Final Exam, Control Systems, 111-1 (2022)	姓名：
Date: Friday, December 23, 2022. Time: 9:30-11:30am.	學號：
Closed books, closed notes, no calculators. Only pens and erasers are allowed.	系級：

(1) (20%=5%+5%+10%)

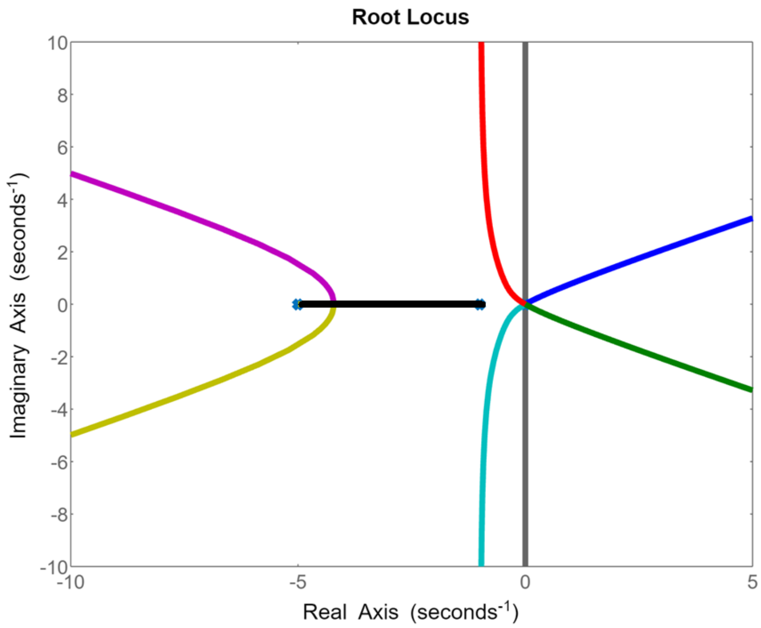
For the characteristic equation:

$$1 + \frac{K}{s^4(s+1)(s+5)} = 0$$

(a) Draw the real-axis segments of the corresponding root locus.
(b) Sketch the asymptotes of the locus for $K \to \infty$.
(c) Sketch the locus.

Solution:

(a)
The characteristic function has 6 poles: s = 0, 0, 0, 0, -1, -5 and no zeros.
For Rule 2, the locus is on the real axis to the left of an odd number of poles and zeros.
That is, at -5 < s < -1, as the BLACK line on the real axis between s = -5 and s = -1, shown in the following figure.

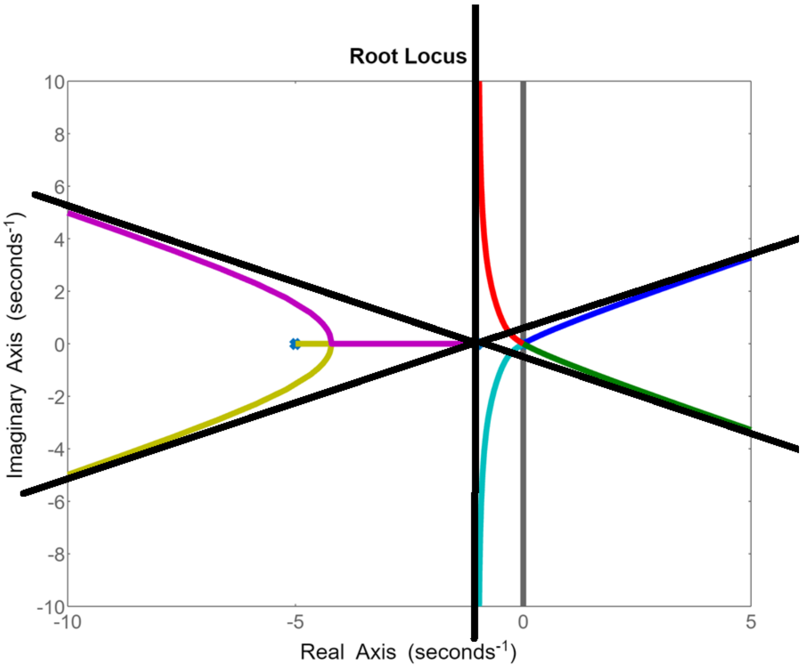


(b)
For Rule 3, n = 6, m = 0.
Thus,

$$\phi_l = \frac{180^o + 360^o(l-1)}{n-m} = \frac{180^o + 360^o(l-1)}{6}$$
$$= \pm 30^o, \ \pm 90^o, \ 150^o,$$

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{0+0+0+0+(-1)+(-5) - 0}{6}$$
$$= -1$$

There are 6 asymptotes centered at s = -1 and at the angles $\pm 30^o, \ \pm 90^o, \ 150^o,$.
As the 6 BLACK lines, centered at s = -1, shown in the following figure.



(c)
For Rule 4, the branches depart from the pole at s = 0 (multiplicity = 4) at the angles:

$$q\ \phi_{l,dep} = \sum \psi_i - \sum_{i \neq l,dep} \phi_i - 180^o - 360^o(l-1)$$
$$4\ \phi_{l,dep} = -180^o - 360^o(l-1)$$
$$\phi_{l,dep} = \pm 45^o, \ \pm 135^o$$

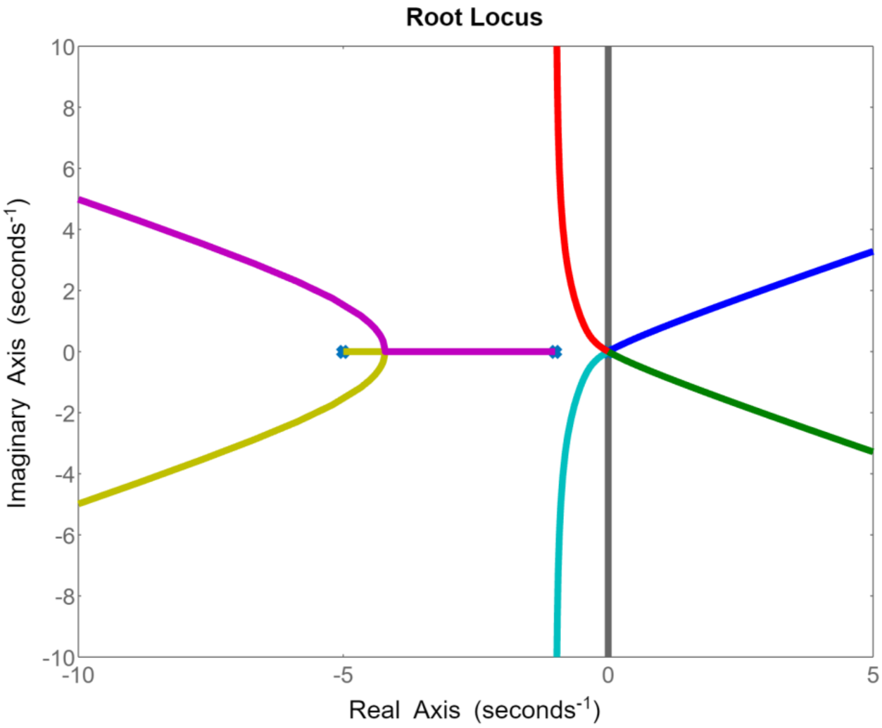
Another branch departs from s = -1 (multiplicity = 1) at the angle:

$$\phi_{l,dep} = 0^o + 4 \times 180^o - 180^o - 360^o(l-1)$$
$$= 180^o,$$

The other branch departs from s = -5 (multiplicity = 1) at the angle:

$$\phi_{l,dep} = 5 \times 180^o - 180^o - 360^o(l-1)$$
$$= 0^o,$$

The locus is shown in the following figure.



(2) (20%=10%+10%)

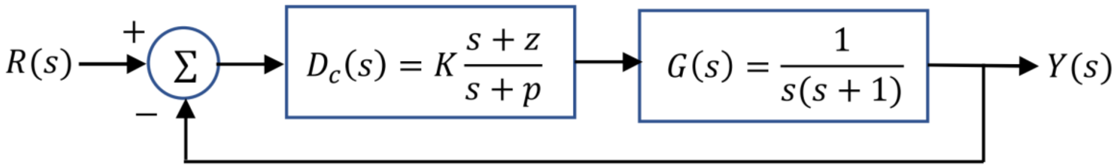
Suppose the unity feedback systems of the following block diagram has an open-loop plant given by

$$G(s) = \frac{1}{s(s+1)}.$$

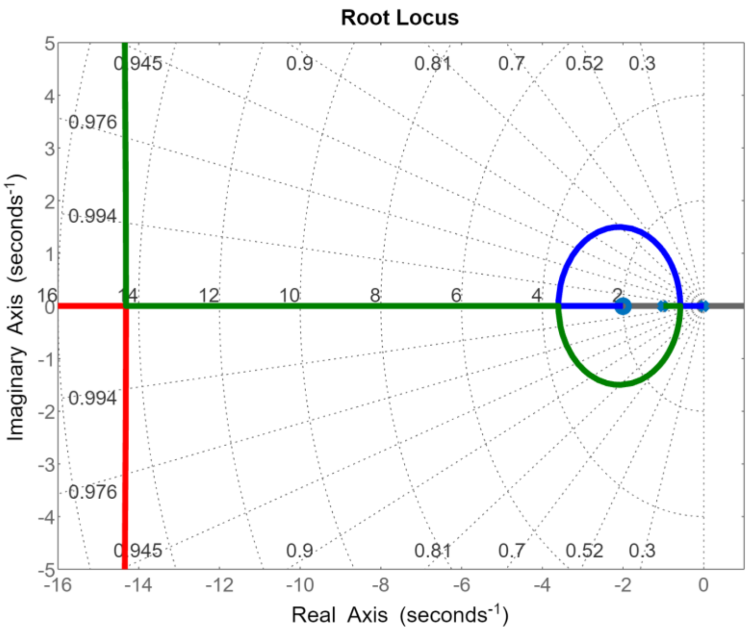
Design a lead compensation

$$D_c(s) = K\frac{(s+z)}{(s+p)}$$

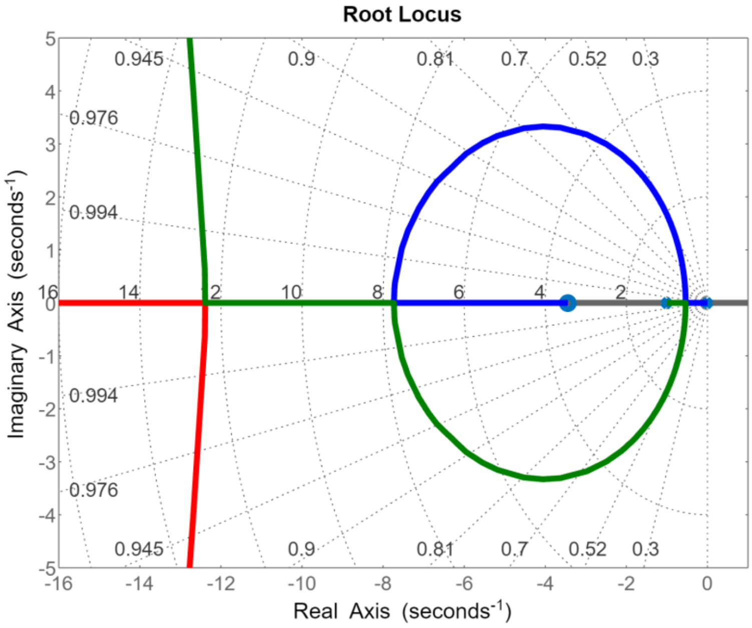
to be added in cascade with the plant so that the dominant poles of the closed-loop system are located at $s = -3.2 \pm 3.2j$.



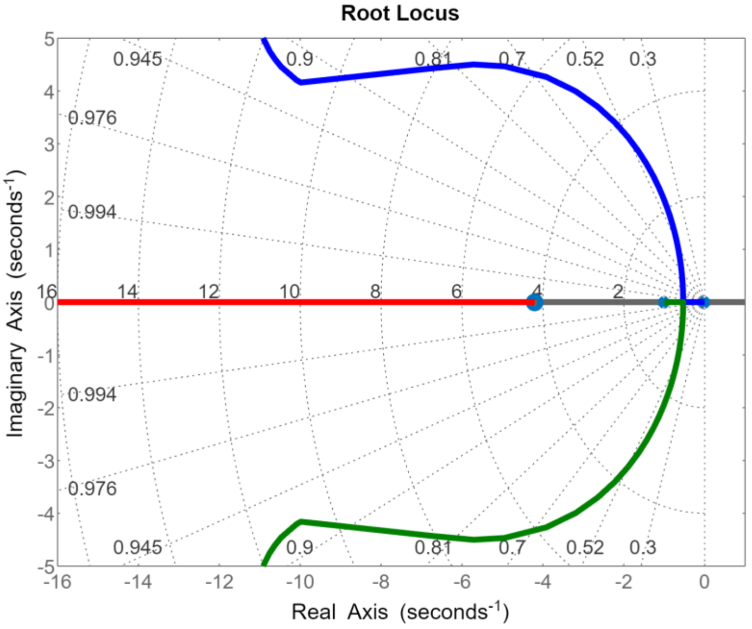
The root loci of the three different designs are shown in the following figures.



$$D_{c1}(s) = K\frac{(s+2)}{(s+30)}$$



$$D_{c2}(s) = K\,\frac{(\,s + 3.44\,)}{(\,s + 30\,)}$$



$$D_{c3}(s) = K\,\frac{(\,s + 4.2\,)}{(\,s + 30\,)}$$

(a) Please identify which design can fulfill the requirement and describe your reason.

(b) For the design, please compute the (rough) value of K.

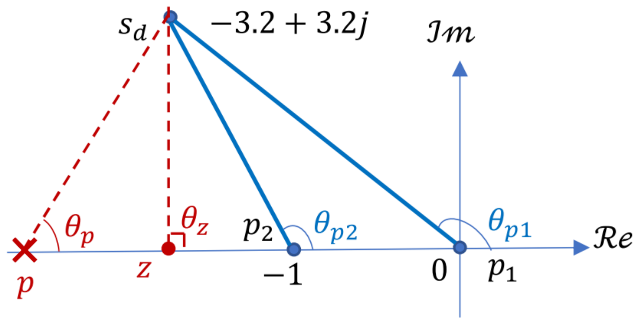
Solution:

(a)

The possible design is $D_{c2}(s) = K\,\frac{(\,s + 3.44\,)}{(\,s + 30\,)}$

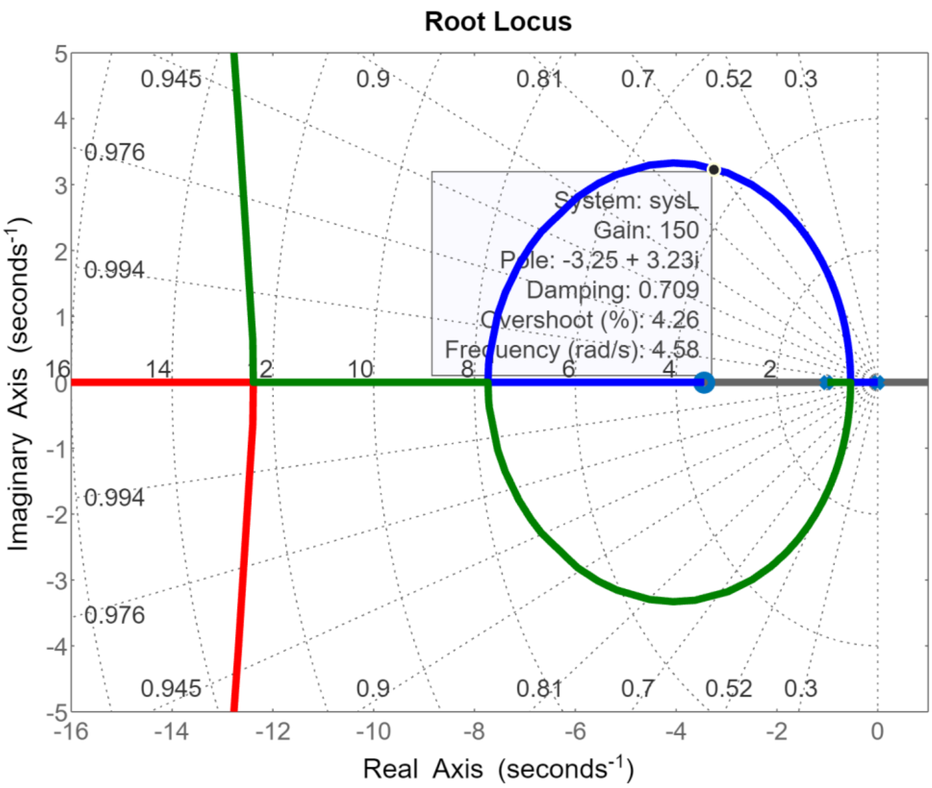
because the locus goes through $s = -3.2 \pm 3.2j$ at some value of K.

(b)



$$K = \frac{(\sqrt{3.2^2 + 3.2^2})(\sqrt{3.2^2 + 2.2^2})(\sqrt{3.2^2 + 26.8^2})}{(3.2)}$$

$$\approx 150$$



(3) (20%=10%+10%)

For the open-loop transfer function of unity feedback control system:

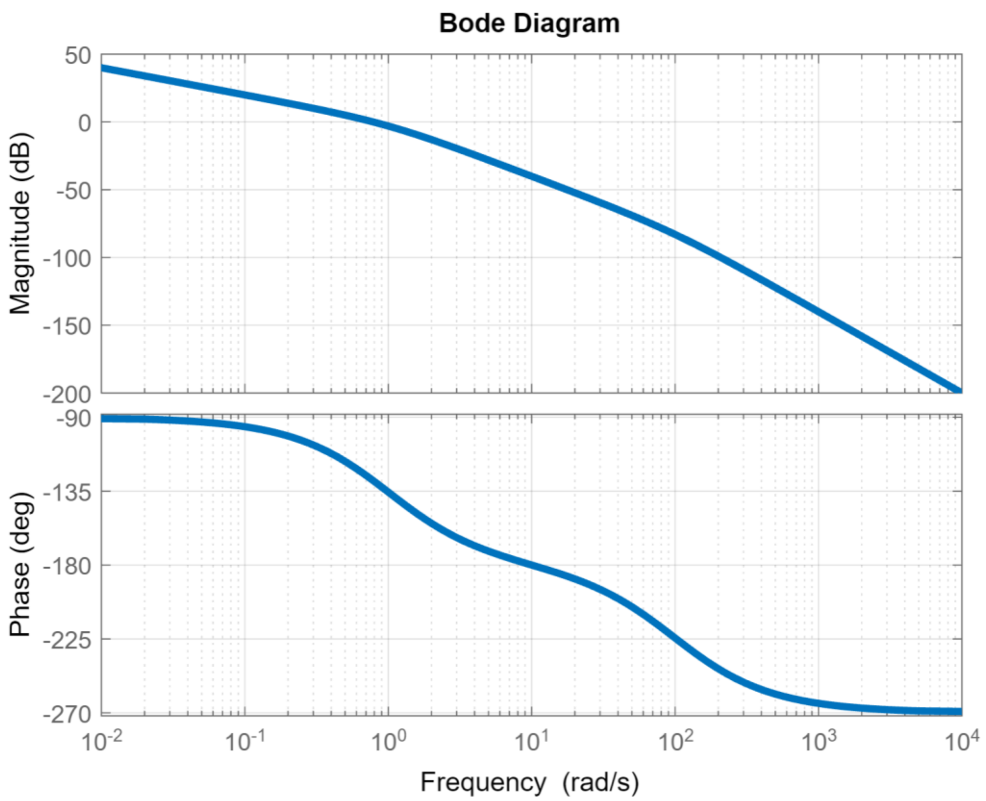
$$L(s) = \frac{100}{s\,(s+1)\,(s+100)}.$$

(a) Sketch the Bode magnitude and phase plots.

(b) Find the gain margin, gain crossover frequency, phase margin, and phase crossover frequency.

Solution:

(a)



(b)

GM = 101, at w = 10 rad/s (roughly, GM = 100, at w=10)

PM = 51.3776, at w = 0.7861 rad/s (roughly, PM = 45, at w=1)

(4) (20%=10%+10%)

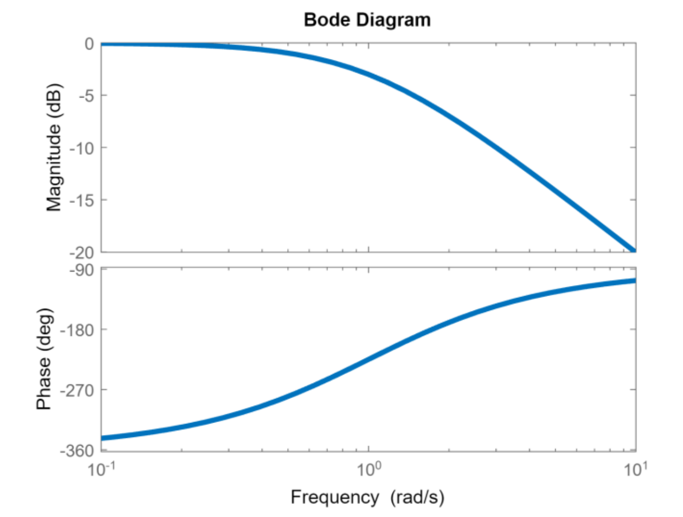
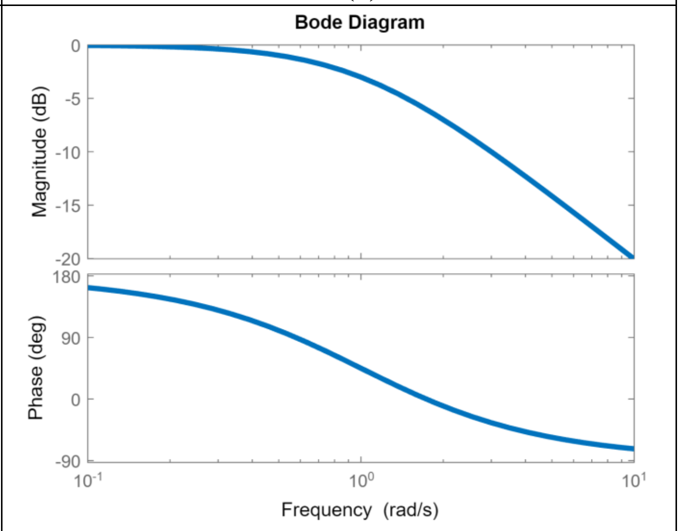
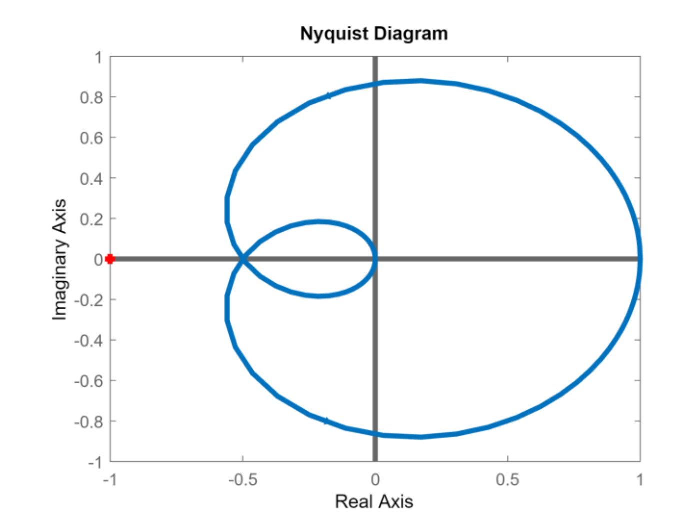
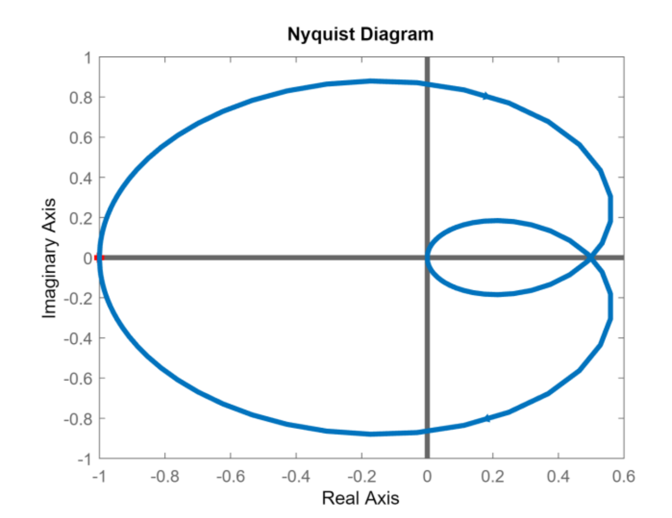
Consider the following two transfer functions:

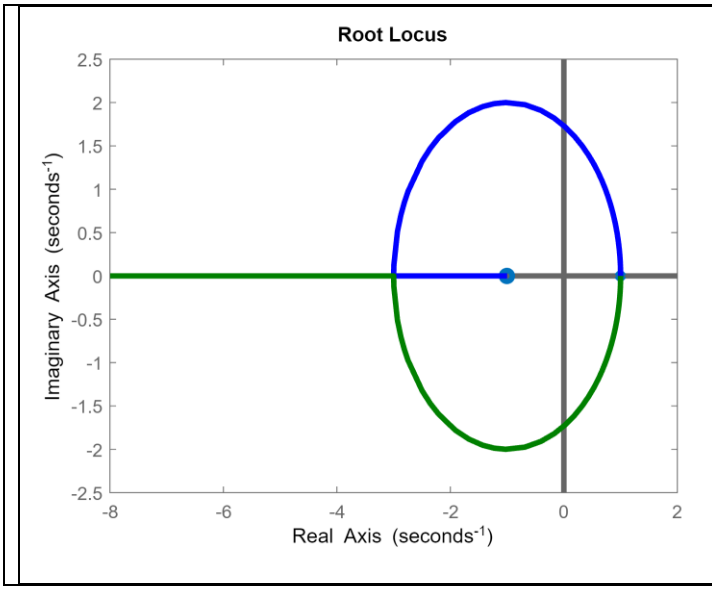
$$G_1(s) = \frac{s+1}{(s-1)^2} \qquad\qquad G_2(s) = \frac{s-1}{(s+1)^2}$$

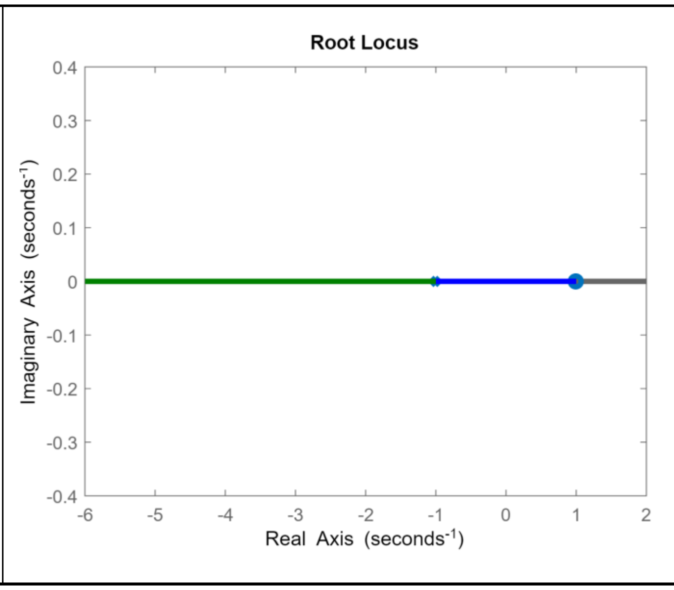
The Bode plot, Nyquist plot, root locus plot of these two transfer functions are shown in the following plots. Please find the detailed answers for the following two questions.

(a) For *G1*(s), please use these plots to determine the ranges of *K* in *K* >0 for which *KG1*(s) is STABLE or UNSTABLE.

(b) For *G2*(s), please use these plots to determine the ranges of *K* in *K* >0 for which *KG2*(s) is STABLE or UNSTABLE.

G1(s)	*G2*(s)
	
	





Solution:

(a) For *G1(s)*

The curve is the case when $K = 1$ and it crosses the real axis at -0.5 and 1.
From the Nyquist plot we can observe that

(1) $-\frac{1}{K} < -\frac{1}{2}$

In this case, we have $0 < K < 2, N = 0, P = 2$, so $Z = 2$.
That is, when $0 < K < 2$, the system is unstable and there are two closed-loop roots in RHP.

(2) $-\frac{1}{2} < -\frac{1}{K} < 0$

In this case, we have $K > 2, N = -2, P = 2$, so $Z = 0$.
That is, when $K > 2$, the system is stable and there are no closed-loop roots in RHP.

(3) $0 < -\frac{1}{K} < 1$

In this case, we have $K < -1, N = -1, P = 2$, so $Z = 1$.
That is, when $K < -1$, the system is unstable and there is one closed-loop root in RHP.

(4) $1 < -\frac{1}{K}$

In this case, we have $-1 < K < 0, N = 0, P = 2$, so $Z = 2$.
That is, when $-1 < K < 0$, the system is unstable and there are two closed-loop roots in RHP.

(e) We can use Routh's criterion to verify that the closed-loop system of KG_1 is stable if $K > 2$. The steps are shown in Figure 5.

Closed-loop transfer function:

$$\frac{KG_1(s)}{1+KG_1(s)} = \frac{K\frac{s+1}{(s-1)^2}}{1+K\frac{s+1}{(s-1)^2}} = \frac{K(s+1)}{s^2-(2-K)s+(1+K)}$$

Characteristic equation: $s^2-(2-K)s+(1+K)=0$

Using Routh's criterion:

$$\begin{array}{lll} s^2: & 1 & 1+K \\ s^1: & -2+K & 0 \\ s^0: & 1+K & \end{array}$$

For stability, $-2+K>0$ and $1+K>0$.

That is, $K>2$ and $K>-1$.

$\therefore$ $K>2 \Rightarrow$ stable, otherwise $\Rightarrow$ unstable.

Figure 5. The steps of using Routh's criterion to verify the condition for stability of $KG_1(s)$.

(b) For *G2(s)*

The curve is the case when $K = 1$ and it crosses the real axis at 0.5 and -1.

From the Nyquist plot we can observe that

(1) $0 < K < 1$

In this case, we have $N = 0, P = 0$, so $Z = 0$.

That is, the system is stable and there are no closed-loop roots in RHP.

(2) $K > 1$

In this case, we have $N = 1, P = 0$, so $Z = 1$.

That is, the system is unstable and there is one closed-loop root in RHP.

(3) $K < -2$

In this case, $N = 2, P = 0$, so $Z = 2$.

That is, the system is unstable and there are two closed-loop roots in RHP.

(4) $-2 < K < 0$

In this case, we have $N = 0, P = 0$, so $Z = 0$.

That is, the system is stable and there are no closed-loop roots in RHP.

(e) We can use Routh's criterion to verify that the closed-loop system of KG_2 is stable if $-2 < K < 1$. The steps are shown in Figure 10.

Closed-loop transfer function :

$$\frac{KG_2(s)}{1+KG_2(s)} = \frac{K\frac{s-1}{(s+1)^2}}{1+K\frac{s-1}{(s+1)^2}} = \frac{K(s-1)}{s^2+(2+K)s+(1-K)}$$

Characteristic equation : $s^2+(2+K)s+(1-K)=0$

Using Routh's criterion:

$$\begin{array}{lll} s^2: & 1 & 1-K \\ s^1: & 2+K & 0 \\ s^0: & 1-K & \end{array}$$

For stability, $2+K>0$ and $1-K>0$.

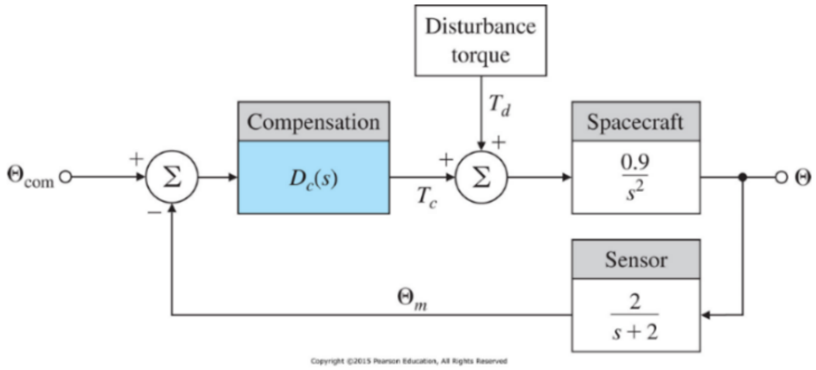
That is, $-2<K<1$.

$\therefore$ $-2<K<1 \Rightarrow$ stable , otherwise $\Rightarrow$ unstable

Figure 10. The steps of using Routh's criterion to verify the condition for stability of $KG_2(s)$.

(5) (20%=10%+10%)

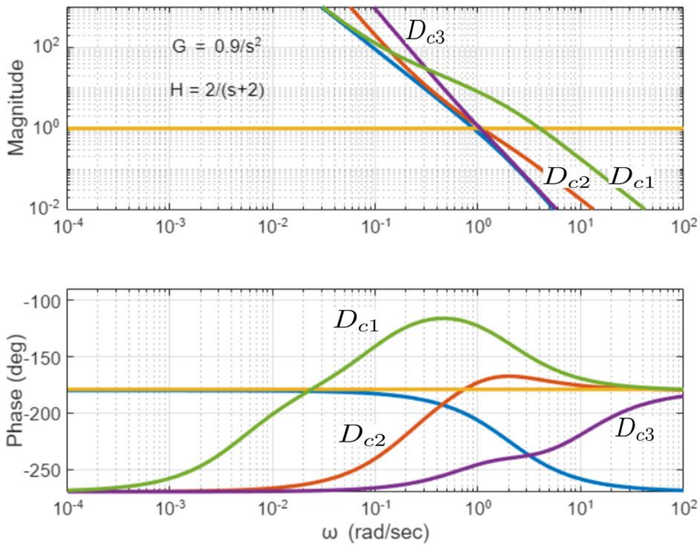
Consider the following system:



where the compensation is the PID controller of the form:

$$D_c(s) = \frac{K}{s}\left[(T_D s + 1)\left(s + \frac{1}{T_I}\right)\right]$$

The following two plots show the Bode magnitude and phase plots of the systems with different PID controllers with different PID gains (K's, TD's, TI's), that is, Dc1(s), Dc2(s), and Dc3(s), respectively.



(a) From the Bode plots, what are possible values of the controller D_{C2}:

(A) $1/T_D=1$, $1/T_I=0.2$ (B) $1/T_D=10$, $1/T_I=5$ (C) $1/T_D=10$, $1/T_I=1$

(D) $1/T_D=0.1$, $1/T_I=0.005$

And please justify your answer by describing proper reason.

(b) From the Bode plots, if PM=60 is needed, which controller will be suitable:

(A) K= 10 and D_{C2} (B) K= 0.5 and D_{C3} (C) K= 0.5 and D_{C1} (D) K= 0.05 and D_{C1}

And please justify your answer by describing proper reason.

Solution:

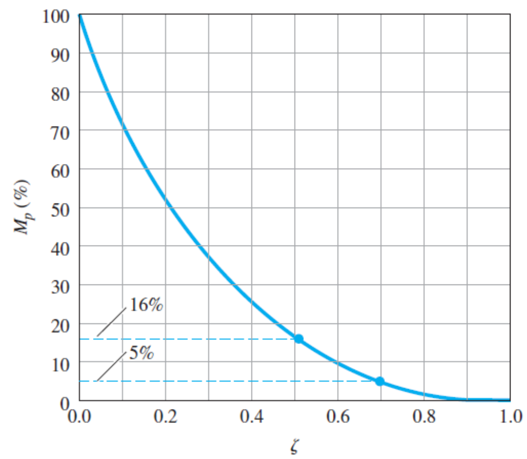
(a) **A**

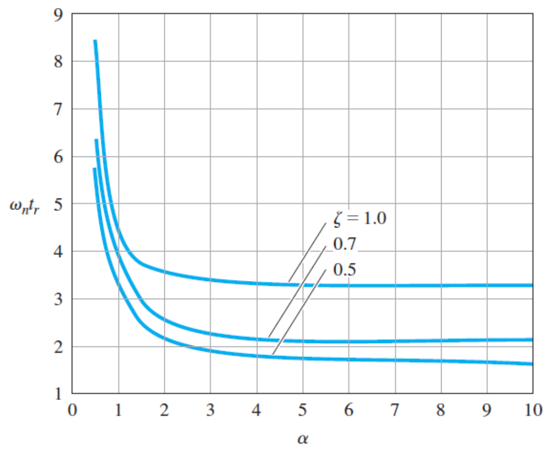
(b) **D**

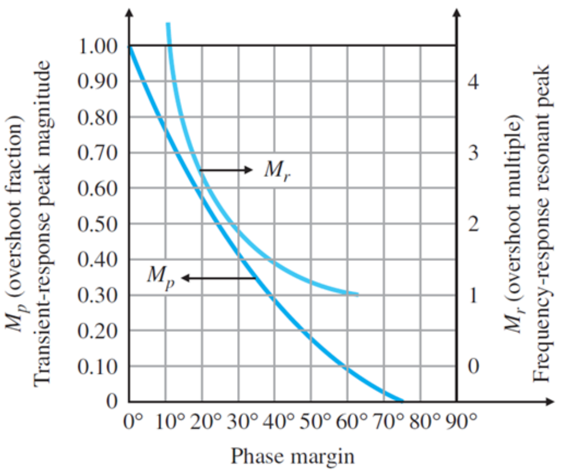
[Helpful Information]

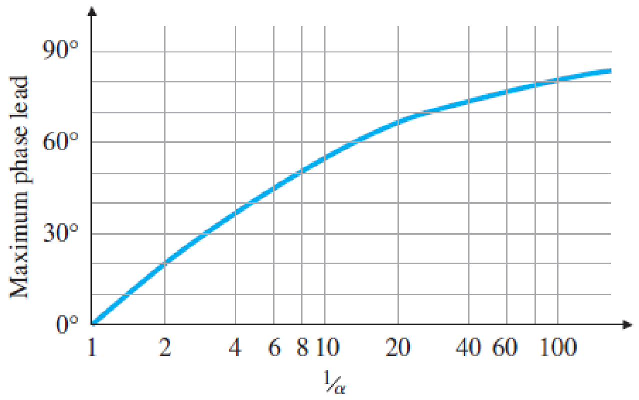
$\lim\limits_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
$\lim\limits_{s\to 0} sF(s)$	$\lim\limits_{t\to\infty} f(t)$	Final Value Theorem

$$t_r \cong \frac{1.8}{w_n},\quad t_p = \frac{\pi}{w_d},\quad M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}},\quad t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma}$$









Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$