

Control System: Homework 03 for Units 3D, 3E, 3F: Dynamic Response

Assigned: October 15, 2021

Due: October 28, 2021 (23:59)

1. (Effect of zeros and additional poles)

41. ▲ Sketch the step response of a system with the transfer function

$$G(s) = \frac{s/2 + 1}{(s/40 + 1)[(s/4)^2 + s/4 + 1]}.$$

Justify your answer on the basis of the locations of the poles and zeros. (Do not find inverse Laplace transform.) Then compare your answer with the step response computed using MATLAB.

2. (Effect of zeros and additional poles)

46. Consider the two nonminimum phase systems,

$$G_1(s) = -\frac{2(s-1)}{(s+1)(s+2)}, \quad (1)$$

$$G_2(s) = \frac{3(s-1)(s-2)}{(s+1)(s+2)(s+3)}. \quad (2)$$

- Sketch the unit step responses for $G_1(s)$ and $G_2(s)$, paying close attention to the transient part of the response.
- Explain the difference in the behavior of the two responses as it relates to the zero locations.
- Consider a stable, strictly proper system (that is, m zeros and n poles, where $m < n$). Let $y(t)$ denote the step response of the system. The step response is said to have an undershoot if it initially starts off in the “wrong” direction. Prove that a stable, strictly proper system has an undershoot if and only if its transfer function has an *odd* number of *real* RHP zeros.

3. (Stability)

51. A measure of the degree of instability in an unstable aircraft response is the amount of time it takes for the *amplitude* of the time response to double (see Fig. 3.65), given some nonzero initial condition.

(a) For a first-order system, show that the **time to double** is

$$\tau_2 = \frac{\ln 2}{p},$$

where p is the pole location in the RHP.

(b) For a second-order system (with two complex poles in the RHP), show that

$$\tau_2 = \frac{\ln 2}{-\zeta\omega_n}.$$

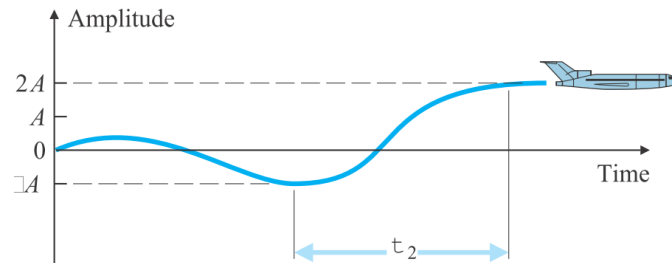


Figure 3.65: Time to double

4. (Stability)

55. The transfer function of a typical tape-drive system is given by

$$KG(s) = \frac{K(s+4)}{s[(s+0.5)(s+1)(s^2+0.4s+4)]},$$

where time is measured in milliseconds. Using Routh's stability criterion, determine the range of K for which this system is stable when the characteristic equation is $1 + KG(s) = 0$.