Midterm Exam, Control System, 110-1 (2021) 姓名：

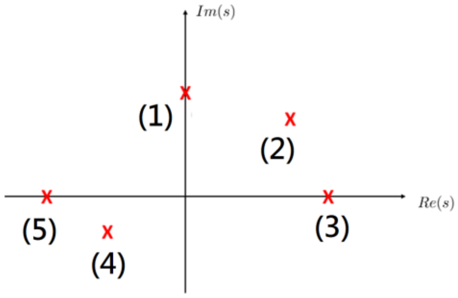
Date: Friday, November 12, 2021. Time: 9am-11am. 學號：

Closed books, closed notes, no calculators. 系級：

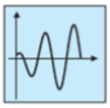
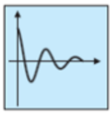
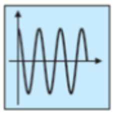
Only pens and erasers are allowed.

Part A. [30%] Find the best choice. (評分標準：只需寫出正確選項)

For the pole locations shown in the figure below, please identify the corresponding responses.

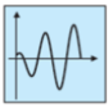
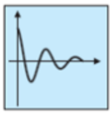
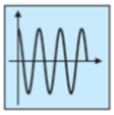


(A1) The response corresponds to Position (1) is:

(A)  ; (B)  ; (C)  ; (D)  .

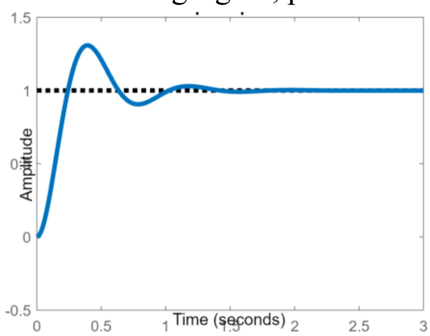
=> D

(A2) The response corresponds to Position (4) is:

(A)  ; (B)  ; (C)  ; (D)  .

=> C

(A3) For the step response shown in the following figure, please find the best possible transfer function:



(A) $\frac{73}{(s^2+6s+73)}$; (B) $\frac{72}{(s^2+17s+72)}$; (C) $\frac{400}{(s^2+6s+409)}$; (D) $\frac{400}{(s^2+40s+375)}$;

=> A

DC gains: (A) 1, (B) 1, (C) 400/409, (D) 400/375, => (A) and (B) are OK.

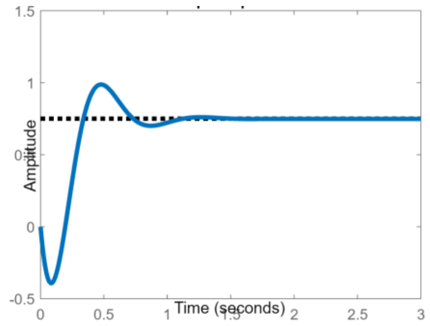
Poles: (A) -3 +- 8j, (B) -8, -9, (C) -3 +- 20j, (D) -15, -25

There are oscillations. Hence, (A) and (C) are possible.

Oscillation Period is around 1 sec. Oscillation Frequency = 2*Pi/(1) = 6.28.

⇨ **(A) is the answer.**

(A4) For the step response shown in the following figure, please find the best possible transfer function:



(A) $\frac{(60-10s)}{(s^2+8s+80)}$; (B) $\frac{(20-10s)}{(s^2+8s+80)}$; (C) $\frac{(20-10s)}{(s^2+12s+32)}$; (D) $\frac{(60-10s)}{(s^2+12s+32)}$;

=> A

Initially, go to negative direction. All have Unstable zero. All answers are possible.

DC gains: (A) 60/80, (B) 20/80, (C) 20/32, (D) 60/32, => (A) and (C) are OK.

Poles: (A) -4 +- 8j, (B) -4 +- 8j, (C) -8, -4, (D) -8, -9,

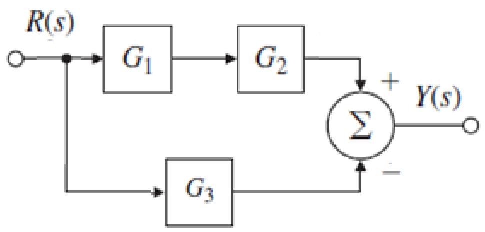
There are oscillations. Hence, (A) and (B) are possible.

Oscillation Period is around 1 sec. Oscillation Frequency = 2*Pi/(1) = 6.28.

=> (A) and (B) are OK.

(A) is the answer.

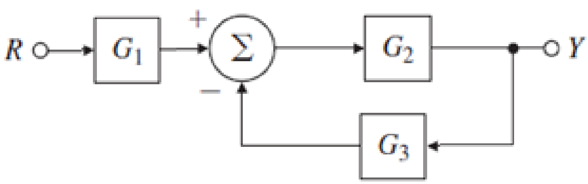
(A5) For the block diagram shown in the following figure, please find the transfer function from R to Y:



(A) $G_1 + G_2 + G_3$; (B) $G_1G_2 - G_3$; (C) $G_1 + G_2G_3$; (D) $G_1G_2 + G_3$;

=> B

(A6) For the block diagram shown in the following figure, please find the transfer function from R to Y:



(A) $\frac{G_1G_2}{1+G_1G_2G_3}$; (B) $\frac{G_1G_2}{1+G_2G_3}$; (C) $\frac{G_1G_2G_3}{1+G_1G_2G_3}$; (D) $\frac{G_1}{1+G_2G_3}$;

=> B

Part B. [70%] Write down proper description for the following problems.

(B1) (10%=5%+5%)

A unit negative feedback system has the following open-loop transfer function:

$$G(s) = \frac{1}{(a\,s + b)}$$

(a) Compute the sensitivity of the closed-loop transfer function to changes in parameter *a*.
(b) Compute the sensitivity of the closed-loop transfer function to changes in parameter *b*.

Solution:

(a)

$$G(s) = \frac{1}{(a\,s + b)}$$

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{\frac{1}{(a\,s+b)}}{1 + \frac{1}{(a\,s+b)}} = \frac{1}{a\,s + (b + 1)}$$

$$S_a^T = \frac{a}{T}\frac{dT}{da} = \frac{a(a\,s + (b + 1))}{1}\,\frac{-s}{(a\,s + (b + 1))^2}$$

$$= \frac{-\,a\,s}{a\,s + (b + 1)}$$

(b)

$$S_b^T = \frac{b}{T}\frac{dT}{db} = \frac{b(a\,s + (b + 1))}{1}\,\frac{-1}{(a\,s + (b + 1))^2}$$

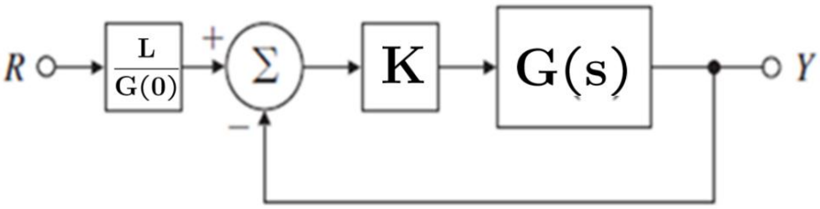
$$= \frac{-\,b}{a\,s + (b + 1)}$$

(B2) (30%=5%*6)

For a second-order system with transfer function:

$$G(s) = \frac{16}{s^2 + s + 16}.$$

Assume that System *G(s)* is added into the following block diagram, where *K* and *L* are two constants:



(a) Find the poles and zeros of system *G(s)*.
(b) Determine whether the system is stable and why?
(c) Determine the overshoot and rise time of system *G(s)* if unit-step input is applied.
(d) Determine the closed-loop transfer function *T(s)* from *R* to *Y*, in terms of *K* and *L*.
(e) Determine *L* such that the steady-state value of the of the closed-loop system *T(s)* is equal to 1 if *R* is the unit-step input and *K=3*.
(f) Determine the overshoot and rise time of the closed-loop system *T(s)* if *R* is the unit-step input and *K=3*.

Solution:

(a)

$$G(s) = \frac{16}{s^2 + s + 16}$$

Poles: $s^2 + s + 16 = 0$, s = -0.5 +-3.97j

Zeros: none

(b)

Real(poles) < 0, then the open-loop system is stable.

(c)

$$G(s) = \frac{16}{s^2 + s + 16} = \frac{w_n{}^2}{s^2 + 2\,\zeta\, w_n\, s + w_n{}^2}$$

$$\Rightarrow w_n = 4, \quad \zeta = 0.125$$

From the M_p vs ζ plot, M_p is about 63%

$$\Rightarrow t_r \cong \frac{1.8}{w_n} = \frac{1.8}{4} = 0.45(sec)$$

(d)

$$G(0) = \frac{16}{16} = 1$$

$$\frac{Y}{R} = T(s) = \frac{\frac{L}{G(0)} K G(s)}{1 + K G(s)} = \frac{LK \frac{16}{s^2+s+16}}{1 + K \frac{16}{s^2+s+16}}$$

$$= \frac{16LK}{s^2 + s + 16 + 16K}$$

(e)

$$Y(s) = R(s)\, T(s)$$

$$= \frac{1}{s} \frac{16LK}{s^2 + s + 16 + 16K}$$

$$y_{ss} = \lim_{s \to 0} s \frac{1}{s} \frac{16LK}{s^2 + s + 16 + 16K}$$

$$= \frac{16LK}{16 + 16K} = \frac{16 \times 3 \times L}{16 + 16 \times 3} = 1$$

$$\to L = 4/3$$

(f)

$$\frac{Y}{R} = T(s) = \frac{\frac{L}{G(0)} K G(s)}{1 + K G(s)} = \frac{16LK}{s^2 + s + 16 + 16K}$$

$$= \frac{48L}{s^2 + s + 64} = \frac{{w_n}^2}{s^2 + 2\,\zeta\, w_n\, s + {w_n}^2}$$

$$\to 2\,\zeta\, w_n = 1, \quad w_n^2 = 64$$

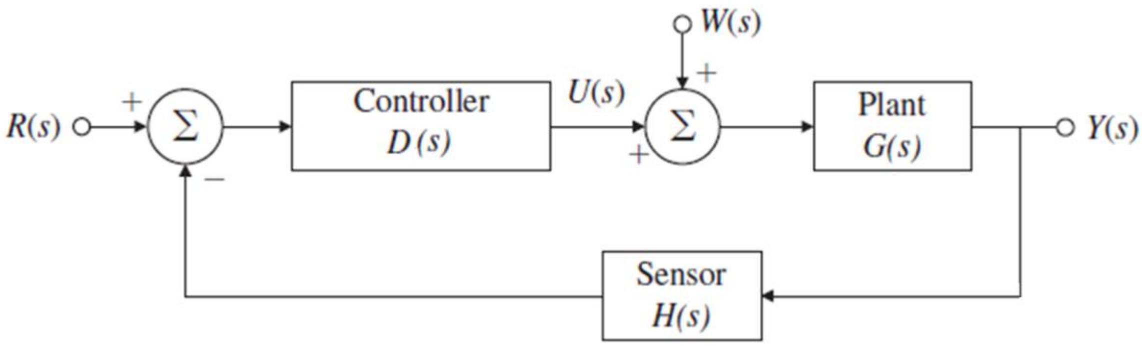
$$\to w_n = 8 \quad \zeta = 1/16 = 0.0625$$

From the M_p vs ζ plot, M_p is about 80%

$$\Rightarrow t_r \cong \frac{1.8}{w_n} = \frac{1.8}{8} = 0.225(sec)$$

(B3) (30%=5%*6)

A standard feedback control block diagram is shown as follows:



where $G(s) = \dfrac{2}{s}; \quad D(s) = \dfrac{(s+8)}{(s+2)}; \quad H(s) = \dfrac{100}{(s+100)}.$

(a) Let $W = 0$ and compute the transfer function from R to Y.
(b) Let $R = 0$ and compute the transfer function from W to Y.
(c) What is the tracking error if R is a unit-step input and $W = 0$?
(d) What is the tracking error if R is a unit-ramp input and $W = 0$?
(e) What is the tracking error if R is a unit-step input and W is a 0.1-unit-step input?
(f) What is the system type with respect to the reference inputs R and the corresponding error coefficients?

Solution:

(a)

$$G(s) = \frac{2}{s}; \quad D(s) = \frac{(s+8)}{(s+2)}; \quad H(s) = \frac{100}{(s+100)}$$

$$T_{RY}(s) = \frac{D(s)\,G(s)}{1 + D(s)\,G(s)\,H(s)} = \frac{\frac{(s+8)}{(s+2)}\frac{2}{s}}{1 + \frac{(s+8)}{(s+2)}\frac{2}{s}\frac{100}{(s+100)}}$$

$$= \frac{2s^2 + 216s + 1600}{s^3 + 102s^2 + 400s + 1600}$$

(b)

$$T_{WY}(s) = \frac{G(s)}{1 + D(s)\,G(s)\,H(s)} = \frac{\frac{2}{s}}{1 + \frac{(s+8)}{(s+2)}\frac{2}{s}\frac{100}{(s+100)}}$$

$$= \frac{2s^2 + 204s + 400}{s^3 + 102s^2 + 400s + 1600}$$

(c)

$$E(s) = R(s) - Y(s) = R(s) - T_{RY}(s)\,R(s)$$

$$= R(s)\,(1 - T_{RY}(s)\,)$$

$$= R(s)\,(1 - \frac{2s^2 + 216s + 1600}{s^3 + 102s^2 + 400s + 1600})$$

$$= R(s)\,(\frac{s^3 + 100s^2 + 184s}{s^3 + 102s^2 + 400s + 1600})$$

$$e_{ss} = \lim_{s \to 0} s\,\frac{1}{s}\,(\frac{s^3 + 100s^2 + 184s}{s^3 + 102s^2 + 400s + 1600}) = 0$$

(d)

$$e_{ss} = \lim_{s \to 0} s\,\frac{1}{s^2}\,(\frac{s^3 + 100s^2 + 184s}{s^3 + 102s^2 + 400s + 1600}) = \frac{184}{1600} = 0.115$$

(e)

$$Y(s) = Y_R(s) + Y_W(s) = T_{RY}(s)\,R(s) + T_{WY}(s)\,W(s)$$

$$= T_{RY}(s)\,\frac{1}{s} + T_{WY}(s)\,\frac{0.1}{s}$$

$$E(s) = R(s) - Y(s)$$

$$= \frac{1}{s} - T_{RY}(s)\,\frac{1}{s} + T_{WY}(s)\,\frac{0.1}{s}$$

$$= \frac{1}{s}\,(1 - T_{RY}(s) - 0.1\,T_{WY}(s))$$

$$= \frac{1}{s}\,(1 - \frac{2s^2 + 216s + 1600}{s^3 + 102s^2 + 400s + 1600} - \frac{0.1\,(\,2s^2 + 204s + 400\,)}{s^3 + 102s^2 + 400s + 1600})$$

$$= \frac{1}{s}\,(\frac{s^3 + 99.8s^2 + 163.6s - 40}{s^3 + 102s^2 + 400s + 1600})$$

$$e_{ss} = \lim_{s \to 0} s\,\frac{1}{s}\,(\frac{s^3 + 99.8s^2 + 163.6s - 40}{s^3 + 102s^2 + 400s + 1600}) = \frac{-40}{1600} = -0.025$$

(f)

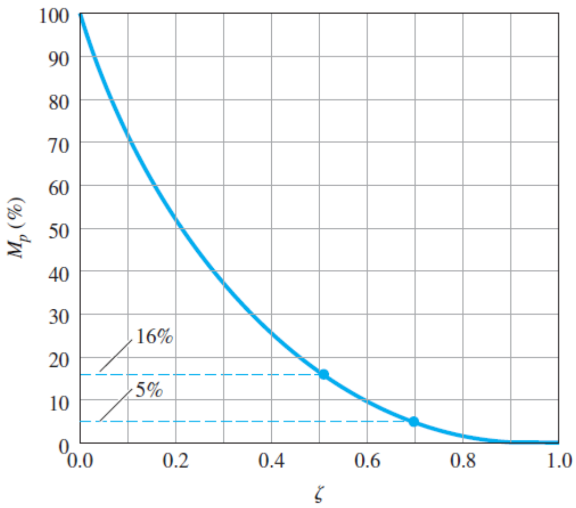
From (c) and (d), the system is System Type 1.

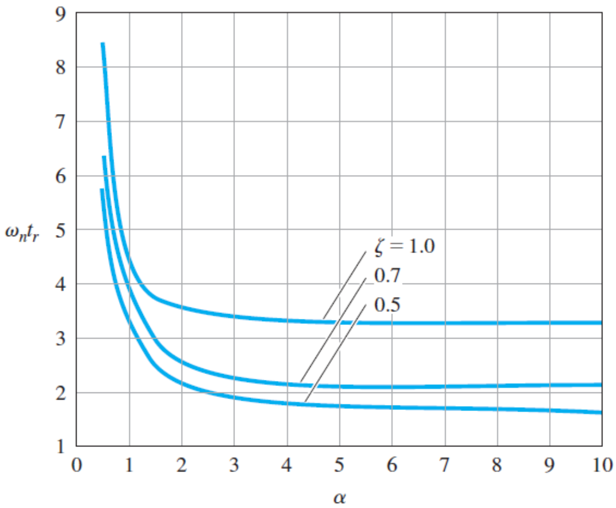
$$k_v = \frac{1}{|e_{ramp}|} = \frac{1}{|0.115|} = 8.7$$

[Helpful Information]

$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem

$$t_r \cong \frac{1.8}{w_n},\quad t_p = \frac{\pi}{w_d},\quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}},\quad t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$





Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

Type of Controller	Optimum Gain
P	$k_P = 1/RL$
PI	$\begin{cases} k_P = 0.9/RL \\ T_I = L/0.3 \end{cases}$
PID	$\begin{cases} k_P = 1.2/RL \\ T_I = 2L \\ T_D = 0.5L \end{cases}$

Ziegler-Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$\begin{cases} k_P = 0.45K_u \\ T_I = \frac{P_u}{1.2} \end{cases}$
PID	$\begin{cases} k_P = 1.6K_u \\ T_I = 0.5P_u \\ T_D = 0.125P_u \end{cases}$

Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$