $G(s) = \frac{1}{s(s+3)}$.



(a) Design a lead compensation $D_c(s) = K\frac{s+z}{s+p}$ to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -6.5 \pm 6.5\,j$. (Use Matlab to sketch the root locus plot.)

(b) According to (a), use Matlab to sketch the step response of the system. Find out or calculate the overshoot, the settling time, and also the steady-state error for a unit ramp input of the system.

(c) Consider a case of adding a pole of the plant at s = -6, i.e. $G(s) = \frac{1}{s(s+3)(s+6)}$. What would happen if we use the same design as in (a)?

(d) According to (c), try to design a lag compensation in order to meet the following requirements:

i. The step response settling time is to be less than 5 sec.

ii. The step response overshoot is to be equal or less than 16%.

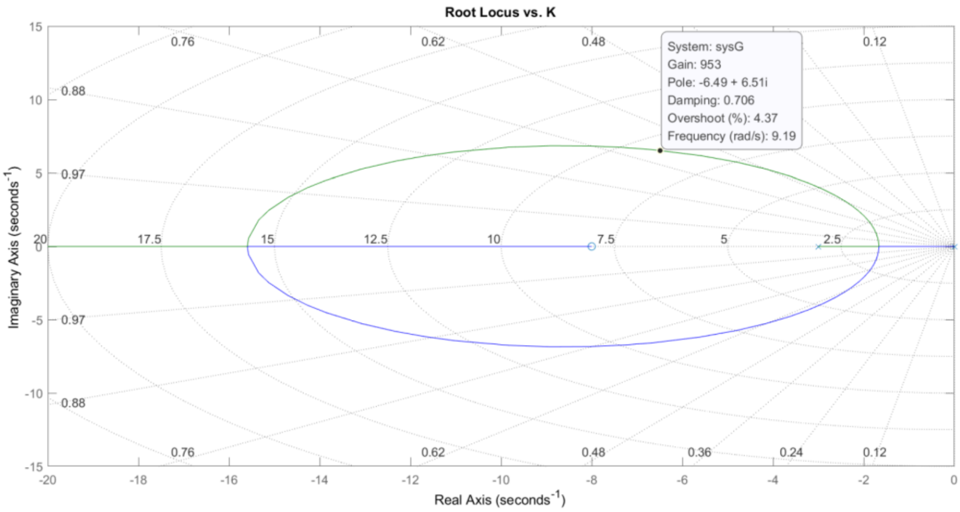
iii. The steady-state error to a unit ramp input must not exceed 10%.

Solutions:

(a) Setting **the pole of D_c at s = -100 and the zero at s = -8** produces a locus with a circle that passes through $s = -6.5 \pm 6.5j$. (For this step, we can either use trial and error or a better way which is to gain experience in each adjustment and to approach the answer, i.e. the position of the zero should be at the left of the pole s = -3 in order to produce a circle.)

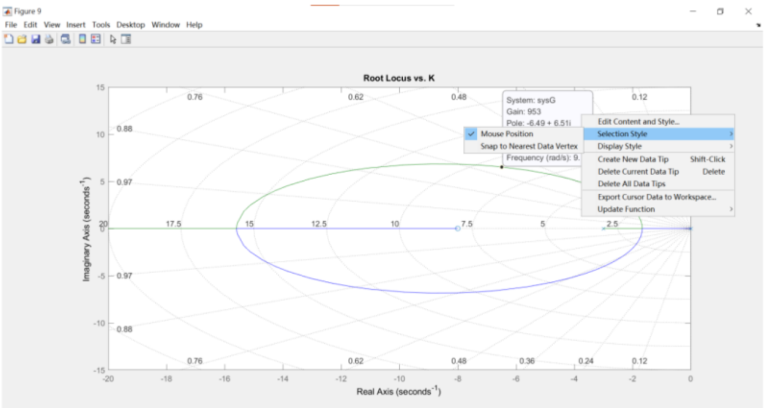
Matlab commands:

```
s = tf('s');
z = 8;
p = 100;
D = (s+z)/(s+p);
G = 1/(s*(s+3)*(s+6));
sysG = G*D
figure
rlocus(sysG);
axis([-20 0 -15 15])        % optional
sgrid;
hold on
title('Root Locus vs. K')
```



Using data tip we can find out that **K = 953** gives the desired root.

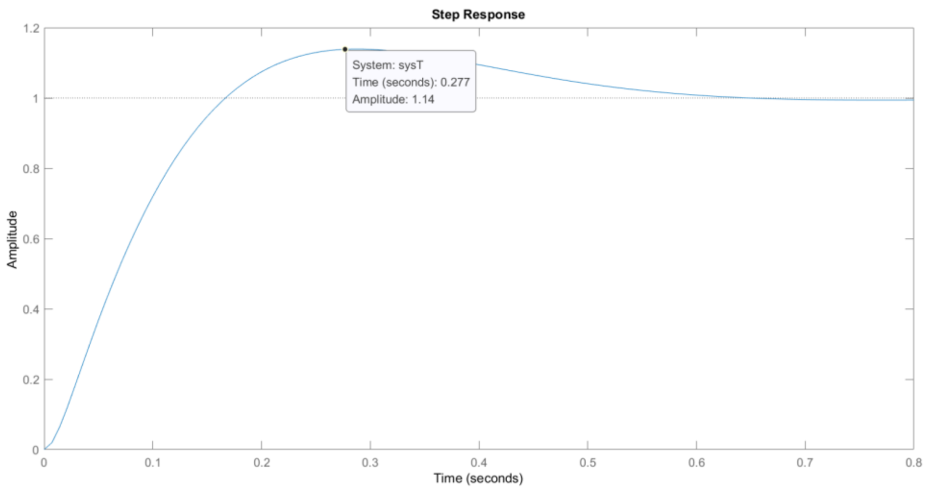
(p.s. If you have trouble with putting data tips on the desired position, you can try to change the selection style to "Mouse Position" as below.



(b) For the feedback system, use Matlab to sketch the step response.

Matlab commands:

```
s = tf('s');
K = 953;
z = 8;
p = 100;
D = (s+z)/(s+p);
G = 1/(s*(s+3));
sysG = G*D
sysL = K*sysG;
[sysT] = feedback(sysL,1);
figure
step(sysT)
```



Overshoot $M_p = 14\%$

Damping ratio $\approx$ 0.5

Settling time $t_s = \frac{4.6}{\sigma} = \frac{4.6}{0.5\times 9.19} = 1.001$

Steady-state error (for a unit ramp inut)

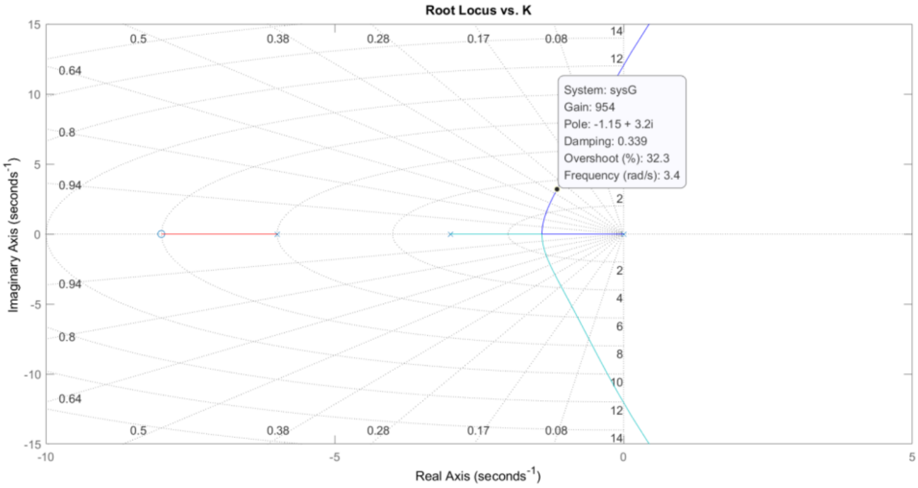
$$e_{ss} = \lim_{s\to 0} s \times \frac{1}{1+GD_c} \times \frac{1}{s^2} = \frac{100\times 3}{953\times 8} = 0.039 = \frac{1}{K_v}$$

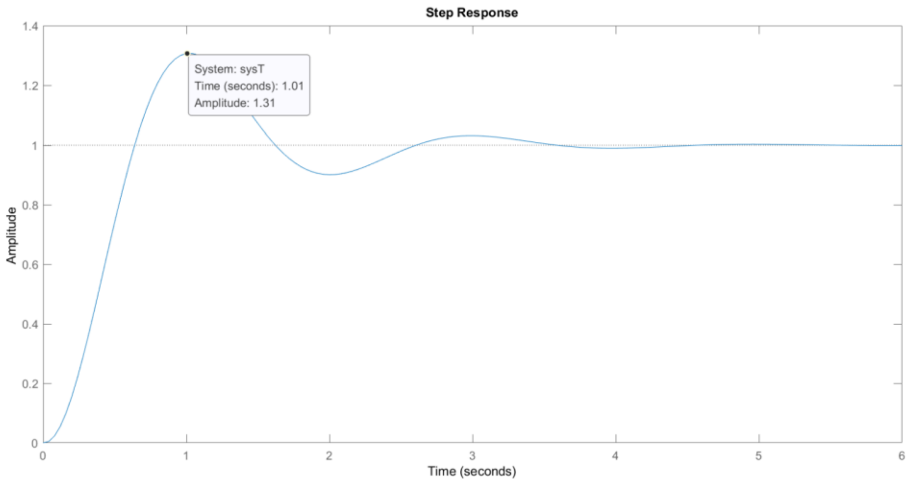
$$K_v = 25.41$$

(c) Use Matlab to get root locus and sketch the step response.

Matlab commands:

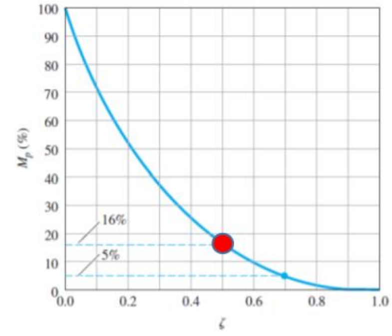
```
s = tf('s');
K = 953;
z = 8;
p = 100;
D = (s+z)/(s+p);
G = 1/(s*(s+3)*(s+6));
sysG = G*D
sysL = K*sysG;
figure
rlocus(sysG);
axis([-10 5 -15 15])
sgrid;
hold on
title('Root Locus vs. K')
[sysT] = feedback(sysL,1);
figure
step(sysT)
```





We can easily observe that the overshoot turns to 31%, which is a lot larger than before. Also, the settling time is greater than before.

(d) For overshoot ≦ 16%, the damping ratio should be equal or greater than 0.5.

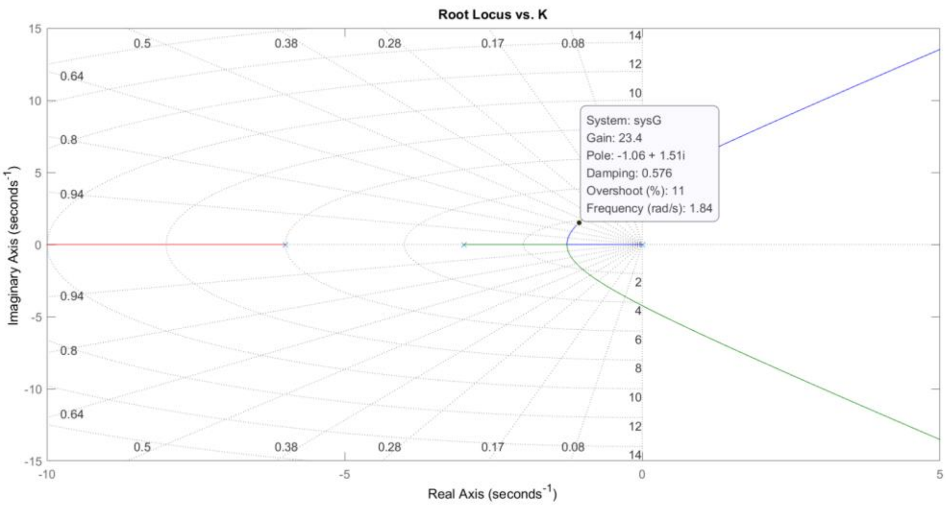


For settling time $t_s \leq 5\,sec,\ \omega_n \geq \frac{4.6}{0.5\times 5} = 1.84\,rad/s.$

Use Matlab to get the root locus.

Matlab commands:

```
s = tf('s');
G = 1/(s*(s+3)*(s+6));
sysG = G
figure
rlocus(sysG);
axis([-10 5 -15 15])
sgrid;
hold on
title('Root Locus vs. K')
```



According to Matlab plot of root locus, we select the gain **K = 23.4**.

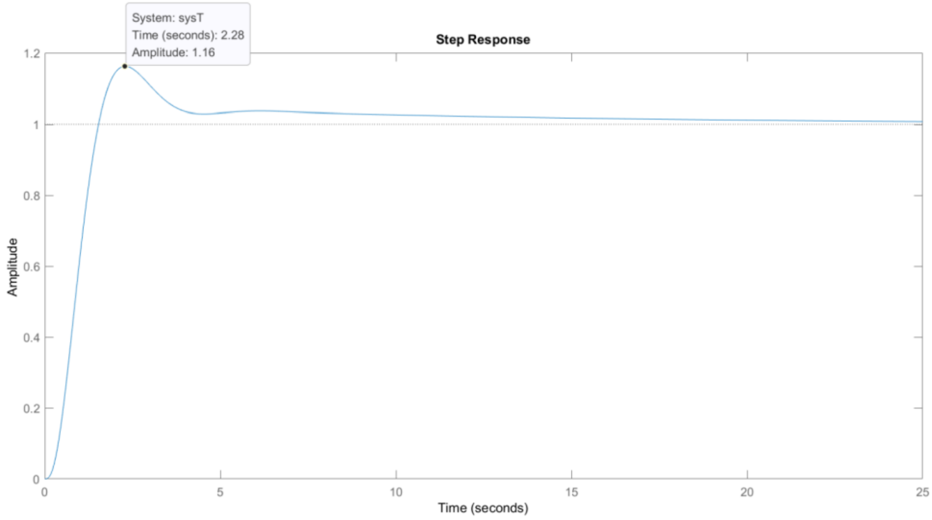
Then, $K_v = \frac{23.4}{3\times6} = 1.3$.

For steady-state error not exceeds 10%, we need $K_v = 10$.

Thus, we need a lag gain of $\frac{10}{1.3} \approx 7.7$.

To meet the overshoot specifications, it is better to select a smaller K.

For $\frac{(s+z)}{(s+p)}$, we set **p = 0.01 and z = 0.077** and get the following result, which meets the requirements. (Other choices of p, z are also possible.)



<u>Matlab commands:</u>

```
s = tf('s');
K = 23.4;
z = 0.077;
p = 0.01;
D = (s+z)/(s+p);
G = 1/(s*(s+3)*(s+6));
sysL = K*sysG*D;
[sysT] = feedback(sysL,1);
figure
step(sysT)
```

Control System: Homework 07 for Units 5C, 5D, 5E, 5F: Root Locus	**Control Systems, Fall 2021, NTU-EE**
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1.1 Problem 1

For a feedback system shown in **Figure(a)**, $G(s) = \frac{1}{s^2+3s+2}$

(a) Sketch the root locus with respect to K for the characteristic equation $1 + KL(s) = 0$.

(b) Determine that whether the system is stable or not by two different methods.

(c) What is the step response of $L(s)$?

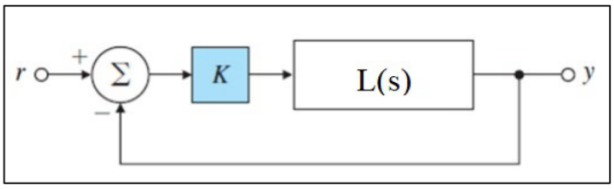
(d) Our goal is to adjust the overshoot and rise time. Which method can be applied?

(e) We need to let overshoot $\leq$ 25% and rise time $\leq$ 0.4s, how to choose the value of ζ and ω_n ?

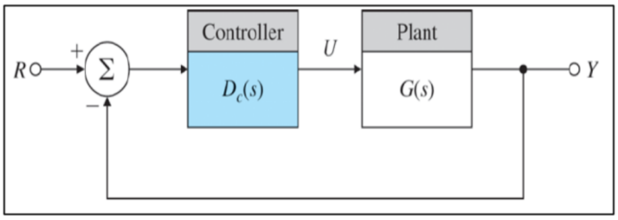
(f) Design $D_C(s)$ to meet the requirements in (e).

(g) Sketch the root locus of the new with respect to K for the characteristic equation $1 + G(s)D_C(s) = 0$.

(h) What is the step response of the modified $L(s)$? Does the overshoot and rise time meet the requirement after applying compensation?



Figure(a): block diagram of the feedback system



Figure(b): feedback system with compensation

Solution:

(a) The characteristic equation is

$$1 + K\frac{1}{s^2 + 3s + 2} = 0$$
$$s^2 + 3s + 2 + K = 0$$
$$s^2 + 3s + (K + 2) = 0 \quad (1)$$

The root locus is the set of values of s for which for which $1 + KL(s) = 0$ is satisfied as the real parameter K varies from 0 to ∞. Use MATLAB to get the root locus of the system, and the answer is shown in **Figure (c)**.

(b) One method is by **observing** the root locus, we can find that there are poles (s= -1, -2) on the **LHP**, so the system is **stable**. Another method is by **Routh's Stability** Criterion, a system is stable if and only if all the elements in the first column of the Routh array are positive. The characteristic equation is eq.(1) $s^2 + 3s + (K + 2) = 0$. The following is Routh array：

s^2	1	$K+2$
s	3	0
1	$-\frac{\det\begin{bmatrix}1 & K+2\\ 3 & 0\end{bmatrix}}{1} = 3(K+2)$	0

We can find that the first column of Routh array is always positive for all non-negative K, so the system is **stable** Q.E.D.

(c) By partial fraction method,

$$\text{Assume } L(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \quad (2)$$

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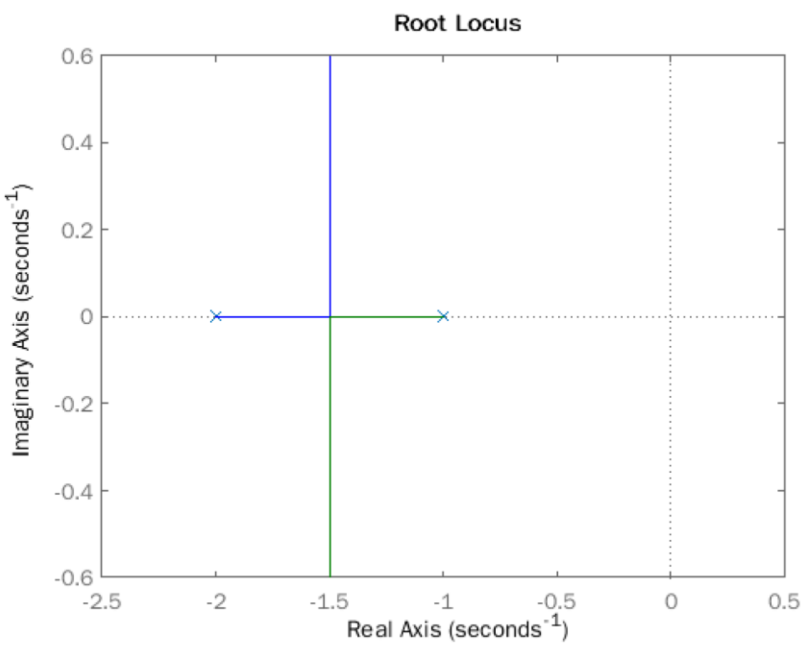
$$A(s+2)+B(s+1)=(A+B)s+(2A+B)=1$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$\mathrm{L(s)}=\frac{1}{s^2+3s+2}=\frac{1}{s+1}+\frac{-1}{s+2}$$

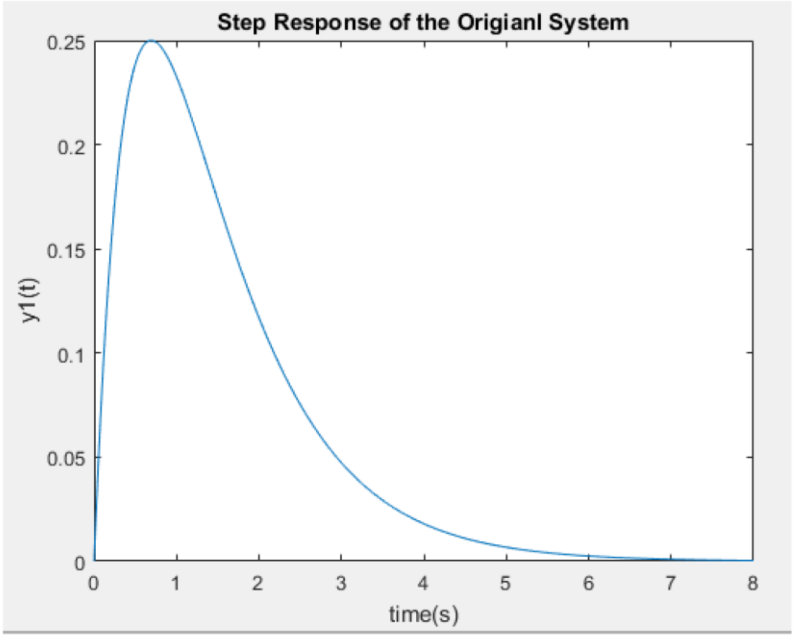
Do inverse Laplace transform, we get the step response $y_1(t)$. Use MATLAB to get the step response of the system, and the answer is shown in **Figure(d)**.

$$y_1(t)=e^{-t}-e^{-2t} \quad (3)$$



Figure(c): Root Locus of the original system

3



Figure(d): step response of the original L(s) by inverse Laplace transform

MATLAB code

```
sys = tf([0,0,1],[1,3,2]);
rlocus(sys)
t=0:0.01:8;
y=1.*exp(-t)-1.*exp(2.*-t);
figure;
plot(t,y);
title('Step Response of the Origianl System');
xlabel('time(s)');
ylabel('y1(t)');
```

(d) In general, **lead compensation** uses PD control in order to speed up a response by **lowering rise time and decreasing the transient overshoot.** In contrast, lag uses PI control in order to improve the steady-state accuracy of the system. However, improve the steady-state accuracy is not our goal. Besides, notch compensation is often used to achieve stability for systems with lightly damped flexible modes. Our feedback system has already been stable. Therefore, notch compensation is not a suitable choice. In summary, **lead compensation is the best option.**

(e) We need to let overshoot $\leq$ 25% and rise time $\leq$ 0.4s, how to choose the value of ζ and ω_n.

overshoot $M_P = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \leq 25\% = \frac{1}{4}$ $-\frac{\pi\zeta}{\sqrt{1-\zeta^2}} \leq \ln(\frac{1}{4}) = -\ln(4)$ $-\pi\zeta \leq -\ln(4)\sqrt{1-\zeta^2}$ $\ln(4)\sqrt{1-\zeta^2} \leq \pi\zeta$ $(\ln 4)^2(1-\zeta^2) \leq \pi^2\zeta^2$ $\zeta \geq \sqrt{\frac{(\ln 4)^2}{\pi^2 + (\ln 4)^2}} = 0.4037$	(4)

rise time $t_r \cong \frac{1.8}{\omega_n} \leq 0.4$ sec $\omega_n \geq \frac{1.8}{0.4} = 4.5$	(5)

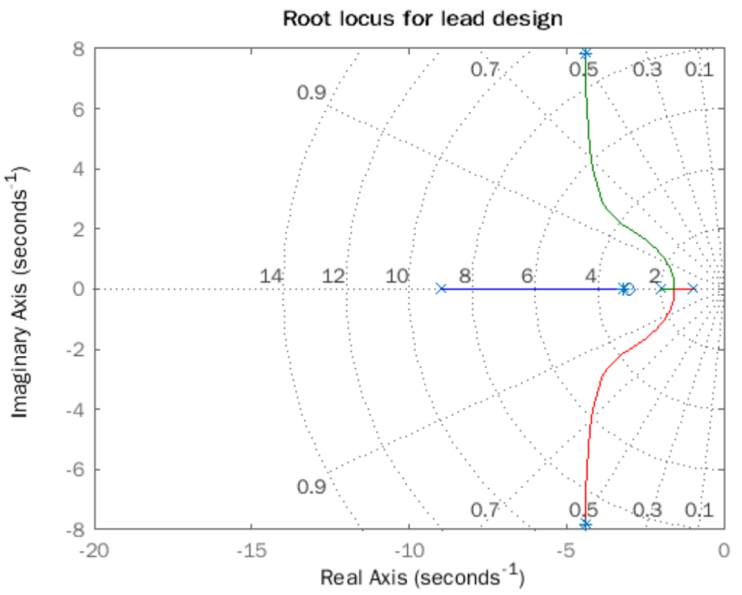
(f) Compensation with a transfer function of the form $D_C(s)= K\frac{s+z}{s+p}$. Since we need to implement **lead compensation,** so choose z<p. We choose K=80, z=3, p=9, and obtain

$D_C(s) = K\frac{s+z}{s+p} = 80\frac{s+3}{s+9}$	(6)

The characteristic equation becomes

$$1 + KL(s) = 0$$ $$1 + 80\frac{1}{(s^2+3s+2)}\frac{s+3}{s+9} = 0$$ $$(s^2+3s+2)(s+9) + 80(s+3) = 0$$ $$(s^2+3s+2)(s+9) + 80(s+3) = 0$$ $$s^3 + 3s^2 + 2s + 9s^2 + 27s + 18 + 80s + 240 = 0$$ $$s^3 + 12s^2 + 109s + 258 = 0$$	(7)

(g) Use MATLAB to get the root locus, and the answer is shown in **Figure (e).**



Figure(e): Root Locus of the new system

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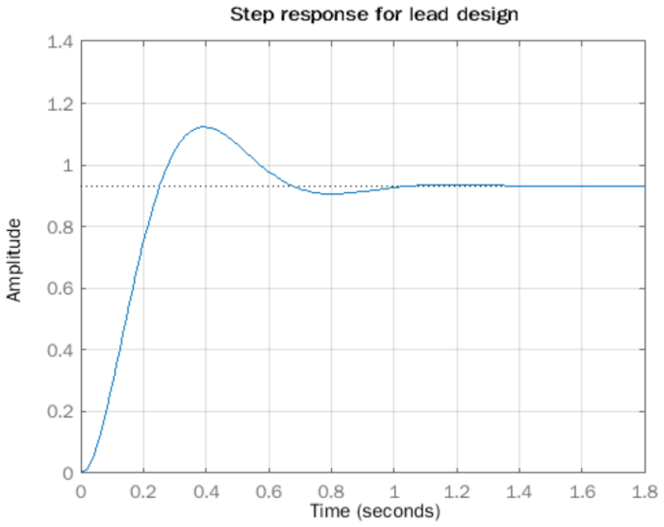
MATLAB code

```
s=tf('s');
sys4=(s+3)/((s+2)*(s+1)*(s+9));
rlocus(sys4)
axis([-20 0 -8 8])
title('Root locus for lead design')
hold on
r=roots([1 12 109 258]);
plot(r,'*')
z=0.1:.2:.9;
wn=2:2:15;
sgrid(z, wn)
```

(h) Use MATLAB to get the step response of the system, and the answer is shown in **Figure(f)**. By observing the step response before and after lead compensation (**Figure(d)** and **Figure(f)**), we can conclude that the answer is YES. The overshoot and rise time are certainly changed to meet the requirement after applying compensation.



Figure(f): step response of the new L(s) by inverse Laplace transform

MATLAB code

```
s=tf('s');
sys4=(s+3)/((s+2)*(s+1)*(s+9));
K=80;
sysCL=feedback(K*sys4,1);
step(sysCL)
grid on
axis([0 1.8 0 1.4])
title('Step response for lead design')
```

(Revised from problem 2)

2. A servomechanism position control has the plant transfer function

$$G(s) = \frac{10}{s(s+1)(s+10)}$$

You are to design a series compensation transfer function Dc(s) in the unity feedback configuration to meet the following closed loop specifications:

- The response to a reference step input is to have no more than 16% overshoot.
- The response to a reference step input is to have a rise time of no more than 0.4(sec).
- The steady-state error to a unit ramp at the reference input must be less than 0.05.

a. Find the smallest value of K for the following two lead compensators to meet the dynamic response specifications, ignoring the error requirement.

(1) $Dc_1(s) = K\frac{s+1}{s+60}$

(2) $Dc_2(s) = K\frac{s+1}{s+120}$

b. What is the relation between the pole location of Dc(s) and gain K?

c. From (a), give the respective MATLAB plots of root locus with lead compensators and the step response after specifying each K.

d. Find the respective value of p for lag compensators to be used in series with the lead you have designed to cause the system to meet the steady-state error specification. Lag compensators are in the following form,

$$Dc'(s) = \frac{s+0.4}{s+p}$$

e. Are two p values almost the same? Why or why not?

f. From (a)(d), give the respective MATLAB plots of root locus with both lead and lag compensators(p as calculated in (d)) and the step response after specifying K.

Sol.

<u>(a)-(1)</u>

Using Control System Designer in MATLAB

```
s = tf('s');
sysG = 10/(s*(s+1)*(s+10));
controlSystemDesigner(sysG);
```

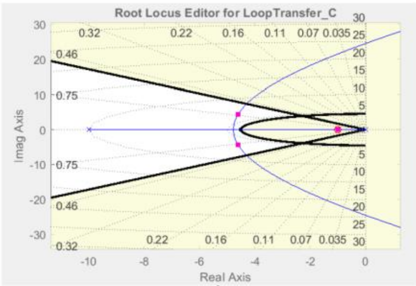
In block C, type `tf([1 1],[1 60])`.

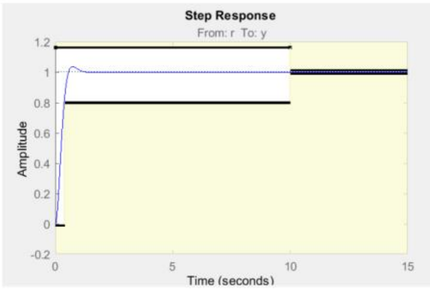
From approximation of standard second order system, we have $\zeta \geq 0.5$ and $\omega_n \geq 4.5$.

Then, specify the reasonable region in root locus.

Now, strictly changing K in locus by viewing step response to meet required t_r and M_p.

Finally, from plots and data browser, we have **K = 245**.

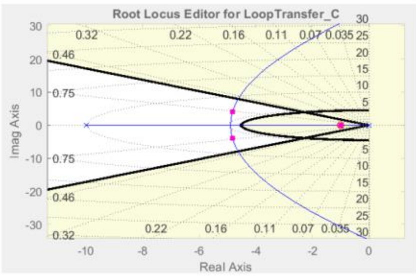


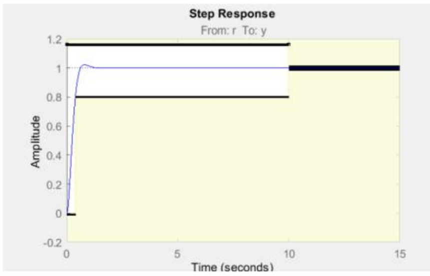


<u>(a)-(2)</u>

Similar to above, type `tf([1 1],[1 120])` in block C.

From plots and data browser, we have **K = 470**.





<u>(b)</u>

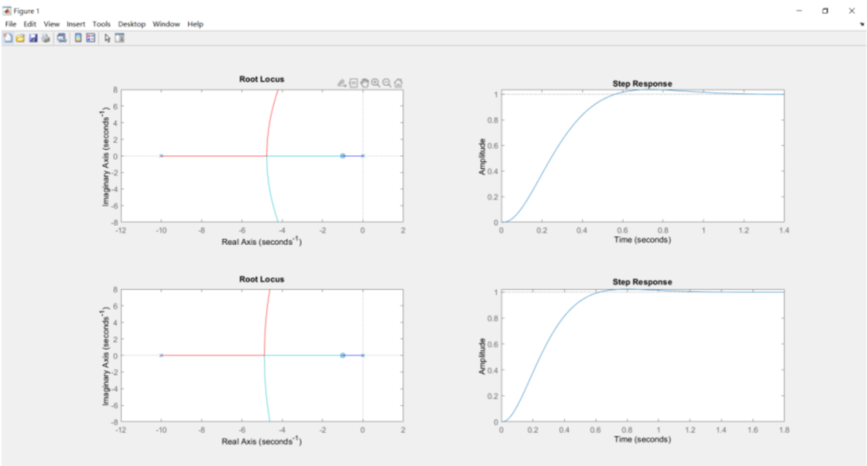
As the pole location s = -120 in (2) becomes twice as large as s = -60 in (1), the gain K = 470 in (2) becomes almost twice as large as K = 245 in (1).

(c)

MATLAB code without using Control System Designer

```
s = tf('s');
sysG = 10/(s*(s+1)*(s+10));
D1 = (s+1)/(s+60);
figure
subplot(2,2,1);
rlocus(sysG*D1);
axis([-12 2,-8,8])
sys1 = feedback(245*sysG*D1,1);
subplot(2,2,2);
step(sys1);
S = stepinfo(sys1)
```

```
D2 = (s+1)/(s+120);
subplot(2,2,3);
rlocus(sysG*D2);
axis([-12 2,-8,8])
sys2 = feedback(470*sysG*D2,1);
subplot(2,2,4);
step(sys2);
S = stepinfo(sys2)
```



Upper plots are with $Dc_1(s)$. RiseTime: 0.3495(sec) Overshoot: 3.6352 %

Lower plots are with $Dc_2(s)$. RiseTime: 0.3788(sec) Overshoot: 2.2043 %

(d)

$$\mathrm{Kv}_1 = \lim_{s\to 0} \mathrm{sGDc}_1 = \lim_{s\to 0} s\frac{10}{s(s+1)(s+10)}\frac{245(s+1)}{(s+60)} = 4.083$$

To meet requirement, we need a new Kv_1=20, which is an increase of a factor of 4.898.

We have $\boldsymbol{p_1} = \frac{\mathbf{0.4}}{\mathbf{4.898}} = \mathbf{0.0817}$.

$$\mathrm{Kv}_2 = \lim_{s\to 0} \mathrm{sGDc}_2 = \lim_{s\to 0} s\frac{10}{s(s+1)(s+10)}\frac{470(s+1)}{(s+120)} = 3.917$$

To meet requirement, we need a new Kv_2=20, which is an increase of a factor of 5.106.

We have $\boldsymbol{p_2} = \frac{\mathbf{0.4}}{\mathbf{5.106}} = \mathbf{0.0783}$.

(e)

Two p values are almost the same.

Lead compensators are used to speed up a response by lowering rise time and decreasing the transient overshoot, not to improve the state-state accuracy.

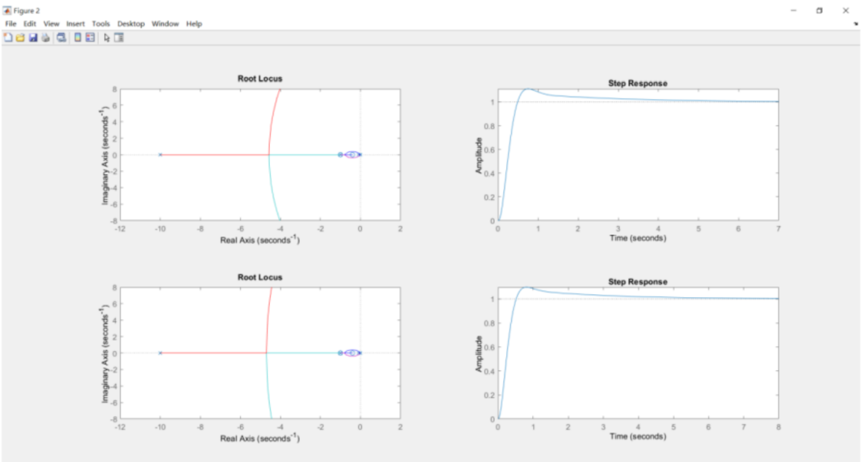
Hence, different lead compensators do not affect state-state error, which means two lag compensators used to improve the state-state accuracy should be the same.

(f)

MATLAB code without using Control System Designer, continued from (c)

```
D3 = (s+0.4)/(s+0.0817);
figure
subplot(2,2,1);
rlocus(sysG*D1*D3);
axis([-12 2,-8,8])
sys3 = feedback(245*sysG*D1*D3,1);
subplot(2,2,2);
step(sys3);
```

```
D4 = (s+0.4)/(s+0.0783);
subplot(2,2,3);
rlocus(sysG*D2*D4);
axis([-12 2,-8,8])
sys4 = feedback(470*sysG*D2*D4,1);
subplot(2,2,4);
step(sys4);
```



Upper plots are with $Dc_1(s)$ and $Dc_1'(s)$.

Lower plots are with $Dc_2(s)$ and $Dc_2'(s)$.

4/4

HW 7	Digital Control Systems, Fall 2021, NTU-EE
Name: 劉宥妤 B08901076	Date: 12/02, 2021

Ref: Feedback control of dynamic systems Chp5-4 problem 5.23

A servomechanism control system has the plant transfer function $G(s) = \frac{400}{s(s+2)(s^2+s+400)}$

Design a series compensation which will make this system satisfied following conditions:

1. Mp <= 5%
2. Rise Time <= 0.6 sec
3. Steady-state error <= 0.003

(a) Find a lead compensation to make the system meet the dynamic response specifications.

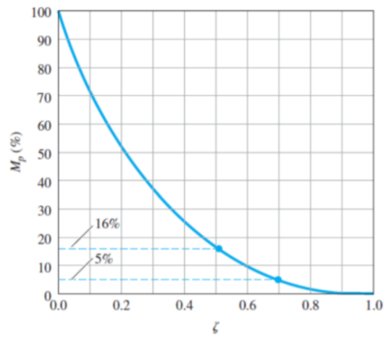
(b) What is the velocity constant K_v for your design?
Does it meet the error specification?
If not, find a lag compensation used in series with the lead you have found in (a).
Then give the Matlab plot of the root locus of the series compensation.

(c) Find a notch compensation in series with the lead and lag found in (a), (b).
Give the plot of step response with and without the notch filter you designed.

HW 7	Digital Control Systems, Fall 2021, NTU-EE
Name: 劉宥妤 B08901076	Date: 12/02, 2021

(sol.)

(a) From figure showed on the right, $\zeta \geq 0.7$,

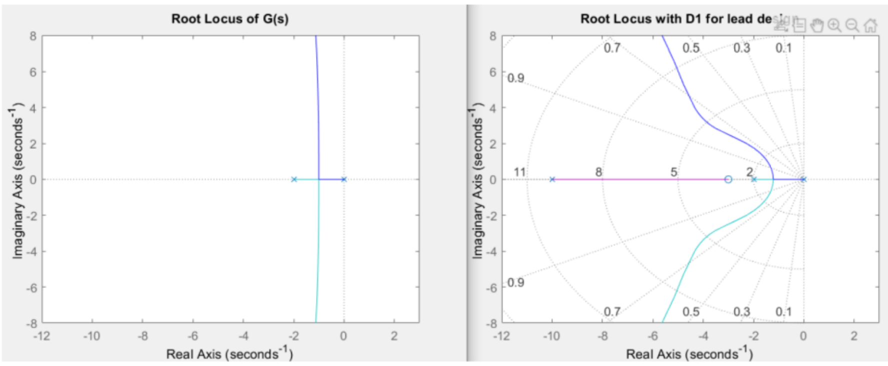


$$\omega_n \approx \frac{1.8}{0.4} = 4.5 \rightarrow \omega_n \geq 5$$

$D_{c1}(s) = K\frac{s+z_1}{s+p_1}$, $z_1 < p_1$

Choose $z_1 = 3,\ p_1 = 10$, set K = 1 first.

The root loci are showed in following figure.



And then we choose K, from 'Root Locus for lead design', choose K = 45 can satisfy the condition. $D_{c1}(s) = 45 \cdot \frac{s+3}{s+10}$

2

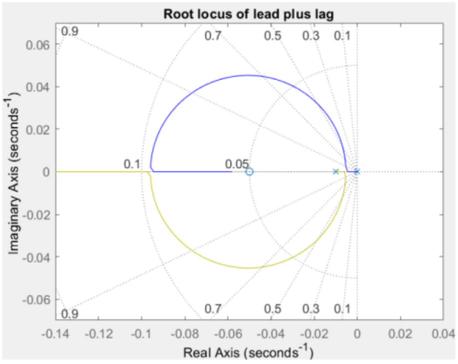
HW 7	**Digital Control Systems, Fall 2021, NTU-EE**
Name: 劉宥妤 B08901076	**Date: 12/02, 2021**

(b) $K_v = \lim_{s\to 0} sGD_{c1} = \lim_{s\to 0} s \cdot \frac{1600}{s(s+2)(s^2+s+1600)} \cdot 45 \cdot \frac{s+3}{s+10} = 67.5 \le \frac{1}{0.003} \approx 333$

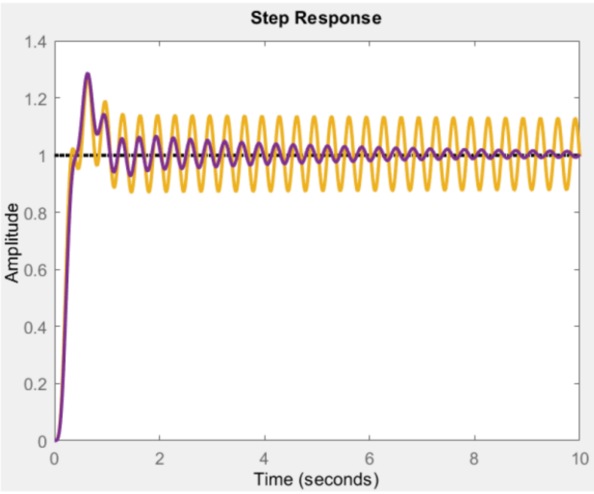
To meet the error specification, K_v should be larger than 333, which is an increase of

factor of 5, so we need a lag compensator $D_{c2}(s) = \frac{s+z_2}{s+p_2}$, where $z_2 > p_2$

choose $z_2 = 0.05,\ p_2 = 0.01$



(c) $D_{notch}(s) = \frac{s^2+2\zeta\omega_0 s+\omega_0{}^2}{(s+\omega_0)^2} = \frac{s^2+2\cdot 0.7\cdot 21^2 s+21^2}{(s+21)^2}$

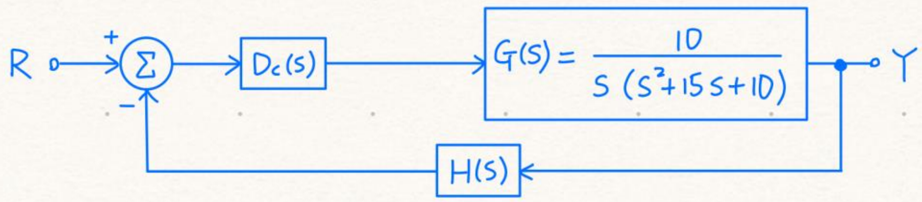


討論：
當 notch 設計完成後震盪還持續一陣子，這算是一個好的設計嗎？
（即使已經遠小於原本的情形）

3

Control System : Homework 07 B08901111 電機三 簡宏哲

For the system below :



(a) We design a lead compensation ,

let $H(S) = 1$, $D_c(S) = K \frac{S+Z}{S+5Z}$, $K > 0$, please find the minimum of K

such that the velocity constant is more than (or equal) 10 , i.e., $K_v \geq 10$.

(b) We design a lag compensation ,

let $H(S) = 1$, $D_c(S) = K \frac{S+5P}{S+P}$, $K > 0$, please find the minimum of K

such that the velocity constant is more than (or equal) 10 , i.e., $K_v \geq 10$.

(c) If we use $K = 100$ in (a), please draw the root locus versus Z $(Z > 0)$

(d) If we use $K = 100$, $Z = 4$ in (a) , please find the dominant roots

(the roots nearest the origin) of the closed-loop system and find the

corresponding damping ratio (ζ) and overshoot (M_p) .

(e) We design a tachometer feedback ,

let $H(S) = 1 + K_T S$, $K_T > 1$, $D_c(S) = K$, $K > 0$, please show that

the closed-loop system is always stable using Routh's Stability

Criterion .

Solutions :

(a) We use $K_v = \lim_{s \to 0} s\, D_c(s)\, G(s) = \lim_{s \to 0} s \cdot K \frac{s+z}{s+5z} \cdot \frac{10}{s(s^2+15s+10)}$

$= \frac{K}{5} \geq 10 \Rightarrow K \geq 50$, so the minimum K is 50

(b) We use $K_v = \lim_{s \to 0} s\, D_c(s)\, G(s) = \lim_{s \to 0} s \cdot K \frac{s+5p}{s+p} \cdot \frac{10}{s(s^2+15s+10)}$

$= 5K \geq 10 \Rightarrow K \geq 2$, so the minimum K is 2

(c) For $K=100$, we have $1+100\,\frac{s+Z}{s+5Z}\cdot\frac{10}{s(s^2+15s+10)}=0$

$\Rightarrow 1+Z\,\frac{1000+5s(s^2+15s+10)}{1000s+s^2(s^2+15s+10)}=0$

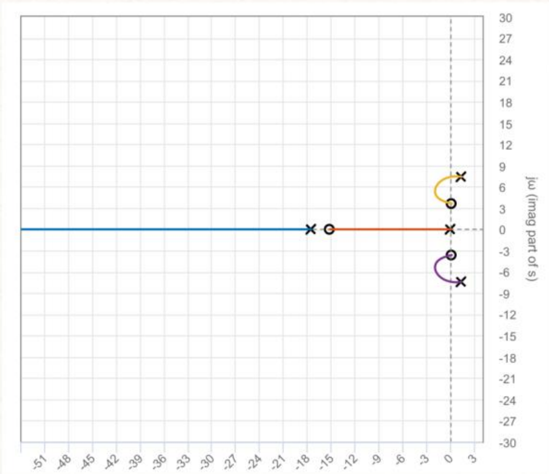


Fig for (C)

Matlab Code for (C)

```
s = tf( 's' )
sysL =(1000+5*s*(s^2+15*s+10))/
(1000*s+s*s*(s^2+15*s+10));
rlocus( sysL );
[ K, p ] = rlocfind( sysL );
```

(d) For $K=100$, $Z=4$, the roots are -23.8 and $-1.39\pm4.24j$

, so the dominant roots are $-1.39\pm4.24j$

$\Rightarrow \sin^{-1}\zeta=\tan^{-1}\frac{1.39}{4.24}=0.317 \Rightarrow \zeta=0.312 \Rightarrow M_p \fallingdotseq 35\%$

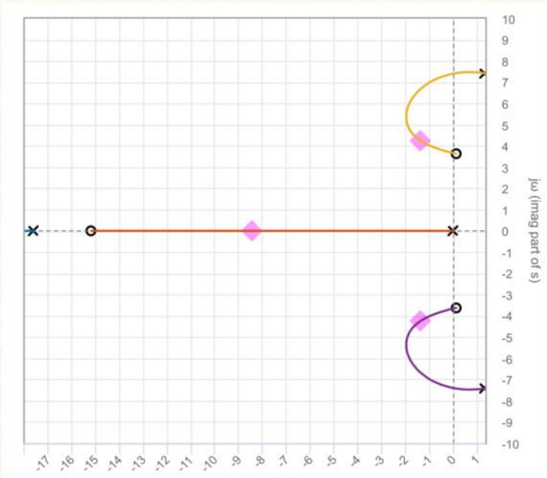


Fig for (D)

(e) The characteristic equation is

$$1 + H(s) D_c(s) G(s) = 1 + (1 + K_T s) K \frac{10}{s(s^2+15s+10)} = 0$$

$$\Rightarrow s^3 + 15s^2 + (10 + 10KK_T)s + 10K = 0$$

Routh's Stability Criterion :

1	$10+10KK_T$
15	$10K$
a_1	0
a_2	0

Where $a_1 = -\frac{1}{15}\begin{vmatrix} 1 & 10+10KK_T \\ 15 & 10K \end{vmatrix} = \frac{1}{15}\left(150 + K(150K_T - 10)\right)$,

$$a_2 = -\frac{1}{a_1}\begin{vmatrix} 15 & 10K \\ a_1 & 0 \end{vmatrix} = 10K.$$

$\therefore$ When $K > 0$, $K_T > 1$, we have $1, 15, a_1, a_2 > 0$, so the system is stable.

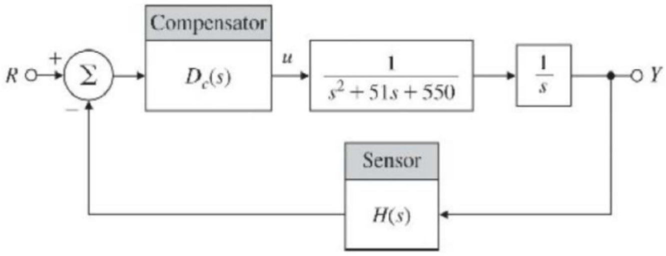
楊子毅 B08901190

HW 7: for Units 5C, 5D, 5E, 5F: Root Locus	Digital Control Systems, Fall 2019, NTU-EE
Name: 楊子毅	Assigned: November 19, 2021

修改自 Problem 3

1. (U5D: Lead-Lag Compensator)

40. Consider the instrument servomechanism with the parameters given in Fig. 5.69. For each of. the following cases, draw a root locus with respect to the parameter K, and indicate the location of the roots corresponding to your own design.



(a) Lead network : Let H(s)=1; D(s)= $K\frac{s+z}{s+p}$ $\frac{p}{z}$=6

Select z and K so that the roots nearest the origin (the dominant roots) yield.

$\zeta \geq 0.4, -\sigma \leq -7, K_v \geq 16\frac{2}{3}sec^{-1}$

(note: 這題與作業的(a)完全相同，但我算出來的答案跟給的解答不一樣)

(b) 承接(a)小題，若 $\frac{p}{z}$= 12 其他條件相同 選取此時的 z 與 K

(c) 承接(a)小題，若 $\frac{p}{z}$= 3 其他條件相同 選取此時的 z 與 K

(d) 由(a),(b),(c)是否有看出什麼規律呢 (about damping and overshoot)

(e) 已知下面四張圖為 H(s)=1;

D(s)分別為：$13750\frac{s+17}{s+25.5}$ $27500\frac{s+17}{s+51}$ $55000\frac{s+17}{s+102}$ $110000\frac{s+17}{s+204}$ 時所繪製出的 step response

(順序已打亂)

圖 a

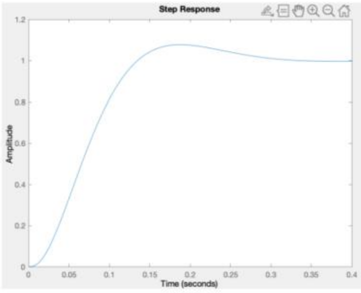


圖 b

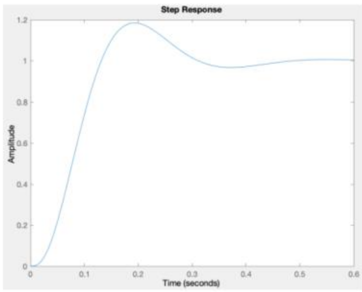


圖 c

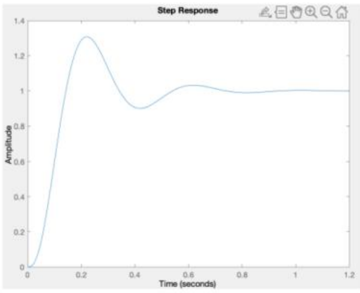
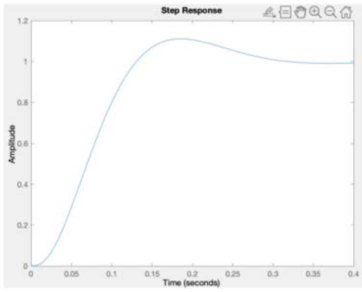


圖 d



請問圖幾為 D(s)=13750$\frac{s+17}{s+25.5}$ 所得之 step response

(f) 承接(e)小題 請問圖幾為 D(s)= 27500$\frac{s+17}{s+51}$ 所得之 step response

Solution:

(a) Setting p = 6z

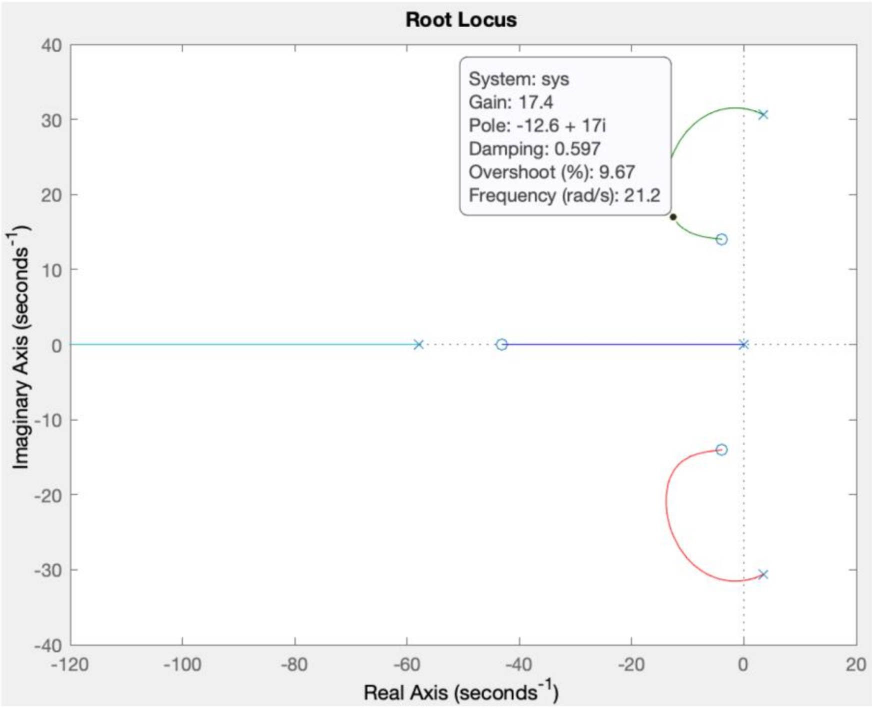
$$K_v = \lim_{s\to 0}(sK * \frac{s+z}{s+6z} * \frac{1}{s} * \frac{1}{s^2+51s+550}) = \frac{K}{6*550} = \frac{K}{3300}$$

From 條件$K_v \geq 16\frac{2}{3}sec^{-1}$ 可得 $K \geq K_v \geq 16\frac{2}{3} * 3300 = 55000$ （原始答案給 35200?）

將上式湊成 1+zL(s)的形式 1+z$\frac{[6s(s^2+51s+550)+K}{s^2(s^2+51s+550)+Ks}$

Let K=55000 1+zL(s) = 1+z$\frac{[6s(s^2+51s+550)+55000}{s^2(s^2+51s+550)+55000s}$

用 matlab 畫出上式對應的 root locus 後



(附上程式碼)

```
s=tf('s');
sys = ( 6*s*(s^2+51*s+550)+55000 )/( s^2*(s^2+51*s+550)+55000*s )
rlocus(sys)
```

用 damping ratio 最大那點當解：得 z = 17.4 (p = 17.4*6 = 104.4)

因此$D_c(s)$ =55000$\frac{s+17.4}{s+104.4}$

3

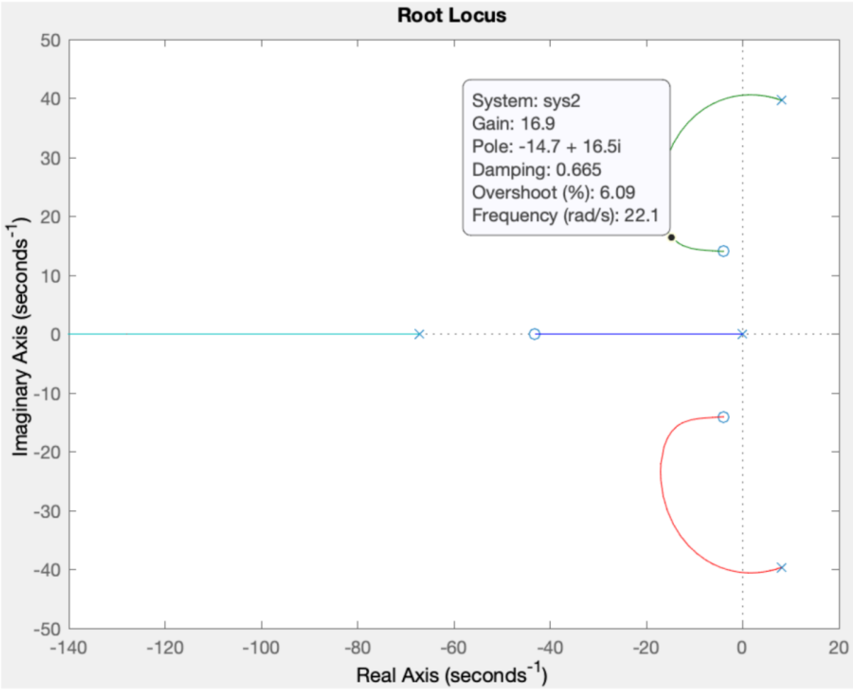
(b) Setting p = 12z

$$K_v = \lim_{s\to 0}(sK * \frac{s+z}{s+12z} * \frac{1}{s} * \frac{1}{s^2+51s+550}) = \frac{K}{12*550} = \frac{K}{6600}$$

From 條件$K_v \geq 16\frac{2}{3}sec^{-1}$ 可得 $K \geq K_v \geq 16\frac{2}{3} * 6600 = 110000$

Let K=110000 1+zL(s) = 1+z$\frac{[12s(s^2+51s+550)+110000}{s^2(s^2+51s+550)+110000s}$

用 matlab 畫出上式對應的 root locus 後



(附上程式碼)

```
s=tf('s');
sys2 = ( 12*s*(s^2+51*s+550)+110000 )/( s^2*(s^2+51*s+550)+110000*s )
rlocus(sys2)
```

用 damping ratio 最大那點當解：得 z = 16.9 (p = 16.9*12 = 104.4)

因此$D_c(s)$ =110000$\frac{s+17.4}{s+202.8}$

4

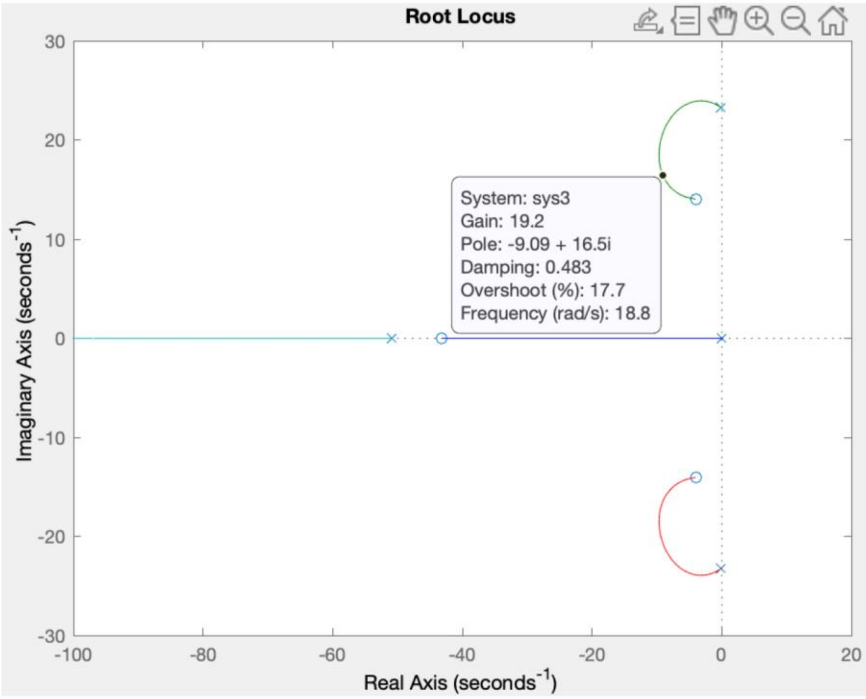
(c) Setting p = 3z

$$K_v = \lim_{s \to 0}(sK * \frac{s+z}{s+6z} * \frac{1}{s} * \frac{1}{s^2+51s+550}) = \frac{K}{3*550} = \frac{K}{1650}$$

From 條件$K_v \geq 16\frac{2}{3} sec^{-1}$ 可得 $K \geq K_v \geq 16\frac{2}{3} * 1650 = 27500$

Let K=27500　1+zL(s) = 1+z$\frac{[3s(s^2+51s+550)+27500}{s^2(s^2+51s+550)+27500s}$

用 matlab 畫出上式對應的 root locus 後



(附上程式碼)

```
s=tf('s');
sys3 = ( 3*s*(s^2+51*s+550)+27500 )/( s^2*(s^2+51*s+550)+27500*s )
rlocus(sys3)
```

用 damping ratio 最大那點當解：得 z = 19.2　(p = 19.2*3 = 57.6)

因此$D_c(s)$ =27500$\frac{s+19.2}{s+57.6}$

(d) 可以發現 當 $\frac{p}{z}$ 越來越大的時候 overshoot 越來越小

damping ratio 越來越大（for damping ratio 最大的點）

(e) 圖 c

$13750\frac{s+17}{s+25.5}$ 為給定四個 D(s)當中 $\frac{p}{z}$ 最小的(等於 1.5)

因此圖為 damping ratio 最小的 也就是震動最明顯的

(f) 圖 b

$27500\frac{s+17}{s+51}$ 為給定四個 D(s)當中 $\frac{p}{z}$ 第二小的(等於 3)

因此圖為 damping ratio 第二小的 也就是震動第二大的

（BTW 圖 d 為 $55000\frac{s+17}{s+102}$ 圖 a 為 $110000\frac{s+17}{s+204}$）

(附上畫出那四張圖的 code)

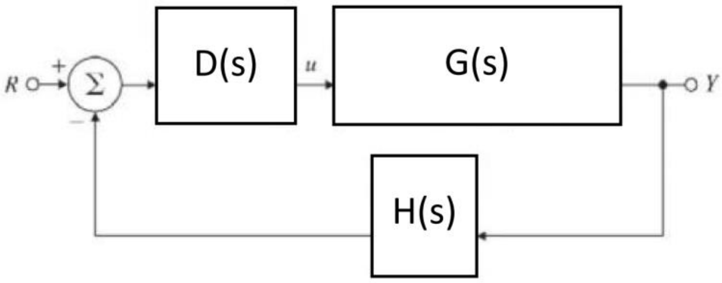
```
s=tf('s');
D1 = 13750 * (s+17) / (s+25.5)
forward1 = D1 * 1/(s*(s^2+51*s+550))
step(forward1/(1+forward1))
stepinfo(forward1/(1+forward1))

D2 = 27500 * (s+17) / (s+51)
forward2 = D2 * 1/(s*(s^2+51*s+550))
step(forward2/(1+forward2))
stepinfo(forward2/(1+forward2))

D3 = 55000 * (s+17) / (s+102)
forward3 = D3 * 1/(s*(s^2+51*s+550))
step(forward3/(1+forward3))
stepinfo(forward3/(1+forward3))

D4 = 110000 * (s+17) / (s+204)
forward4 = D4 * 1/(s*(s^2+51*s+550))
step(forward4/(1+forward4))
stepinfo(forward4/(1+forward4))
```

B0890108 黃靖元

CS-HW7

改編自第 3 題：

Consider the instrument servomechanism with the parameters given below . For each of the following cases, draw a root locus with respect ot the parameter K, and indicate the location of the roots corresponding to your final design.



(a) Briefly explain what lead, lag and notch compensation will do to the system. And what's is their pros and cons?

(b) As you could find in (a), use lead compensator to fulfill the following requirement:

$$\mathrm{H(s)} = 1\,,\mathrm{G(s)} = \frac{1}{s(s+2)}\ \mathrm{D}(s) = K\frac{s+z_1}{s+p_1},\ \frac{z_1}{p_1} = \frac{1}{4}$$

$$\text{Overshoot} \le 10\%\,,\text{Rise time} \le 0.4\ \text{second}$$

Draw the root locus with the constrain and give your z_1, p_1 and K.

Draw the step response and check whether the overshoot and rise time fulfill the statement or not.

Also find the velocity constant after adding the lead compensator.

(c) After adding lead compensator in (b), we find that the steady-state error is too big for us, so as you could find in (a), use lag compensator to fulfill the following requirement:

$$\mathrm{H(s)} = 1\,, \mathrm{G(s)} = \frac{1}{s(s+2)}\ \ \mathrm{D}(s) = K\frac{s+z_1}{s+p_1}\frac{s+z_2}{s+p_2}$$

$$reduce\ velocity\ constant\ by\ a\ factor\ of\ 10$$

$$picking\ the\ lag\ compensation\ zero\ at\ \ -0.1$$

Draw the root locus and give your $z_2,\ p_2$.

(d) Congratulation! You have almost finished designing a system. But there is still a small problem need to be fixed. After real testing, we find a small oscillation in the system.

$$\mathrm{H(s)} = 1\,, \mathrm{G(s)} = \frac{1600}{s(s+2)(S^2+S+1600)}$$

$$\mathrm{D}(s) = K\frac{s+z_1}{s+p_1}\frac{s+z_2}{s+p_2} \times notch\ compensator$$

Design your notch compensator and show the step response before and after adding the notch compensator.

Solution :

(a) Lead compensator : use for speed up a response

- Advantages : Increases response speed and bandwidth and increases the phase margin
- Disadvantages : Steady state error is not improved

Lag compensator : use for improve steady-state error

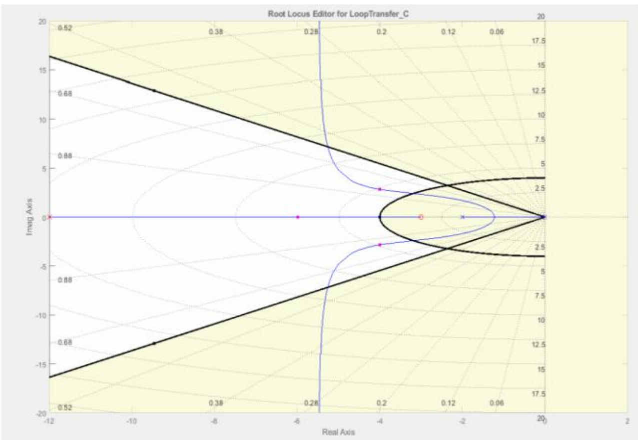
- Advantages : Steady state error is improved
- Disadvantages : decreases the bandwidth and the speed of response

Notch compensator : use for eliminate lightly damped flexible modes

- Advantages : Can eliminate small oscillation
- Disadvantages : Would not have unity gain at zero frequency, and the notch will not be sharp

(b)

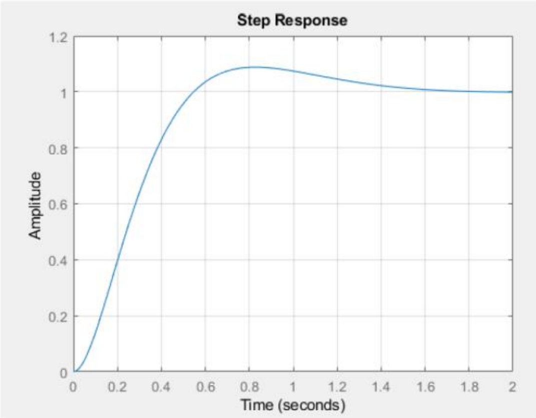
1. Pick $z_1 = 3, p_1 = 12$
2. Use sisotool and adding constrain
   (1) $\text{overshoot} \leq 10\%$
   (2) and by computation $w_n \geq \frac{1.8}{0.45} = 4$



Pick K inside the range, and this time we pick K = 40

$$K_v = \lim_{s \to 0} s \times 40 \times \frac{s+3}{s+12} \times \frac{1}{s(s+1)} = 10$$

Using Matlab to draw step response of the system:




```
S =

  struct with fields:

        RiseTime: 0.3674
    SettlingTime: 1.5633
     SettlingMin: 0.9046
     SettlingMax: 1.0881
       Overshoot: 8.8109
      Undershoot: 0
            Peak: 1.0881
        PeakTime: 0.8224
```

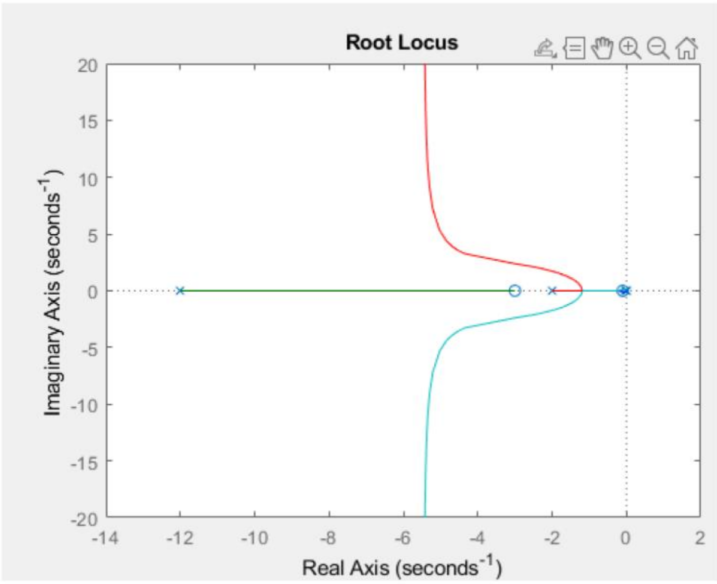
And the overshoot is 8.8109% and risetime is 0.3674 second
Both fulfill the statement.

(c)

1. To reduce velocity constant by a factor of 10, that is, $K_v = 100$

   we need $\frac{z_2}{p_2} = 10$

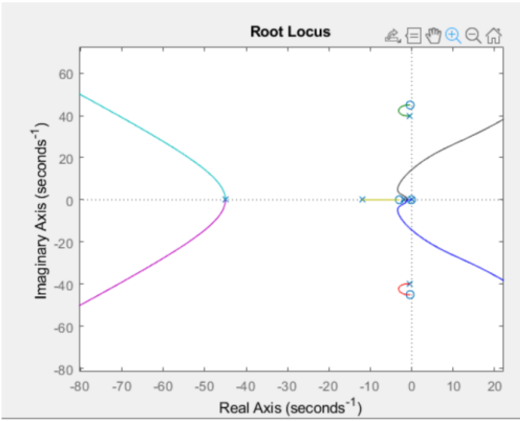
   Since zero has been determined, we pick $z_2 = -0.1$, $p_2 = -0.01$



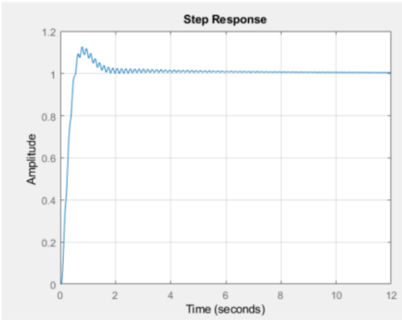
(d) For the root locus of to be in LHP, we can only pick notch compensator with frequency higher than 40.

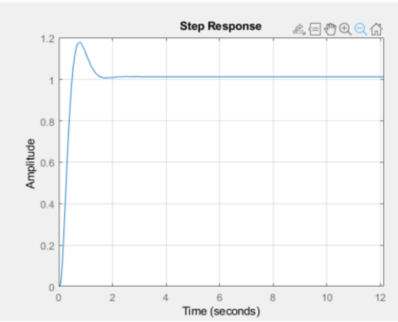
So we pick notch compensator = $\frac{s^2+0.8s+45^2}{(s+45)^2}$

The root locus is shown below :



The step response before and after adding the notch compensator :





We can see that the oscillation is almost eliminate by the notch compensator.