Control System: Homework 05 for Units 4C, 4D, 4E: Feedback Analysis

Assigned: October 29, 2021

Due: 不用交

1. (U4C: Three terms controller)

4.34 Consider the satellite-attitude control problem shown in Fig. 4.45 where the normalized parameters are

$J = 10$ spacecraft inertia, $\text{N·m·sec}^2/\text{rad}$

θ_r = reference satellite attitude, rad.

θ = actual satellite attitude, rad.

$H_y = 1$ sensor scale, factor V/rad.

$H_r = 1$ reference sensor scale factor, V/rad.

w = disturbance torque, N·m.

(a) Use proportional control, P, with $D_c(s) = k_P$, and give the range of values for k_P for which the system will be stable.

(b) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to reference inputs.

(c) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to disturbance inputs.

(d) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to reference inputs.

(e) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to disturbance inputs.

(f) Use PID control, let $D_c(s) = (k_P + \frac{k_I}{s} + k_D s)$, and determine the system type and error constant with respect to reference inputs.

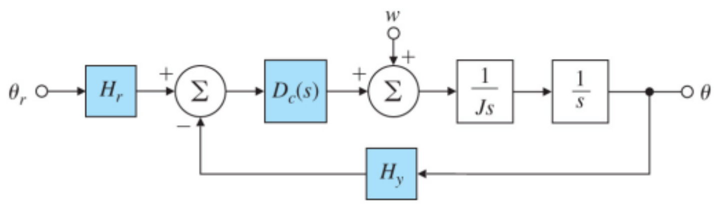


Figure 4.45: Satellite attitude control

Solution:

(a) $D_c(s) = k_P$; The characteristic equation is

$$1 + H_y D_c(s)\frac{1}{Js^2} = 0$$

$$Js^2 + H_y k_P = 0$$

or $s = \pm j\sqrt{\dfrac{H_y k_P}{J}}$ so that no additional damping is provided. The system cannot be made stable with proportional control alone.

(b) Steady-state error to reference steps.

$$\begin{aligned}\frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\frac{1}{Js^2}}{1 + D_c(s)H_y\frac{1}{Js^2}}, \\ &= H_r \frac{(k_P + k_D s)}{Js^2 + (k_P + k_D s)H_y}.\end{aligned}$$

The parameters can be selected to make the (closed-loop) system stable. If $\Theta_r(s) = \dfrac{1}{s}$ then using the FVT (assuming the system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback).

(c) Steady-state error to disturbance steps

$$\frac{\Theta(s)}{W(s)} = \frac{1}{Js^2 + (k_P + k_D s)H_y}.$$

If $W(s) = \dfrac{1}{s}$ then using the FVT (assuming system is stable), the error is $\theta_{ss} = -\dfrac{1}{k_P H_y}$.

(d) The characteristic equation is

$$1 + H_y D_c(s)\frac{1}{Js^2} = 0.$$

With PI control,

$$Js^3 + H_y k_P s + H_y k_I = 0.$$

From the Hurwitz's test, with the s^2 term missing the system will always have (at least) one pole not in the LHP. Hence, this is not a good control strategy.

(e) See (d) above.

(f) The characteristic equation with PID control is

$$1 + H_y \left(k_P + \frac{k_I}{s} + k_D s \right) \frac{1}{Js^2} = 0,$$

or

$$Js^3 + H_y k_D s^2 + H_y k_P s + H_y k_I = 0.$$

There is now control over all the three poles and the system can be made stable.

$$\begin{aligned}
\frac{\Theta(s)}{\Theta_r(s)} &= H_r \frac{D_c(s)\frac{1}{Js^2}}{1 + D_c(s) H_y \frac{1}{Js^2}}, \\
&= \frac{H_r(k_P + \frac{k_I}{s} + k_D s)}{Js^2 + (k_P + \frac{k_I}{s} + k_D s) H_y}, \\
&= \frac{H_r(k_D s^2 + k_P s + k_I)}{Js^3 + (k_D s^2 + k_P s + k_I) H_y}.
\end{aligned}$$

If $\Theta_r(s) = \frac{1}{s}$ then using the FVT (assuming system is stable)

$$\theta_{ss} = \frac{H_r}{H_y},$$

and there is zero steady-state error if $H_r = H_y$ (i.e., unity feedback). In that case, the system is Type 3 and the (Jerk!) error constant is $K_J = \frac{k_I}{J}$.

2. (U4D: Three terms controller and Ziegler–Nichols Tuning)

4.37 A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1},$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

(a) Find the PID-controller parameters using the Ziegler–Nichols tuning rules.

(b) The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 4.48. Find the optimal PID-controller parameters according to the Ziegler–Nichols tuning rules.

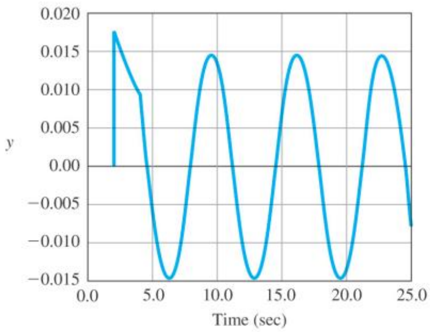


Figure 4.48: Unit impulse response for paper-machine in Problem 4.37

Solution:

(a) From the transfer function: $L = \tau_d \simeq 2$ sec

$$R = \frac{1}{3} \simeq 0.33 \quad \text{sec}^{-1}.$$

From Table 4.1:

Controller Gain P : $K = \frac{1}{RL}1.5,$

PI : $K = \frac{0.9}{RL} = 1.35 \quad T_I = \frac{L}{0.3} = 6.66,$

PID: $K = \frac{1.2}{RL} = 1.8 \quad T_I = 2L = 4 \; T_D = 0.5L = 1.0.$

(b) From the impulse response: $P_u \simeq 7$ sec From Table 4.2:

Controller Gain P : $K = 0.5K_u = 1.52,$

PI : $K = 0.45K_u = 1.37 \quad T_I = \frac{1}{1.2}P_u = 5.83,$

PID : $K = 0.6K_u = 1.82 \quad T_I = \frac{1}{2}P_u = 3.5T_D = \frac{1}{8}P_u = 0.875.$

3. (U4E: Feedforward Control)

4.38 Consider the DC motor speed-control system shown in Fig. 4.49 with proportional control. (a) Add feedforward control to eliminate the steady-state tracking error for a step reference input. (b) Also add feedforward control to eliminate the effect of a constant output disturbance signal, w, on the output of the system.

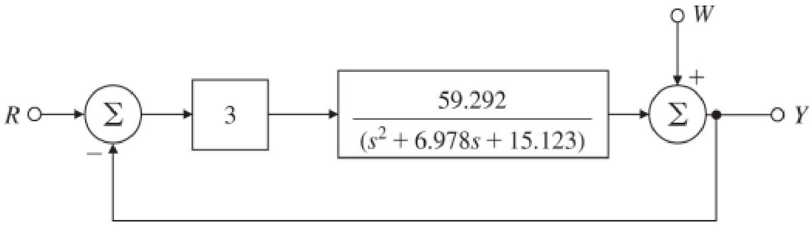
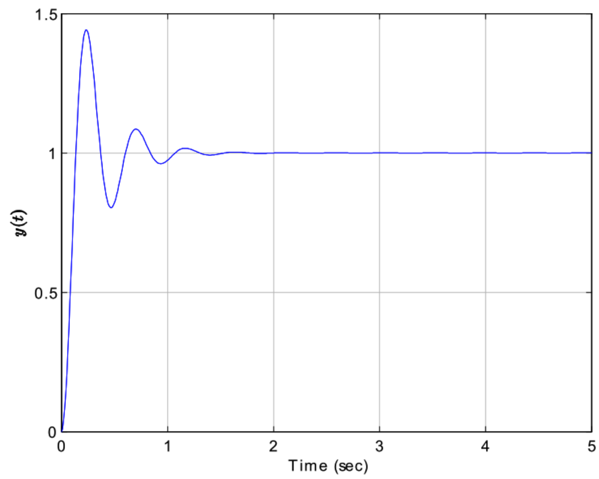


Figure 4.49: Block diagram for Problem 4.38

Solution: (a) In this case the plant inverse DC gain is $G^{-1}(0) = \frac{15.123}{59.292} = 0.2551$. We implement the closed-loop system as shown in Figure 4.22 (a) with $D_c(s) = k_p = 3$. The closed-loop transfer function is

$$\begin{aligned} Y(s) &= G(s)[k_p E(s) + G^{-1}(0)R(s)], \\ E(s) &= R(s) - Y(s), \\ \frac{Y(s)}{R(s)} &= \mathcal{T}(s) = \frac{(G^{-1}(0) + k_p)G(s)}{1 + k_p G(s)}. \end{aligned}$$

Note that the closed-loop DC gain is unity ($\mathcal{T}(0) = 1$). The following figure illustrates the effect of feedforward control in eliminating the steady-state tracking error. The addition of feedforward control results in zero steady-state tracking error for a step reference input.

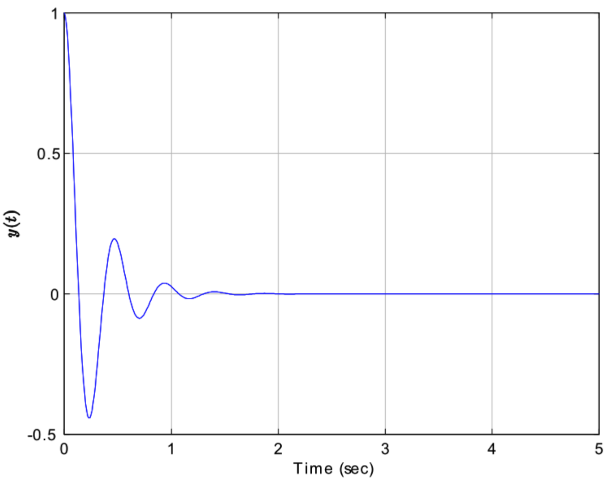


Tracking response with feedforward

(b) Similarly, we implement the closed loop system as shown Figure 4.22 (b). The closed-loop transfer function is

$$
\begin{aligned}
Y(s) &= W(s) + G(s)[k_p E(s) - G^{-1}(0)W(s)], \\
E(s) &= R(s) - Y(s) = 0 - Y(s), \\
\frac{Y(s)}{W(s)} &= \mathcal{T}_w(s) = \frac{1 - G^{-1}(0)G(s)}{1 + k_p G(s)}.
\end{aligned}
$$

Note that the closed-loop DC gain is zero ($\mathcal{T}_w(s) = 0$). The following figure illustrates the effect of feedforward control in eliminating the steady-state error for a step output disturbance.



Disturbance rejection response with feedforward

MATLAB code:

```
%FPE7e Problem 3.38
clf;
% Tracking
s=tf('s');
% plant
G=59.292/(s^2+6.978*s+15.123);
kp=3;
% Closed-loop Transfer function
dcgain1=dcgain(G);
T1=G*(1/dcgain1+kp)/(1+kp*G);
t=0:.01:5;
% Step response

y1=step(T1,t);
figure()
plot(t,y1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;
% Disturbance rejection
kp=3;
Tw1=(1-1/dcgain1*G)/(1+kp*G);
yw1=step(Tw1,t);
figure()
plot(t,yw1);
xlabel('Time (sec)');
ylabel('$y(t)$','interpreter','latex');
nicegrid;
```