Problem Set of HW06 for Units 5A, 5B: Root Locus, Control Systems

November 26, 2021

1. (U5B: Root Locus)

3. For the characteristic equation

$$1 + \frac{K}{s^2(s+1)(s+5)} = 0,$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \to \infty$.

(c) Sketch the locus.

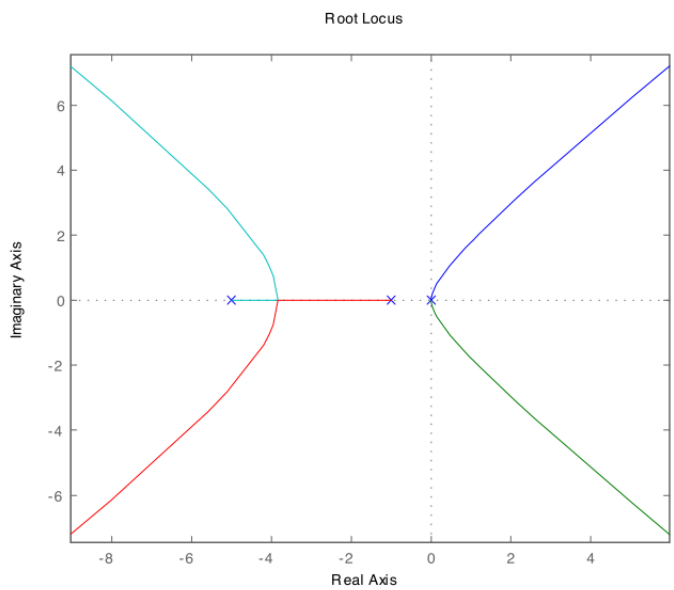
(d) Verify your sketch with a Matlab plot.

Solution:

(a) The real axis segment is $-1 > \sigma > -5$.

(b) $\alpha = -6/4 = -1.5$; $\phi_i = \pm 45°, \pm 135°$

(c) The plot is shown below.



2. (U5B: Root Locus)

4. *Real poles and zeros.* Sketch the root locus with respect to K for the equation $1+KL(s)=0$ and the listed choices for $L(s)$. Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

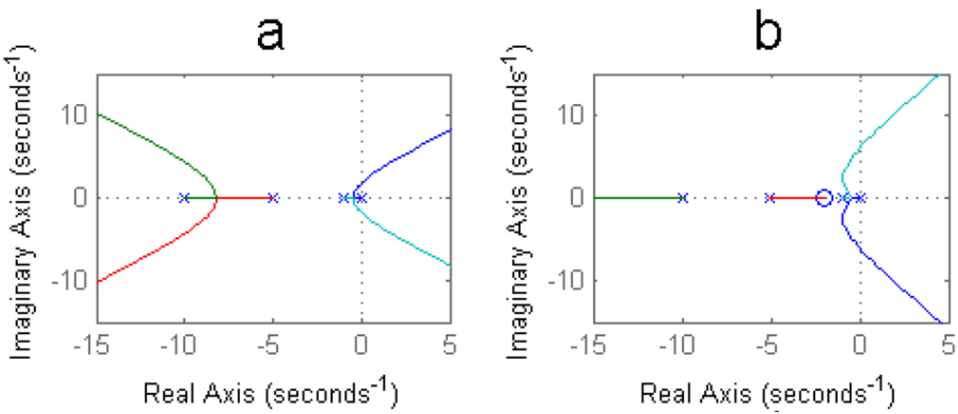
   (a) $L(s) = \dfrac{2}{s(s+1)(s+5)(s+10)}$

   (b) $L(s) = \dfrac{(s+2)}{s(s+1)(s+5)(s+10)}$

Solution:
All the root locus plots are displayed at the end of the solution set for this problem.

(a) $\alpha = -4$; $\phi_i = \pm 45°, \pm 135°$

(b) $\alpha = -4.67$; $\phi_i = \pm 60°, 180°$



3. (U5B: Timing Property and Root Locus)

13. For the system in Fig. 5.53,

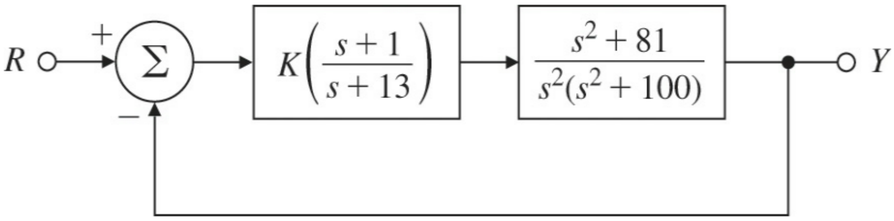


Fig. 5.53 Feedback system for Problem 5.13

(a) Find the locus of closed-loop roots with respect to K.

(b) Is there a value of K that will cause all roots to have a damping ratio greater than 0.5?

(c) Find the values of K that yield closed-loop poles with the damping ratio $\zeta = 0.707$.

(d) Use Matlab to plot the response of the resulting design to a reference step.

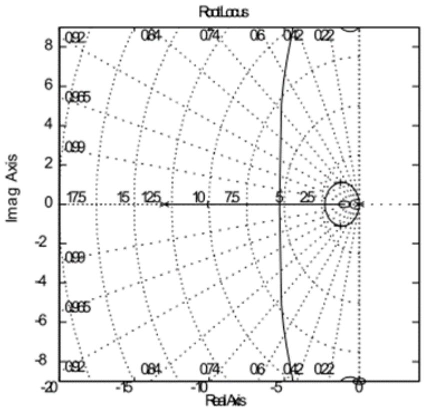
Solution:

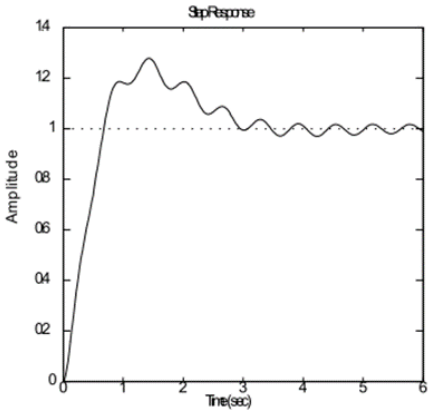
(a) The locus is plotted below

(b) There is a K which will make the 'dominant' poles have damping 0.5 but none that will make the poles from the resonance have that much damping.

(c) Using rlocfind, the gain is about 35.

(d) The step response shows the basic form of a well damped response with the vibration of the response element added.





Homework 06 for Units 5A, 5B: Root Locus, Control Systems

Digital Control System, Fall 2021, NTU-EE

Name: B07901004 電機四 陳恩庭 Date: 11/17, 2021

Problem 1 (revised)

For the characteristic equation

$$1+\frac{K}{s^2(s+5)(s+7)}=0$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K \rightarrow \infty$.

(c) Sketch the locus.

(d) Verify your sketch with a Matlab plot.

(e) Modify the transfer function L(s) by adding an extra pole at s = 0 such that $L(s)=\frac{1}{s^3(s+5)(s+7)}$. Do the same discussion as in (a)~(c), sketch (or use Matlab) to plot the locus, and describe your observation.

(f) Modify the transfer function L(s) by adding a zero at s = -3 such that $L(s)=\frac{s+3}{s^2(s+5)(s+7)}$. Do the same discussion as in (a)~(c), plot the locus, and describe your observation.

(g) How about adding two zeros at s = -3 such that $L(s)=\frac{(s+3)^2}{s^2(s+5)(s+7)}$? What would the locus be like?

Solution:

Use the rules for determining a positive root locus.

Rule 1: The n branches of the locus start at the poles of L(s) and m branches end on the zeros of L(s).

Consider the original characteristic function, there are **four poles at s = 0, 0, -5, -7**, respectively. And there are no zeros.

For (a), (b), (c), we discuss the property of the root locus first and use them together to sketch one picture after (c).

(a)

Rule 2: The loci are on the real axis to the **left** of an **odd** number of poles and zeros.

Following this rule, the real-axis segment of the corresponding root locus is

$\mathbf{-5 > \sigma > -7}$.

(b)

Rule 3: For large s and K, n-m branches of the loci are asymptotic to lines at angles $\emptyset$ radiating out from the point $s = \alpha$ on the real axis, where

$$\emptyset_l = \frac{180^\circ + 360^\circ(l-1)}{n-m}, l = 1, 2, \dots, n-m,$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m}.$$

In this case, n = 4, m = 0. Thus,

$$\emptyset_l = \pm 45^\circ, \pm 135^\circ,$$

$$\alpha = \frac{0 + 0 + (-5) + (-7)}{4} = -\frac{12}{4} = -3.$$

There are 4 asymptotes centered at $\boldsymbol{\alpha = -3}$ and at the angles $\mathbf{\pm 45^\circ, \pm 135^\circ}$.

(c)

Rule 4: The angles of departure of a branch of the locus from a pole of multiplicity q is given by

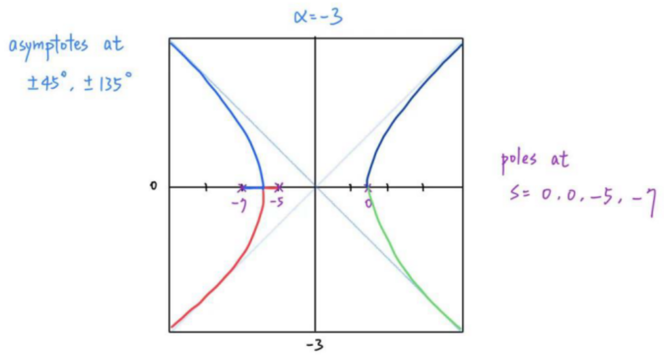
$$q\emptyset_{l,dep} = \sum \psi_i - \sum \emptyset_i - 180^\circ - 360^\circ(l-1),$$

where l = 1, 2, …, q and the angles of arrival of a branch at a zero of multiplicity q is given by

$$q\psi_{l,arr} = \sum \emptyset_i - \sum \psi_i + 180^\circ + 360^\circ(l-1).$$

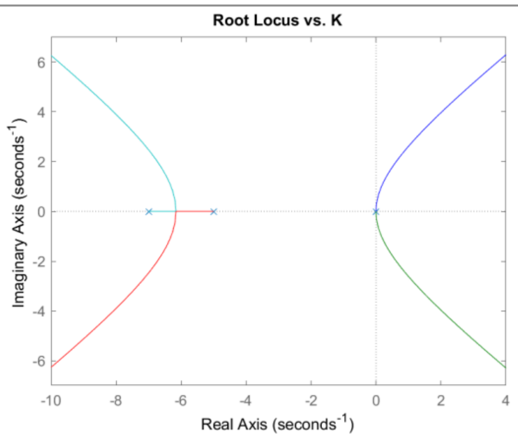
The branches depart from the poles at s = 0 (multiplicity 2) at the angles $\pm 90^\circ$. Another branch departs from s = -5 (multiplicity 1) at the angles 180°, and the other branch departs from s = -7 (multiplicity 1) at the angles 0°.

- Using the discussion above, we can sketch the root locus as below.



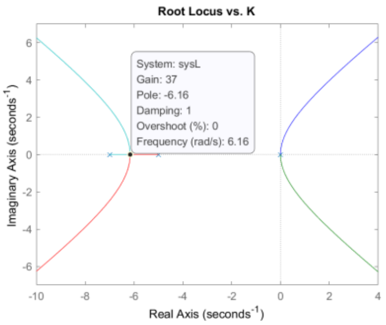
(d) Verify our sketch with Matlab plot.

Matlab commands:	
s=tf('s');	
sysL = 1/(s^2*(s+5)*(s+7));	**% modified in (e)~(g)**
rlocus(sysL);	
axis([-10 4 -7 7])	% optional
hold on	% optional
title('Root Locus vs. K')	% optional
Result:	



It shows that two branches of the locus break vertically from the poles at s = 0, curve to the right and approach the asymptotes at the angles $\pm 45^{\circ}$.

As one branch breaks from the pole at s = -5 going left meets the other one breaks from the pole at s = -7 going right, they form a multiple root at approximately s = -6.16 and break away there at $\pm 90^{\circ}$ (by rule 5, $\frac{180^{\circ}+360^{\circ}(l-1)}{2}$) and approach the asymptotes at the angles $\pm 135^{\circ}$.



We can see that our sketch is similar to the Matlab plot, while using Matlab, we can easily find out more details about the paths, for example, the accurate location of the multiple roots.

(e) Following Rule 2, the real-axis segment of the corresponding root locus is $\mathbf{0 > \sigma > -5 \text{ and } \sigma < -7}$.

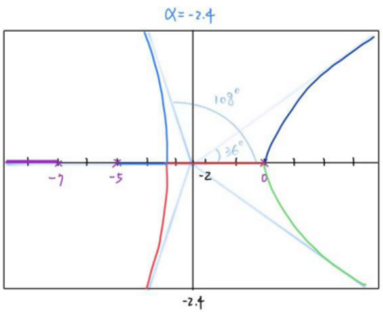
Consider Rule 3, after the modification, n = 5, m = 0. Thus,

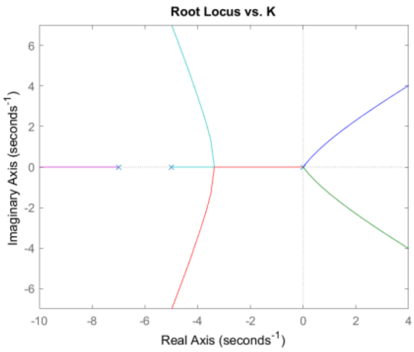
$$\emptyset_l = \pm 36^{\circ}, \pm 108^{\circ}, 180^{\circ},$$

$$\alpha = \frac{0+0+0+(-5)+(-7)}{5} = -\frac{12}{5} = -2.4\,.$$

There are 5 asymptotes centered at $\boldsymbol{\alpha = -2.4}$ and at the angles $\boldsymbol{\pm 36^{\circ}, \pm 108^{\circ}, 180^{\circ}}$.

By Rule 4, the branches depart from the poles at s = 0 (multiplicity 3) at the angles $\pm 60^{\circ}, 180^{\circ}$. Another branch departs from s = -5 (multiplicity 1) at the angles 0°, and the other branch departs from s = -7 (multiplicity 1) at the angles 180°.





✧ After adding a pole at s = 0, the number, the center, and also the angles of asymptotes are changed. Besides, there are now two real-axis segments, and the multiple root which is originally between -5 and -7 is then located between -5 and 0.

(f) Following Rule 2, the real-axis segment of the corresponding root locus is $\mathbf{-3 > \sigma > -5 \text{ and } \sigma < -7}$.

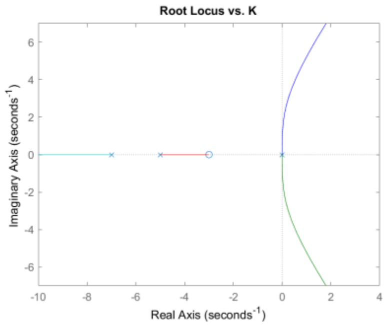
Consider Rule 3, after the modification, n = 4, m = 1. Thus,

$$\emptyset_l = \pm 60^\circ, 180^\circ,$$

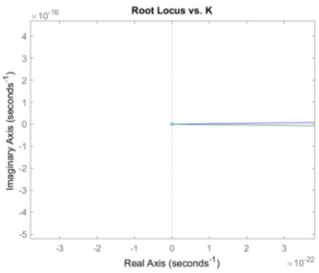
$$\alpha = \frac{0 + 0 + (-5) + (-7) - (-3)}{3} = -\frac{9}{3} = -3\,.$$

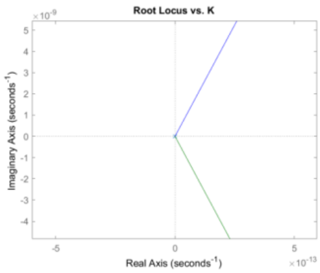
There are 3 asymptotes centered at $\boldsymbol{\alpha = -3}$ and at the angles $\boldsymbol{\pm 60^\circ, 180^\circ}$.

By Rule 4, the branches depart from the poles at s = 0 (multiplicity 2) at the angles 0°. The branch arrives at the zero at s = -3 (multiplicity 1) at the angles 180°. Another branch departs from s = -5 (multiplicity 1) at the angles 0°, and the other branch departs from s = -7 (multiplicity 1) at the angles 180°.



✧ Using Matlab to plot the locus, at first view, the paths are in line with our expectations except for the angles of the branches depart from the poles at s = 0, which seems to be $\pm 90^\circ$ instead of 0°. However, if we scale up the plot, we can get the figures below, finding out that the angles are truly 0° near the poles at s = 0, matching up to our discussion, and they rotates to $\pm 90^\circ$ quickly as s increases.





According to the observation, in this case, it is a little hard for us to sketch the locus by hand without knowing this characteristic of the paths.

(g) Following Rule 2, the real-axis segment of the corresponding root locus is $\mathbf{-5 > \sigma > -7}$.

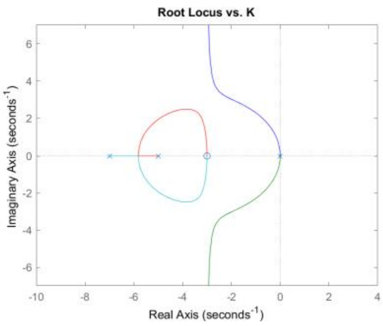
Consider Rule 3, after the modification, n = 4, m = 2. Thus,

$$\emptyset_l = \pm 90^\circ,$$

$$\alpha = \frac{0+0+(-5)+(-7)-(-3)-(-3)}{2} = -\frac{6}{2} = -3\,.$$

There are 2 asymptotes centered at $\boldsymbol{\alpha = -3}$ and at the angles $\boldsymbol{\pm 90^\circ}$.

By Rule 4, the branches depart from the poles at s = 0 (multiplicity 2) at the angles $\pm 90^\circ$. The branches arrive at the zero at s = -3 (multiplicity 2) at the angles $\pm 90^\circ$. Another branch departs from s = -5 (multiplicity 1) at the angles 180°, and the other branch departs from s = -7 (multiplicity 1) at the angles 0°.



✧ In this case, although we can get a few properties of the locus by the rules, it is actually not sufficient for us to sketch the paths to a certain degree of accuracy, especially the curves to the zero. Thus it is better and quite more convenient to use Matlab to see what the locus truly looks like.

陳杰伸 **B08502048**

1. For the characteristic equation of system1

$$1+\frac{K}{s^2(s+1)(s+5)(s+7)}=0$$

(a) Draw the real-axis segments of the corresponding root locus.

(b) Sketch the asymptotes of the locus for $K\rightarrow\infty$.

(c) Sketch the locus.

(d) Verify your sketch with a MATLAB plot.

(e) Now, consider characteristic equation of system2

$$1+\frac{K}{s^2(s+1)(s+5)}=0$$

Overlap the root locus of two systems using MATLAB and find the difference.

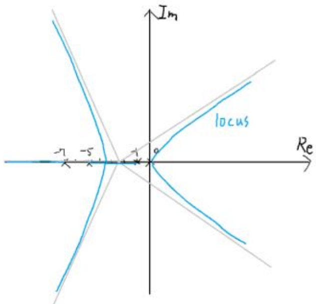
Sol.

(a) L(s) poles: $s=-7,-5,-1,0,0$

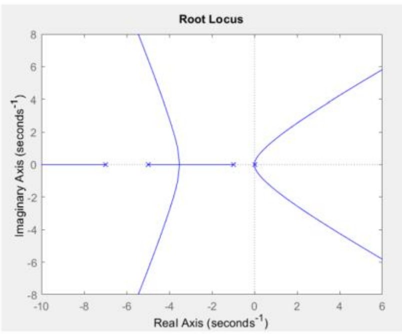
The real axis segment is $-5<\sigma<-1, \sigma<-7$

(b) $\alpha_1=\frac{-13}{5}=-2.6$, $\varphi_1=\frac{180^\circ+360^\circ k}{5}=36^\circ, 108^\circ, 180^\circ, 252^\circ, 324^\circ$

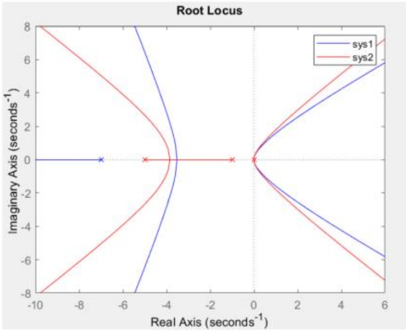
(c) The plot is shown below including (a), (b)



(d) Using MATLAB



(e)



L(s) in system1 with one more pole at $s = -7$

Lead to additional real-axis segment of $\sigma < -7$

Also, the extra pole causes changes in α and φ

$$\begin{cases} \alpha_1 = \dfrac{-6-7}{4+1} & \varphi_1 = \dfrac{180^\circ + 360^\circ k}{4+1} \\ \alpha_2 = \dfrac{-6}{4} & \varphi_2 = \dfrac{180^\circ + 360^\circ k}{4} \end{cases}$$

※MATLAB code for (d)(e)

```
s = tf('s');
figure
sys1 = 1/((s^2)*(s+1)*(s+5)*(s+7));
rlocus(sys1,'b');
axis([-10, 6, -8, 8]);

figure
sys2 = 1/((s^2)*(s+1)*(s+5));
rlocus(sys1,'b',sys2,'r');
axis([-10, 6, -8, 8]);
hold on
legend('sys1','sys2');
hold off
```

HW 6: Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 11/18, 2021

1.1 Question

Sketch the root locus with respect to K for the equation $1 + KL(s) = 0$ and the listed choices for L(s). Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch, verify your results using Matlab. Turn in your hand sketches and the Matlab results on the same scales.

(a) $L(s) = \frac{1}{[(s+2)^2+4](s+4)}$

(b) $L(s) = \frac{(s+1)}{[(s+2)^2+4](s+4)}$

(c) $L(s) = \frac{(s+1)(s+3)}{[(s+2)^2+4](s+4)}$

(d) $L(s) = \frac{(s+1)[(s+3)^2+4]}{[(s+2)^2+4](s+4)}$

1.2 Answer(a)

The real axis segment is $-4 > \sigma > -\infty$

$n = 3$

$m = 0$

$$\alpha = \frac{(-2+2i)+(-2-2i)+(-4)}{3} = -\frac{8}{3}$$

$$\phi_1 = \frac{180^\circ + 360^\circ(1-1)}{3} = 60^\circ$$

$$\phi_2 = \frac{180^\circ + 360^\circ(2-1)}{3} = 180^\circ$$

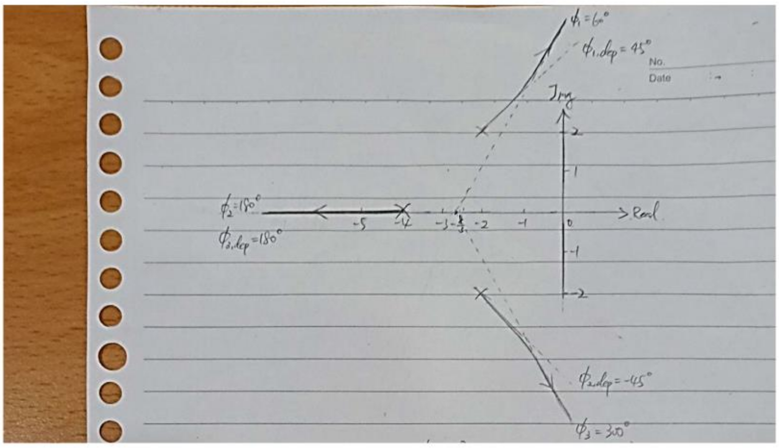
$$\phi_3 = \frac{180^\circ + 360^\circ(3-1)}{5} = 300^\circ$$

$$\phi_{1,\mathrm{dep}} = 0^\circ - 90^\circ - 45^\circ - 180^\circ = -315^\circ = 45^\circ$$

$$\phi_{2,\mathrm{dep}} = 0^\circ - 270^\circ - 315^\circ - 180^\circ = -765^\circ = -45^\circ$$

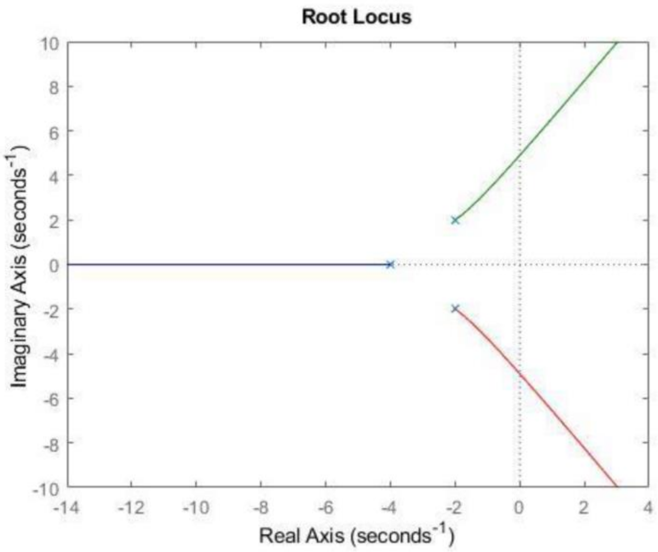
$$\phi_{3,\mathrm{dep}} = 0^\circ - 225^\circ - 135^\circ - 180^\circ = -540^\circ = 180^\circ$$

hand sketch:



HW 6: Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 11/18, 2021

Matlab result:



Matlab code:

```
s = tf( 's' )

sysL1 = 1/(((s+2)^2+4)*(s+4));
rlocus( sysL1 );
```

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1.3 Answer(b)

The real axis segment is $-1 > \sigma > -4$

$n = 3$

$m = 1$

$$\alpha = \frac{(-2+2i)+(-2-2i)+(-4)-(-1)}{2} = -3.5$$

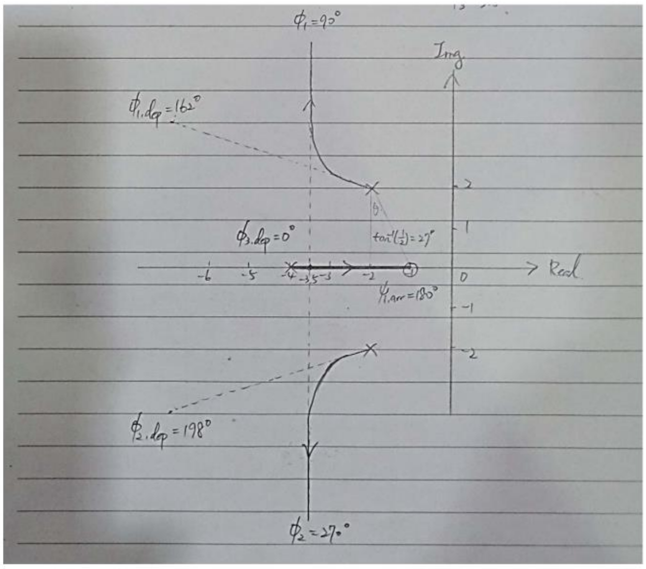
$$\phi_1 = \frac{180^\circ + 360^\circ(1-1)}{2} = 90^\circ$$

$$\phi_2 = \frac{180^\circ + 360^\circ(2-1)}{2} = 270^\circ$$

$$\tan^{-1}(\frac{1}{2}) = 27^\circ$$

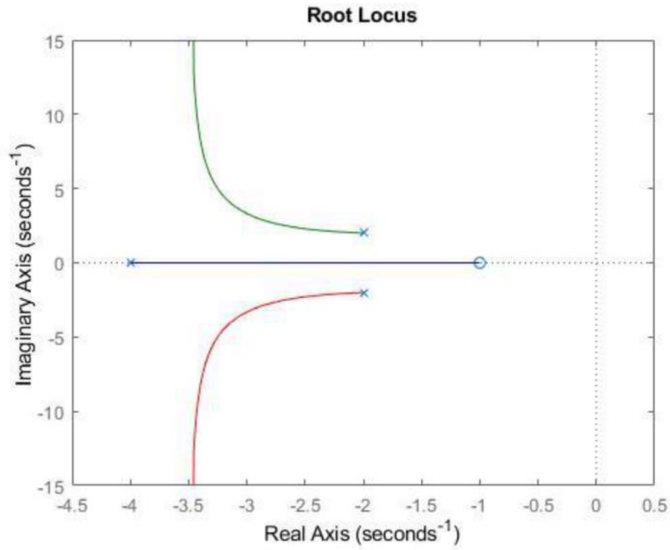
$$\phi_{1,\text{dep}} = 117^\circ - 90^\circ - 45^\circ - 180^\circ = -198^\circ = 162^\circ$$
$$\phi_{2,\text{dep}} = 243^\circ - 270^\circ - 315^\circ - 180^\circ = -522^\circ = 198^\circ$$
$$\phi_{3,\text{dep}} = 180^\circ - 225^\circ - 135^\circ - 180^\circ = -360^\circ = 0^\circ$$
$$\psi_{1,\text{arr}} = 297^\circ + 63^\circ + 0^\circ - 0^\circ + 180^\circ = 540^\circ = 180^\circ$$

hand sketch:



HW 6: Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 11/18, 2021

Matlab result:



Matlab code:

```
s = tf( 's' )

sysL2 = (s+1)/(((s+2)^2+4)*(s+4));
rlocus( sysL2 );
```

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HW 6: Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 11/18, 2021

1.4 Answer(c)

The real axis segment is $-1 > \sigma > -3,\ -4 > \sigma > -\infty$

$n = 3$

$m = 2$

$$\alpha = \frac{(-2+2i) + (-2-2i) + (-4) - (-1) - (-3)}{1} = -4$$

$$\phi_1 = \frac{180^\circ + 360^\circ(1-1)}{1} = 180^\circ$$

$\phi_{1,\mathrm{dep}} = 117^\circ + 63^\circ - 90^\circ - 45^\circ - 180^\circ = -135^\circ = 225^\circ$

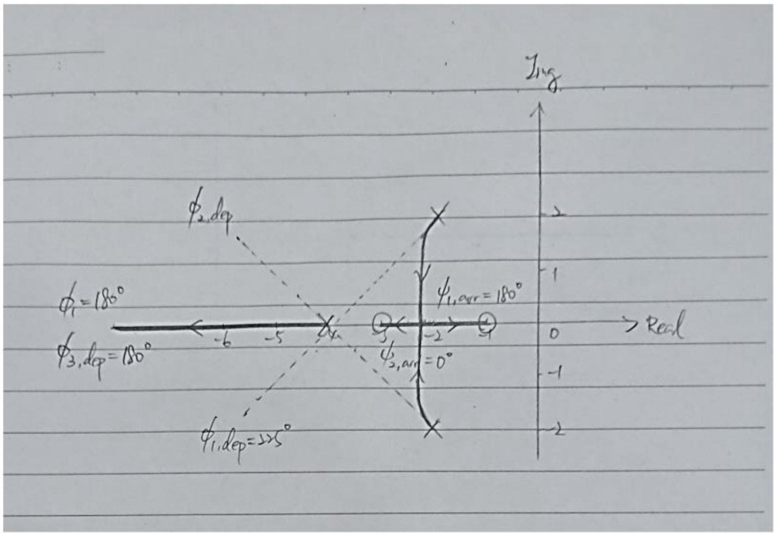
$\phi_{2,\mathrm{dep}} = 243^\circ + 297^\circ - 270^\circ - 315^\circ - 180^\circ = -225^\circ = 135^\circ$

$\phi_{3,\mathrm{dep}} = 180^\circ + 180^\circ - 225^\circ - 135^\circ - 180^\circ = -180^\circ = 180^\circ$

$\psi_{1,\mathrm{arr}} = 297^\circ + 63^\circ + 0^\circ - 0^\circ + 180^\circ = 540^\circ = 180^\circ$

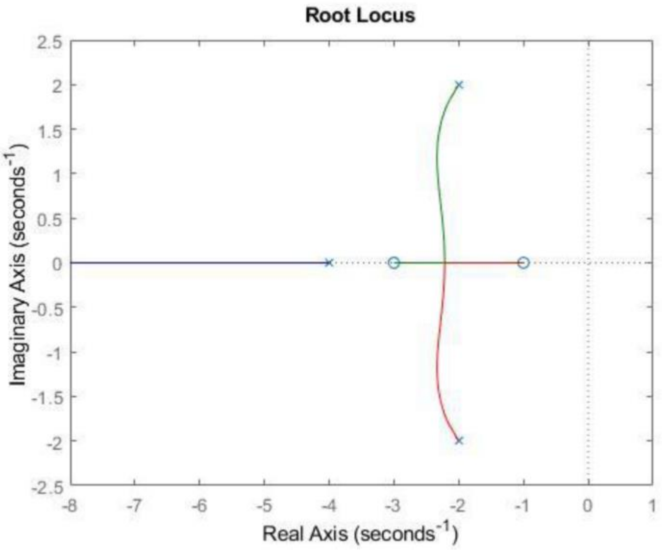
$\psi_{2,\mathrm{arr}} = 243^\circ + 117^\circ + 0^\circ - 180^\circ + 180^\circ = 360^\circ = 0^\circ$

hand sketch:



HW 6: Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 呂元翔 B07901030	Date: 11/18, 2021

Matlab result:



Matlab code:

```
s = tf( 's' )

sysL3 = (s+1)*(s+3)/(((s+2)^2+4)*(s+4));
rlocus( sysL3 );
```

7

1.5 Answer(d)

The real axis segment is $-1 > \sigma > -4$

$n = 3$

$m = 3$

$\tan^{-1}(\frac{4}{1}) = 76^\circ$

$\phi_{1,\text{dep}} = 117^\circ + 0^\circ + 76^\circ - 90^\circ - 45^\circ - 180^\circ = -122^\circ = 238^\circ$

$\phi_{2,\text{dep}} = 243^\circ + 284^\circ + 0^\circ - 270^\circ - 315^\circ - 180^\circ = -238^\circ = 122^\circ$

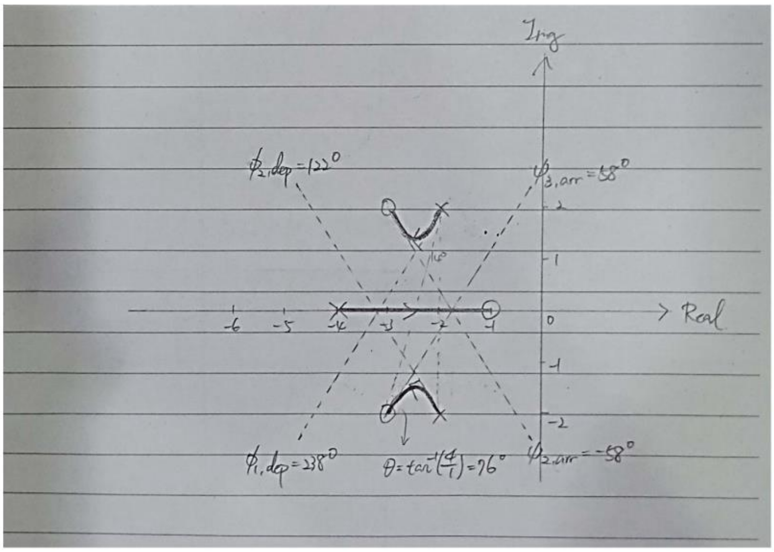
$\phi_{3,\text{dep}} = 180^\circ + 243^\circ + 117^\circ - 225^\circ - 135^\circ - 180^\circ = 0^\circ$

$\psi_{1,\text{arr}} = 297^\circ + 63^\circ + 0^\circ - 315^\circ - 45^\circ + 180^\circ = 180^\circ$

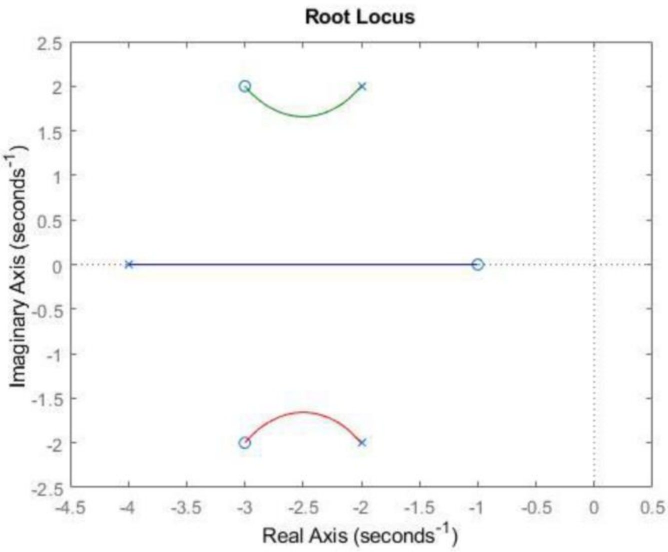
$\psi_{2,\text{arr}} = 180^\circ + 104^\circ + 63^\circ - 135^\circ - 90^\circ + 180^\circ = 302^\circ = -58^\circ$

$\psi_{3,\text{arr}} = 256^\circ + 180^\circ + 297^\circ - 225^\circ - 270^\circ + 180^\circ = 418^\circ = 58^\circ$

hand sketch:



Matlab result:



Matlab code:

```
s = tf( 's' )

sysL4 = (s+1)*((s+3)^2+4)/(((s+2)^2+4)*(s+4));
rlocus( sysL4 );
```

高欣楷 **B07209027**

控制系統HW6

B07209027
高欣楷

2. Real poles and zeros. Sketch the root locus with respect to K for the equation 1 + KL(s) = 0 and the listed choices for L(s). Be sure to give the asymptotes and the arrival and departure angles at any complex zero or pole. After completing each hand sketch verify your results using MATLAB. Turn in your hand sketches and the MATLAB results on the same scales.

   (a) $\mathrm{L}(s) = \frac{2}{s(s+2)^2(s+3)(s+5)^2(s+6)(s+7)^2}$

   (b) $\mathrm{L}(s) = \frac{(s+2)^2}{s(s+4)^2(s+6)}$

Solution:

(a) $\alpha = -30.78;\ \phi_i = \pm 20°, \pm 60°, \pm 100°, \pm 140°, 180$

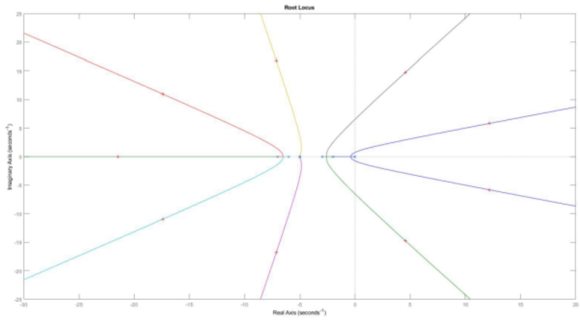
(b) $\alpha = -5.33;\ \phi_i = \pm 60°, 180$

(圖在下一頁)

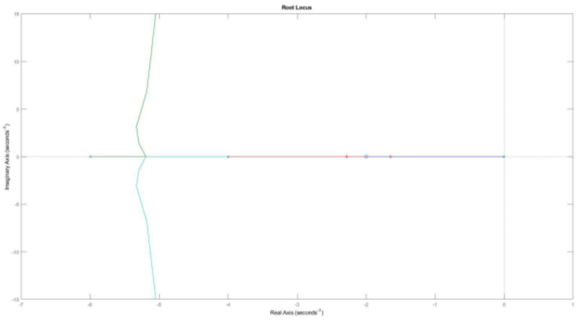
首先(a)的改編其實我就是想看看，如果pole有很多個，那圖會長什麼樣，畢竟pole數量一多手繪的精準性就會降低，好在有matlab的幫忙，可見圖上的漸進線是很漂亮的平分。

至於(b)則是爲了驗證如果zero有重根的話，圖會怎麼走，而結果也是如同上課所講，會有兩個pole最後都走到重根的zero的位置。

Date: November 2021.



(a)



(b)

Control System HW6

電機四 俞建琁 B07901086

改編自第二題

Q: Consider the negative unit feedback system with characteristic equation

1+KL(s)=0, $L(s) = \frac{s+3}{(s+2)^3+27}$, please answer the following questions.

(a) Find all the poles and zeros of L(s).

(b) Find the asymptotes of the locus for $K \to \infty$.

(c) Find the arrival and departure angles of all the zeros and poles.

(d) Find the break-in points and breakaway points of the locus.

(Hint: At break-in/breakaway points, $\frac{dK}{ds} = 0.$)

(e) Sketch the locus.

(f) Verify your sketch with a Matlab plot.

Sol:

(a)

Zero: -3

Pole: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$(s+2)^3 + 3^3 = 0$

$\rightarrow (s+2+3)[(s+2)^2 - 3(s+2) + 3^2] = (s+5)(s^2 + s + 7) = 0$

$$\rightarrow s = -5\ or\ \frac{-1 \pm \sqrt{1 - 4 \times 7}}{2} = -5\ or\ \frac{-1 \pm \sqrt{-27}}{2} = -5\ or\ \frac{-1 \pm 3\sqrt{3}i}{2}$$

Hence, poles are -5, $\frac{-1+3\sqrt{3}i}{2}$, $\frac{-1-3\sqrt{3}i}{2}$.

(b)

$n - m = 3 - 1 = 2$

$$\alpha = \frac{\sum p_i - \sum z_i}{n - m} = \frac{-5 - 1 - (-3)}{2} = -\frac{3}{2}$$

$$\emptyset_l = \frac{180^o + 360^o(l-1)}{n - m} = 90^o, 270^o(or\ -90^o)$$

Thus, the asymptotes is x = -3/2.

(c)

For pole -5, departure angle= $\sum_{zeros}\psi_i - \sum_{other\ poles}\phi_i - 180^o = 0^o$.

For pole $\frac{-1+3\sqrt{3}i}{2}$, departure angle= $\sum_{zeros}\psi_i - \sum_{other\ poles}\phi_i - 180^o = 46.1^o - (30^o + 90^o) - 180^o = -253.9^o = 106.1^o$

For pole $\frac{-1-3\sqrt{3}i}{2}$, departure angle= $\sum_{zeros}\psi_i - \sum_{other\ poles}\phi_i - 180^o = -46.1^o - (-30^o - 90^o) - 180^o = -106.1^o$

For zero -3, arrival angle= $\sum_{poles}\phi_i - \sum_{other\ zeros}\psi_i + 180^o = 180^o$

(d)

$$K = -\frac{1}{L(s)} = -\frac{(s+2)^3 + 27}{s+3}$$

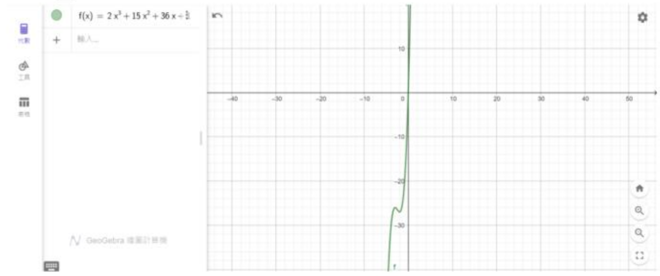
$$\frac{dK}{ds} = -\frac{3(s+2)^2(s+3) - [(s+2)^3 + 27]}{(s+3)^2} = 0$$

$\rightarrow 3(s^2 + 4s + 4)(s+3) - (s^3 + 6s^2 + 12s + 8) - 27 = 0$

$\rightarrow 3s^3 + 21s^2 + 48s + 36 - s^3 - 6s^2 - 12s - 8 - 27 = 2s^3 + 15s^2 + 36s + 1 = 0$ ...(1)

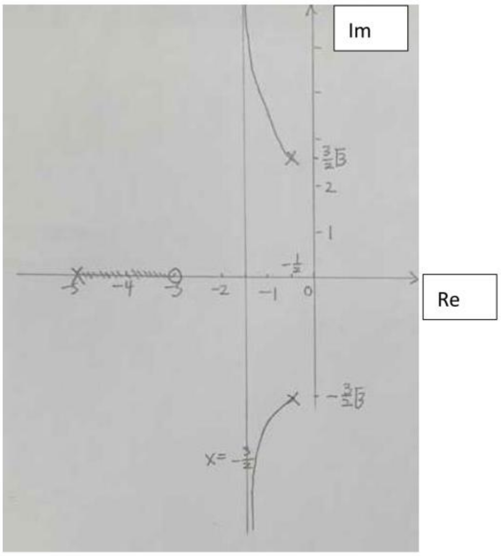
Break-in points and breakaway points are on the real-axis, and K should be positive

$\rightarrow -5 \le s < -3$



We can see that there is no root in this range, so there is no break-in/breakaway point.

(e)

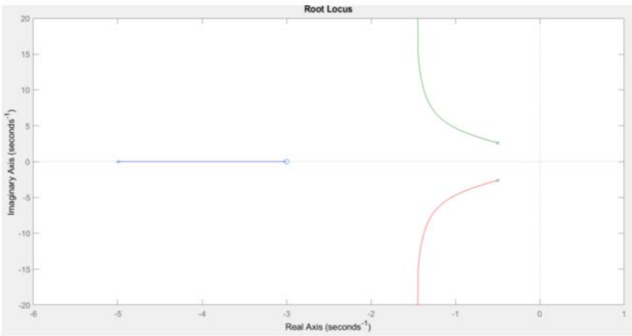


(f)

Matlab code:

```
s = tf('s')
sysL = (s+3)/((s+2)^3+27);
rlocus(sysL);
[K, p] = rlocfind(sysL);
```

Matlab plot:



簡宏哲 B08901111

Control System: Homework 06   B08901111 電機三 簡宏哲

For the system below:

, $L(S) = \dfrac{(S+2)}{(S+2)^4+4}$, $K>0$

(a) Find all the poles and the zeros of $L(S)$.

(b) Find all the asymptotes of the locus for $K \to \infty$.

(C) Find the arrival and departure angles at every zero and pole.

(d) It's known that the breakaway / break-in point can be found by solving $\frac{dK}{ds} = 0$ in $1 + KL(S) = 0$, please find all the breakaway / break-in points on the real axis. ($2^{\frac{1}{2}} \doteqdot 1.41$, $3^{-\frac{1}{4}} \doteqdot 0.76$)

(e) Sketch the locus roughly.

Solution :

(a) For $L(s) = \dfrac{s+2}{(s+2)^4+4} = \dfrac{N(s)}{D(s)}$,

the poles are the roots of $D(s)=0 \Rightarrow s = -1\pm j, -3\pm j$,

and the zero is the root of $N(s)=0 \Rightarrow s=-2$.

(b) $n = \deg D(s) = 4$

$m = \deg N(s) = 1$

$$\Rightarrow \alpha = \frac{\sum \text{pole} - \sum \text{zero}}{n-m} = \frac{(-1+j)+(-1-j)+(-3+j)+(-3-j)-(-2)}{4-1} = -2$$

$$\phi_i = \frac{180^\circ + 360^\circ(i-1)}{n-m} = 60^\circ, 180^\circ, 300^\circ \text{ for } i=1,2,3$$

$\therefore$ The asymptotes are $\mathrm{Im}\{s\} = \pm\sqrt{3}(\mathrm{Re}\{s\}+2)$ and $\mathrm{Im}\{s\} = 0$.

(c) For zero $s=-2$, $\psi_{arr} = \sum \phi_{pole} - \sum \psi_{other\ zero} + 180^\circ$

$= (-45^\circ) + 45^\circ + 135^\circ + (-135^\circ) + 180^\circ = 180^\circ$

For pole $s=-1+j$, $\psi_{dep} = \sum \psi_{zero} - \sum \phi_{other\ pole} + 180^\circ$

$= 45^\circ - 0^\circ - 45^\circ - 90^\circ + 180^\circ = 90^\circ$

For pole $s=-1-j$, $\psi_{dep} = -45^\circ - (-90^\circ) - (-45^\circ) - 0^\circ + 180^\circ = 270^\circ$

For pole $s=-3+j$, $\psi_{dep} = 135^\circ - 180^\circ - 90^\circ - 135^\circ + 180^\circ = -90^\circ$

For pole $s=-3-j$, $\psi_{dep} = -135^\circ - (-135^\circ) - (-90^\circ) - 180^\circ + 180^\circ = 90^\circ$.

(d) $1+\dfrac{K(s+2)}{(s+2)^4+4}=0 \Rightarrow K=-\dfrac{(s+2)^4+4}{s+2}$

$\Rightarrow \dfrac{dK}{ds}=\dfrac{4(s+2)^3(s+2)-((s+2)^4+4)}{(s+2)^2}=\dfrac{3(s+2)^4-4}{(s+2)^2}=0$

$\Rightarrow s=-2\pm\dfrac{2^{\frac{1}{2}}}{3^{\frac{1}{4}}}$ (only need real s) $\fallingdotseq -2\pm 1.41\times 0.76 \fallingdotseq -2\pm 1.07$

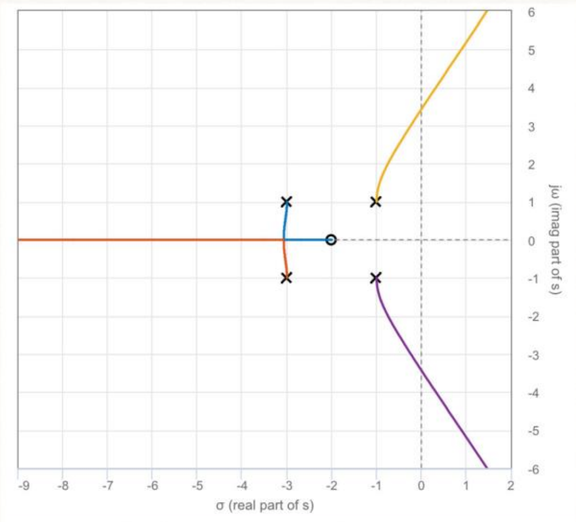
But $K=-\dfrac{(s+2)^4+4}{s+2}$ should be positive, so $s<-2$

$\therefore$ $s\fallingdotseq -3.07$ (break-in point), no breakaway point

(e) Matlab code:

```
s = tf( 's' )
sysL = (s+2)/((s+2)^4+4);
rlocus( sysL );
[ K, p ] = rlocfind( sysL );
```

Locus:



HW 06: Unit 5A, 5B Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 11/18, 2021

Problem (U5B: Root Locus)

For the system in Figure 1-1,

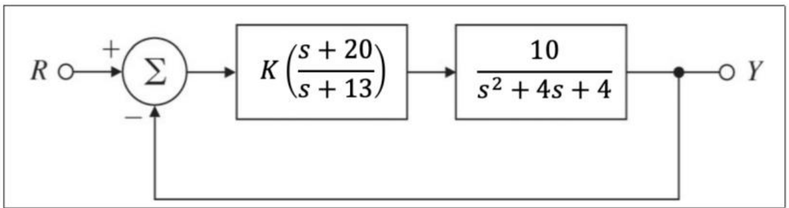


Figure 1-1. Feedback system for this problem

(a) Find the locus of closed-loop roots with respect to K (can use Matlab).

(b) Use `sisotool` in MATLAB to find the range of K such that the system is stable (only need to find an approximate value).

(c) Verify the value you find in (b) using Routh's Criterion.

(d) Use `sisotool` in Matlab to find the range of K such that the system is stable and its settling time is less than 6 seconds (only need to find an approximate value).

(e) Use `sisotool` in Matlab to find the range of K such that the system is stable, its settling time is less than 6 seconds, and its overshoot is less than 20% (only need to find an approximate value).

(f) Fix K to be some value in the range you find in part (e). Use Matlab to plot the response to a reference step.

HW 06: Unit 5A, 5B Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 11/18, 2021

Solution

(a) Using the following MATLAB code, we can draw the desired locus shown in Figure 1-2.

MATLAB code:

```
C = tf([1, 20], [1, 13]);
P = tf([10], [1, 4, 4]);
L = C*P;
rlocus(L);
```

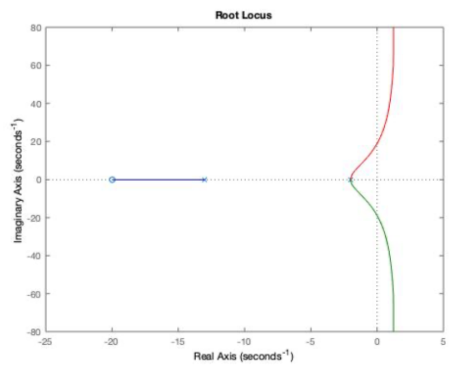


Figure 1-2. the locus of closed-loop roots with respect to K.

(b) Follow the steps below:

(1) Run the following Matlab code:

```
C = tf([1, 20], [1, 13]);
P = tf([10], [1, 4, 4]);
L = C*P;
sisotool
```

(2) Click "Edit Architecture" → Press the button for "C" → Click "C" in the pop-up window → Press "Import" (See Figure 1-3)

2

HW 06: Unit 5A, 5B Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 11/18, 2021

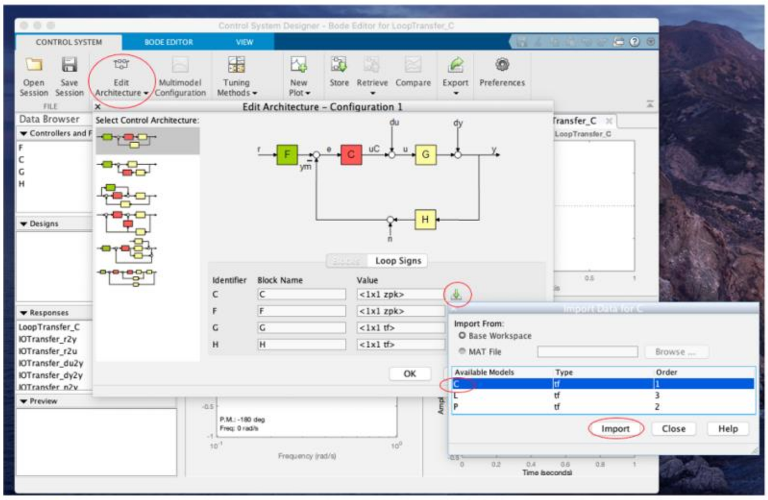


Figure 1-3. Step 2.

(3) Click “Edit Architecture” → Press the button for “G” → Click “P” in the pop-up window → Press “Import” (See Figure 1-4)

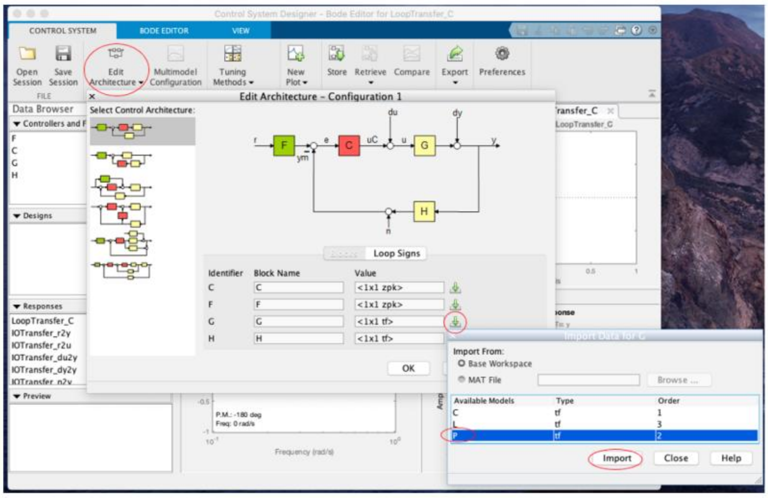


Figure 1-4. Step 3.

(4) Press “OK”, and you shall see the bode-plot, root locus, and step response (see Figure 1-5).

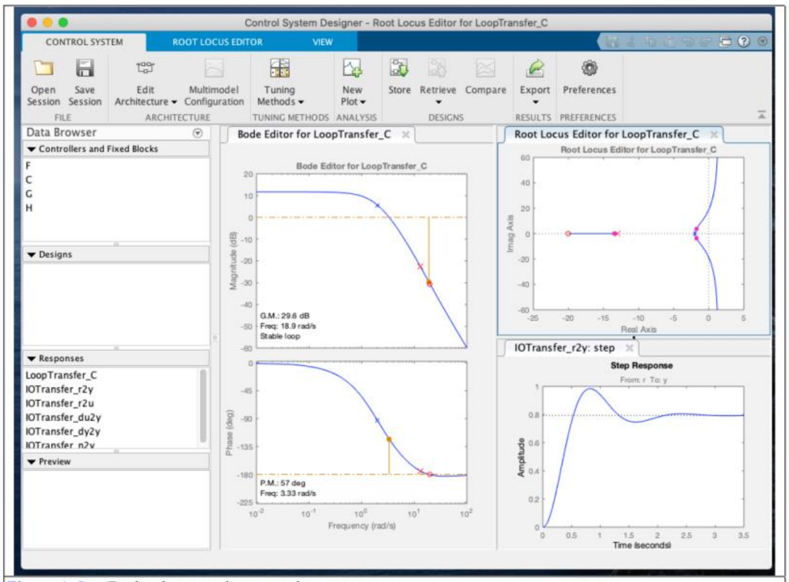


Figure 1-5. Bode plot, root locus, and step response.

(5) Click “C” in the up-left corner, and move the pink cursor in the root locus onto the imaginary axis. Then the corresponding K is the largest value of K. Therefore, in this case, if we want the system to be stable, we need **K < 30** (approximately) (see Figure 1-6).

HW 06: Unit 5A, 5B Root Locus	Control Systems, Fall 2021, NTU-EE
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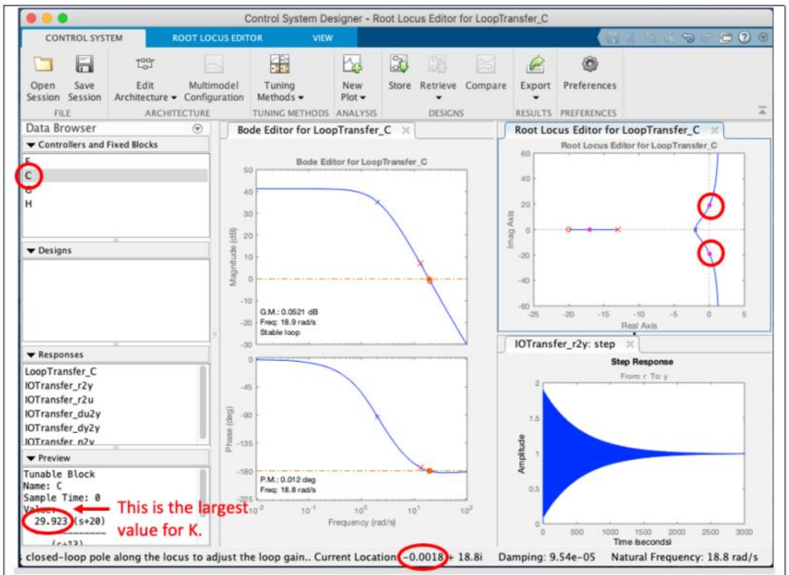


Figure 1-7. Largest value of K for stability.

(c) The steps in Figure 1-8 show that if the system is stable, then we need **K < 30** by Routh's Criterion.

Characteristic equation: $1 + K\left(\frac{s+20}{s+13}\right)\left(\frac{10}{s^2+4s+4}\right) = 0$

$\Rightarrow s^3 + 17s^2 + (56+10K)s + (52+200K) = 0$

Routh array:

$$\begin{array}{ll} 1 & 56+10K \\ 17 & 52+200K \end{array}$$

$$-\frac{52+200K-17(56+10K)}{17} = \frac{900-30K}{17}$$

$\therefore 900 - 30K > 0 \Rightarrow K < 30$

Figure 1-8. The range of K for stability derived by Routh's Criterion

5

(d) Follow the steps below:

(1) Move your mouse onto the root locus in sisotool and press the right button.

(2) Choose "Design Requirements" and press "New…" (see Figure 1-9).

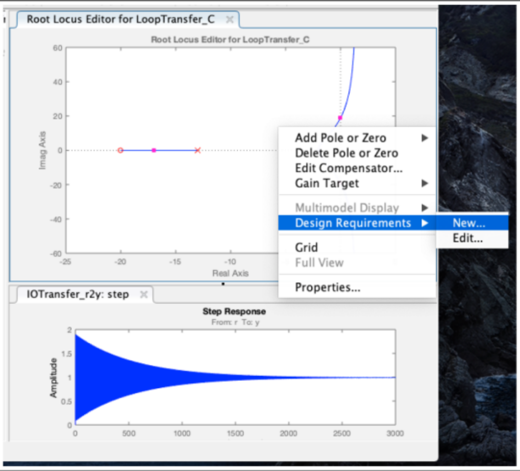


Figure 1-9. Choose "Design Requirements" and press "New…".

(3) Choose the design requirement to be "Settling time", type "6" in the parameter textbox, and press "Enter" (see Figure 1-10).

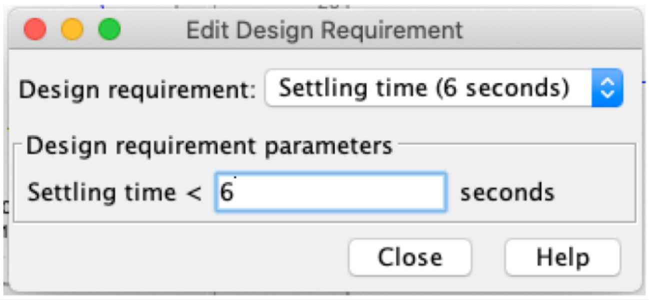


Figure 1-10. Edit Design Requirement.

(4) Click "C" in the up-left corner, and move the pink cursor in the root locus onto the border (the bold black line). Then the corresponding K is the largest value of K. Therefore, in this case, **K < 11** (approximately) (see Figure 1-11).

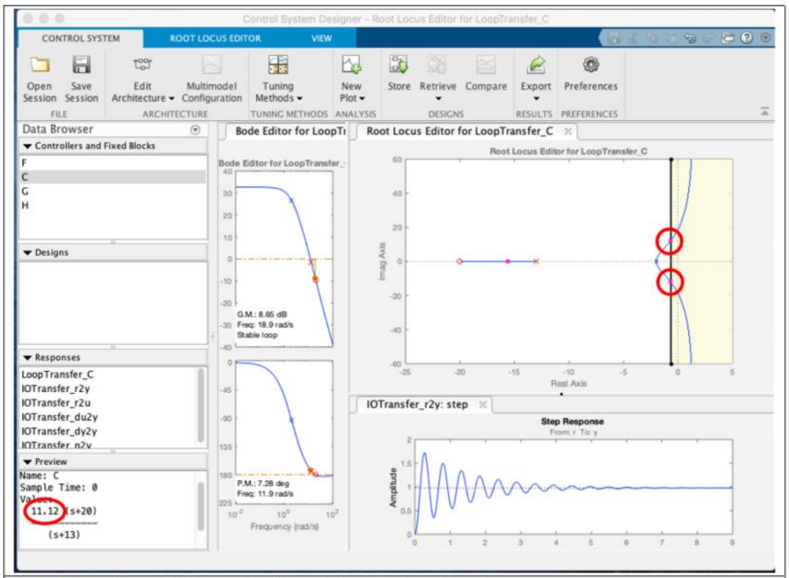


Figure 1-11. Largest Value of K with specification of settling time.

(e) Follow the steps in (d). Only change "Settling time" to "Percent Overshoot" and "6" to "20". Then you will get Figure 1-12 and know that the range of K should be **K < 0.9** (approximately) (see Figure 1-12).

HW 06: Unit 5A, 5B Root Locus	Control Systems, Fall 2021, NTU-EE
Name: 邱泓翔 B08901095	Date: 11/18, 2021

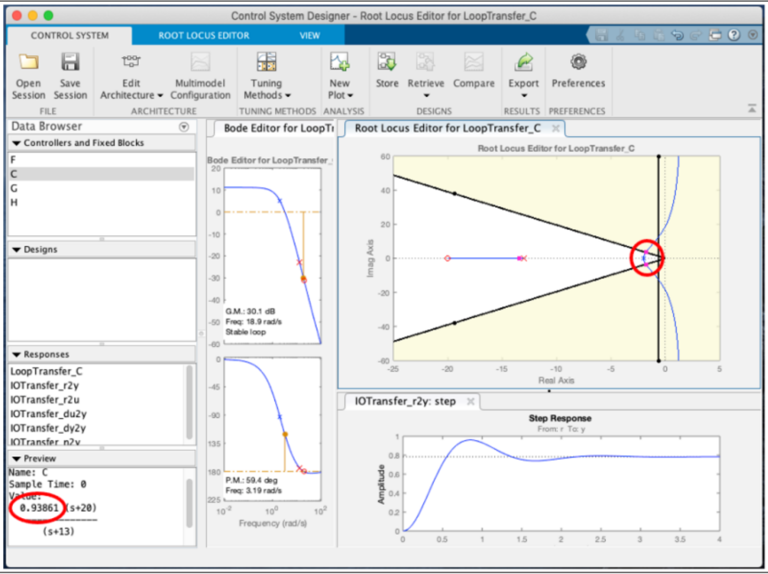


Figure 1-12. Largest Value of K with specification of settling time and overshoot.

(f) Fix K = 0.5. Run the following Matlab code. Then you will get the step response as in Figure 1-13.

```
C = tf([0.5, 10], [1, 13]);
P = tf([10], [1, 4, 4]);
L = C*P;
sys = L/(1+L);
step(sys)
```

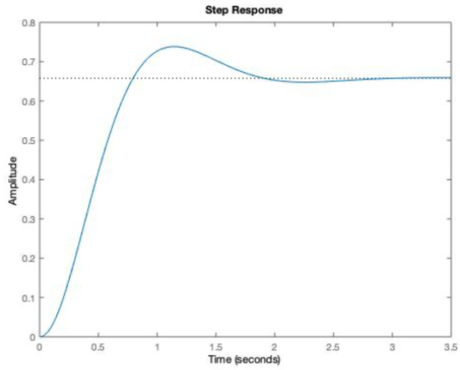


Figure 1-12. Step Response with K = 0.5.

HW06 for Units 5A, 5B: Root Locus, Control Systems	Control Systems, Fall 2021, NTU-EE
Name: 李婕莘 B08901208	Date: 11/16, 2021

1.1 Problem 1

For a feedback system shown in **Figure(a)**, $L(s) = \frac{5s+1}{s^2-3s+2}$

(a) Sketch the root locus with respect to K for the characteristic equation $1 + KL(s) = 0$.

(b) Determine that whether the system is stable or not by observation.

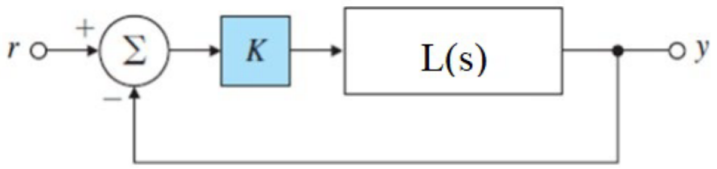
(c) Prove or disapprove that the system is stable.

(d) What is the step response of $L(s)$?

(e) If the system is not stable, how to modify it to make the system stable? (Only modify one coefficient of $L(s)$,the solution is not unique.)

(f) Sketch the root locus of the new with respect to K for the characteristic equation $1 + KL(s) = 0$.

(g) What is the step response of the modified $L(s)$?



Figure(a): block diagram of the feedback system

Solution:

(a) The characteristic equation is

$$1 + K\frac{5s+1}{s^2-3s+2} = 0$$
$$s^2 - 3s + 2 + K(5s+1) = 0$$
$$s^2 + (5K-3)s + (K+2) = 0 \quad (1)$$

1

HW06 for Units 5A, 5B: Root Locus, Control Systems	Control Systems, Fall 2021, NTU-EE
Name: 李婕華 B08901208	Date: 11/16, 2021

The root locus is the set of values of s for which for which $1 + KL(s) = 0$ is satisfied as the real parameter K varies from 0 to ∞. Use MATLAB to get the root locus of the system, and the answer is shown in **Figure(b)**.

(b) Observing the root locus, we can find that there are poles (s=1,2) on the RHP, so the system is **unstable**.

(c) By Routh's Stability Criterion, a system is stable if and only if all the elements in the first column of the Routh array are positive. The characteristic equation is eq.(1) $s^2 + (5K - 3)s + (K + 2) = 0$. The following is Routh array：

s^2	1	$K + 2$
s	$5K - 3$	0
1	$-\dfrac{\det\begin{bmatrix} 1 & K+2 \\ 5K-3 & 0 \end{bmatrix}}{1}$ $= (5K - 3)\ (K + 2)$	0

We can find that the first column of Routh array is not always positive for all non-negative K, so the system is **unstable** Q.E.D.

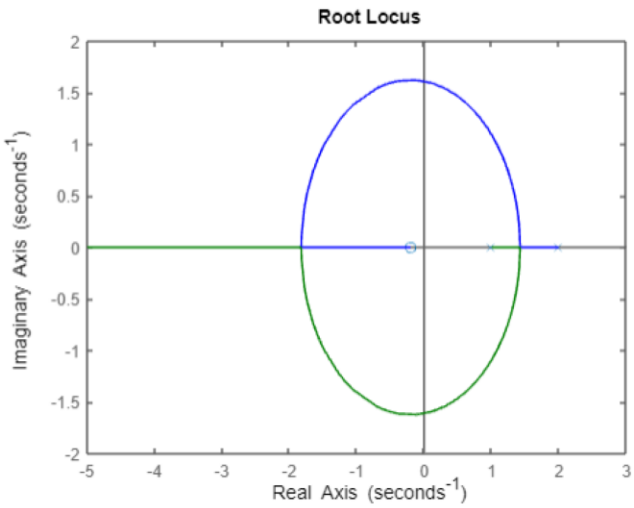
(d) By partial fraction method,

$$L(s) = \frac{5s + 1}{s^2 - 3s + 2} = \frac{-6}{s - 1} + \frac{11}{s - 2} \qquad (2)$$

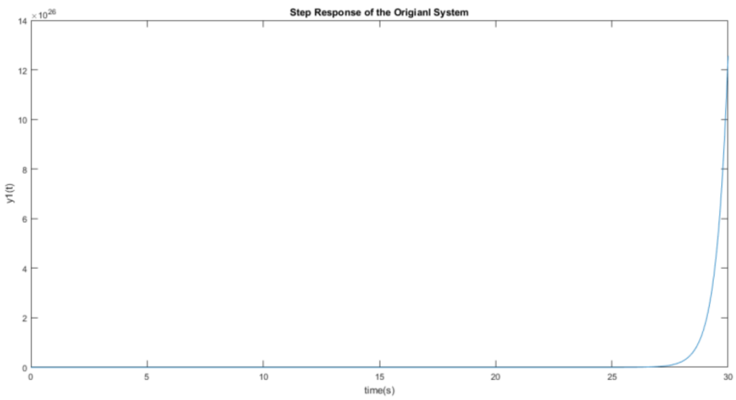
Do inverse Laplace transform, we get the step response$y_1(t)$. Use MATLAB to get the root locus of the system, and the answer is shown in **Figure(c)**.

2

$$y_1(t) = -6e^{t} + 11e^{2t} \quad (3)$$



Figure(b): Root Locus of the original system



Figure(c): step response of the original L(s) by inverse Laplace transform

MATLAB code

```
sys = tf([0,5,1],[1,-3,2]);
rlocus(sys)
tt=0:0.01:30;
yy=-6.*exp(tt)+11.*exp(2.*tt);
figure;
plot(tt,yy);
title('Step Response of the Origianl System');
xlabel('time(s)');
ylabel('y1(t)');
```

(e) We can just modify the denominator to make the poles located at LHP. The new loop gain $L_{new}(s) = \frac{5s+1}{s^2+3s+2}$ .The new characteristic equation is

$$1 + K\frac{5s+1}{s^2+3s+2} = 0$$
$$s^2 + 3s + 2 + K(5s+1) = 0$$
$$s^2 + (5K+3)s + (K+2) = 0 \quad (4)$$

(f) The root locus is the set of values of s for which for which $1 + KL(s) = 0$ is satisfied as the real parameter K varies from 0 to ∞. Use MATLAB to get the root locus of the system, and the answer is shown in **Figure(d)**. Observing the root locus, we can find that there are poles (s=-1,-2) on the LHP, so the system is **stable**.

(g) By Routh's Stability Criterion, a system is stable if and only if all the elements in the first column of the Routh array are positive. The characteristic equation is eq.(4) $s^2 + (5K+3)s + (K+2) = 0$. The following is Routh array：

4

HW06 for Units 5A, 5B: Root Locus, Control Systems	Control Systems, Fall 2021, NTU-EE
Name: 李婕苹 B08901208	Date: 11/16, 2021

s^2	1	$K+2$
s	$5K+3$	0
1	$-\frac{\det\begin{bmatrix}1 & K+2\\ 5K+3 & 0\end{bmatrix}}{1}$ $=(5K+3)\ (K+2)$	0

We can find that the first column of Routh array is always positive for all non-negative K, so the system is **stable** Q.E.D.

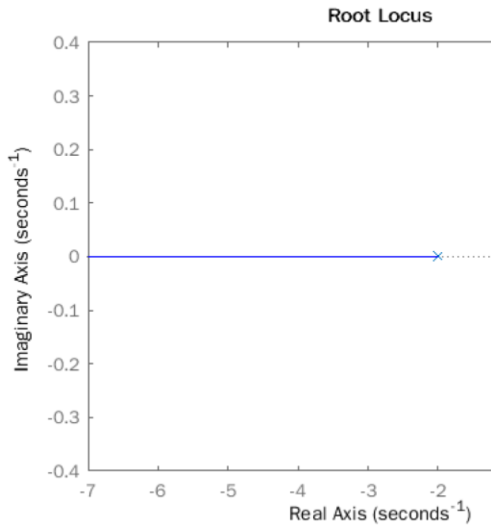
(h) By partial fraction method,

$L(s)=\frac{5s+1}{s^2+3s+2}=\frac{-4}{s+1}+\frac{9}{s+2}$	(5)

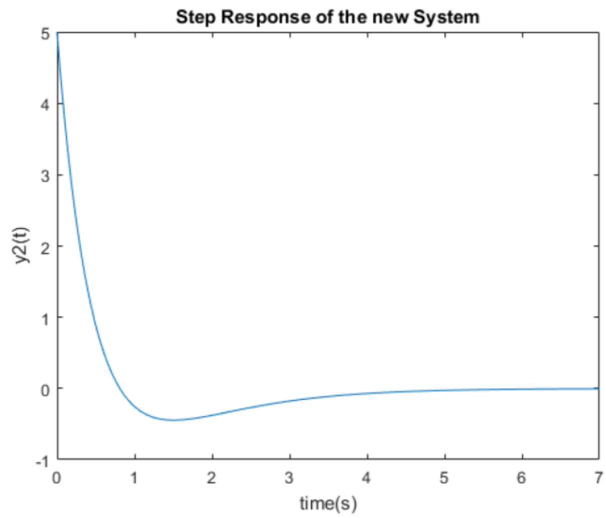
Do inverse Laplace transform,we get the step response$y_1(t)$. Use MATLAB to get the root locus of the system, and the answer is shown in **Figure (e)**.

$y_1(t)=-4e^{-t}+9e^{-2t}$	(6)

5



Figure(d): Root Locus of the new system



Figure(e): step response of the new $L_{new}(s)$ by inverse Laplace transform

HW06 for Units 5A, 5B: Root Locus, Control Systems	Control Systems, Fall 2021, NTU-EE
Name: 李婕莘 B08901208	**Date: 11/16, 2021**

MATLAB code

```
sys = tf([0,5,1],[1,3,2]);
rlocus(sys)
ttt=0:0.01:7;
yNew=-4.*exp(-ttt)+9.*exp(-2.*ttt);
figure;
plot(ttt,yNew);
title('Step Response of the new System');
xlabel('time(s)');
ylabel('y2(t)');
```

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