Fall 2021 (110-1)

控制系統
Control Systems

Unit 6J
PI Compensation and Lag Compensation

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NTU-EE

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- In many problems,

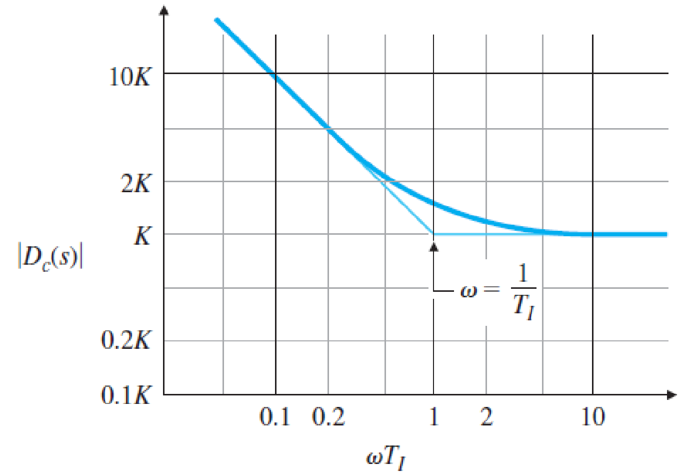
  it is important to keep the bandwidth low

  and also to reduce the steady-state error

- For this purpose,

  a PI Controller

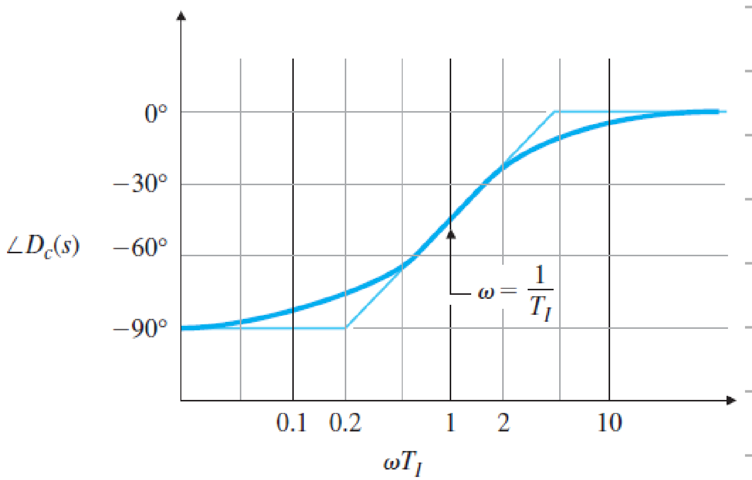
  or Lag Compensator is useful



- PI Controller:

$$D_c(s) = K\left(1 + \frac{1}{T_I}\frac{1}{s}\right)$$

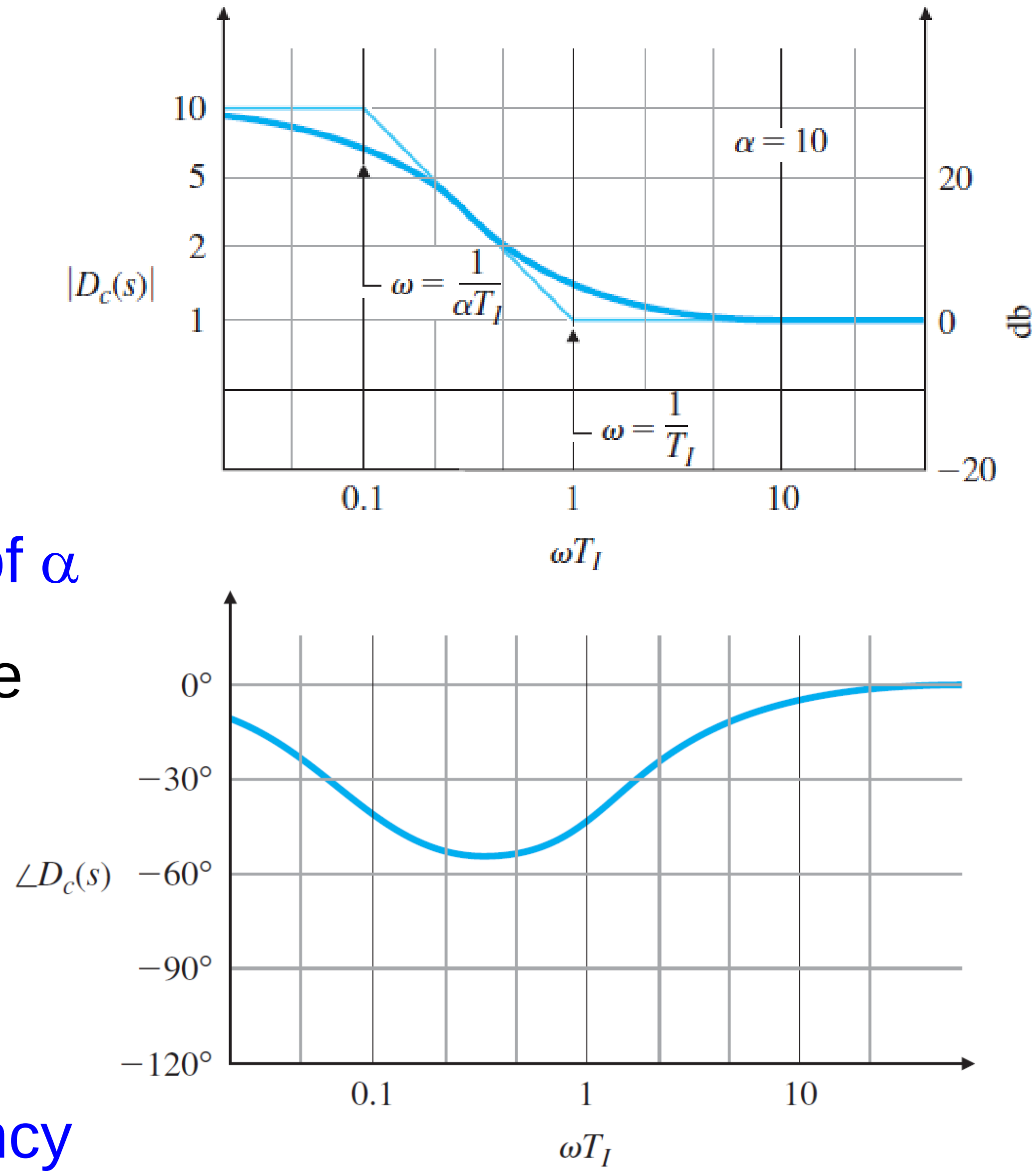
$$= \frac{K}{s}\left(s + \frac{1}{T_I}\right)$$



Lag Compensation

$$D_c(s) = \alpha \frac{T_I s + 1}{\alpha T_I s + 1}, \qquad \alpha > 1$$

- Low frequency:
  - Amplitude: increase
  - Phase: decrease
- Features:
  - Provide additional gain of α in low-frequency range
  - Leave the system sufficient PM
  - Put pole/zero at much lower frequency



Design Procedure for Lag Compensation

1. Determine OL gain K that meet the PM requirements
2. Draw the Bode Plot of the uncompensated system
   with crossover frequency from Step 1, and
   evaluate the low-frequency gain
3. Determine α to meet low-frequency gain error requirement
4. Choose the corner frequency $\omega = 1/T_I$, (the zero)
   to be one octave to one decade below new ω_c
5. The other corner frequency $\omega = 1/\alpha T_I$, (the pole)
6. Iterate on the design. Adjust poles/zeros/gain.

- Example 6.18: Lag-Compensation Design for Temperature Control System

$$K\,G(s) = \frac{K}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{1}+1\right)\left(\frac{s}{2}+1\right)}$$

- $K_p = 9$
- $\text{PM} > 40^o$

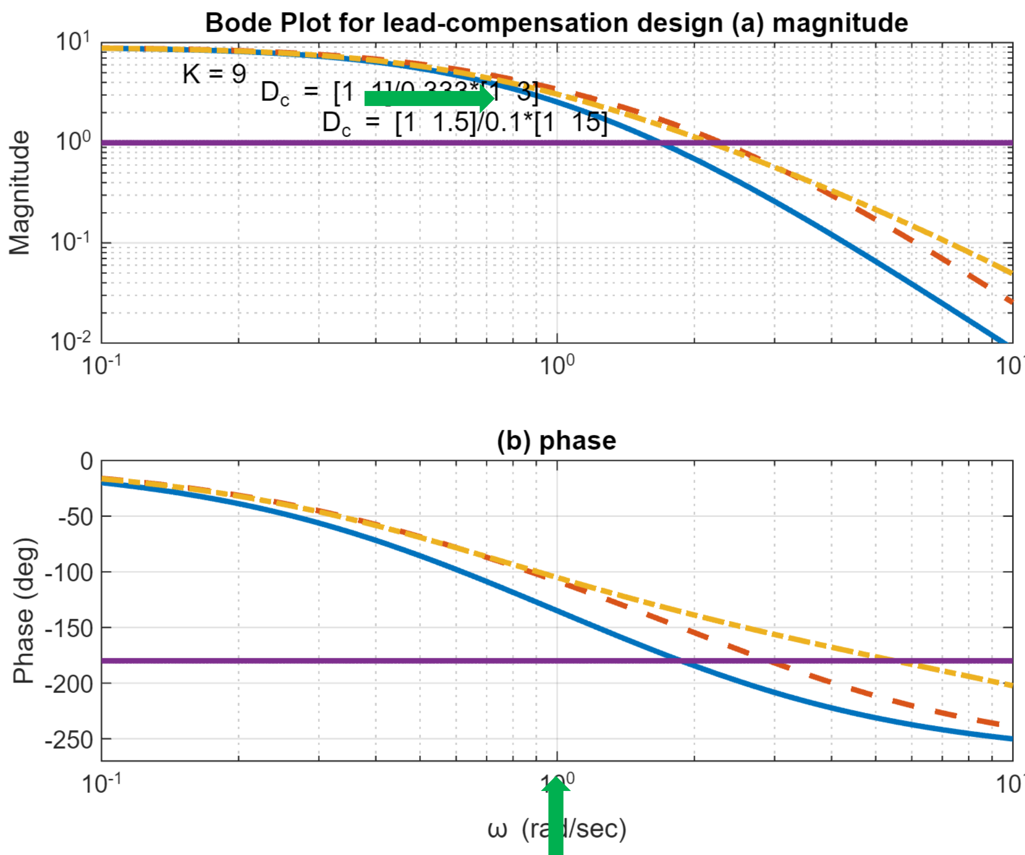
- (1)

  In Ex. 6.16, K = 9

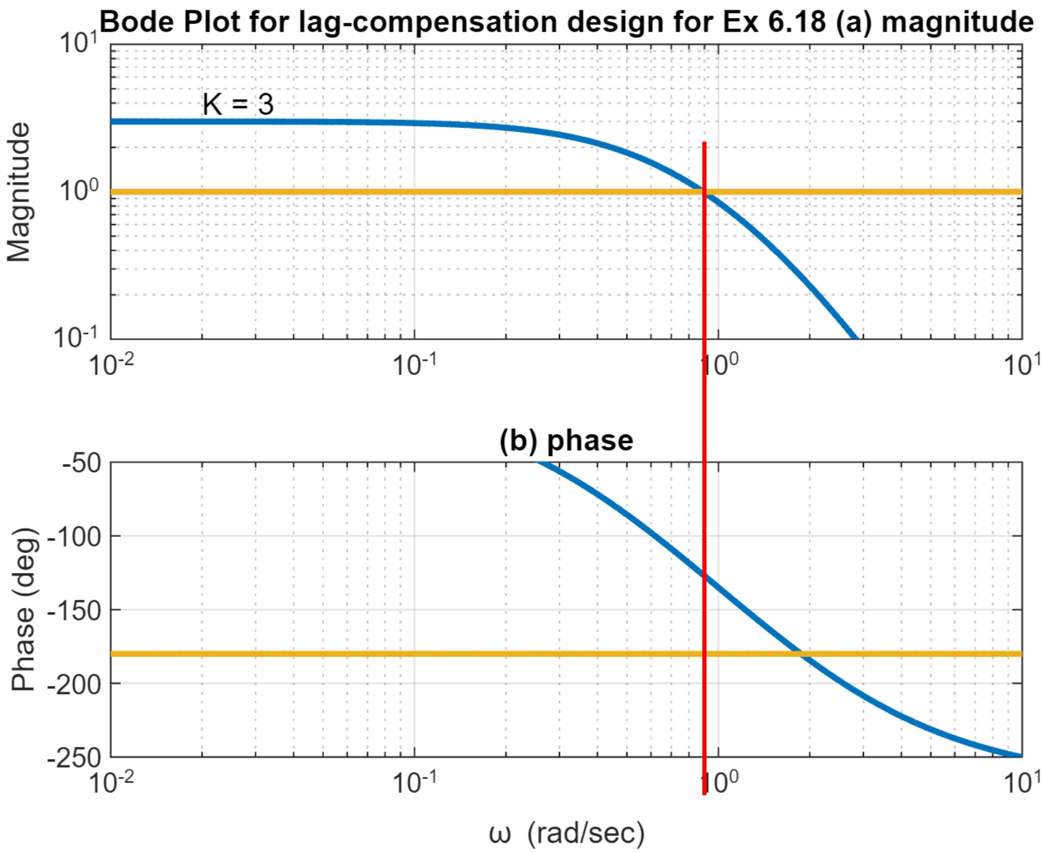
- For $\text{PM} > 40^o$,

  at ω_c ~< 1

- Mag ~= 3
- K = 3



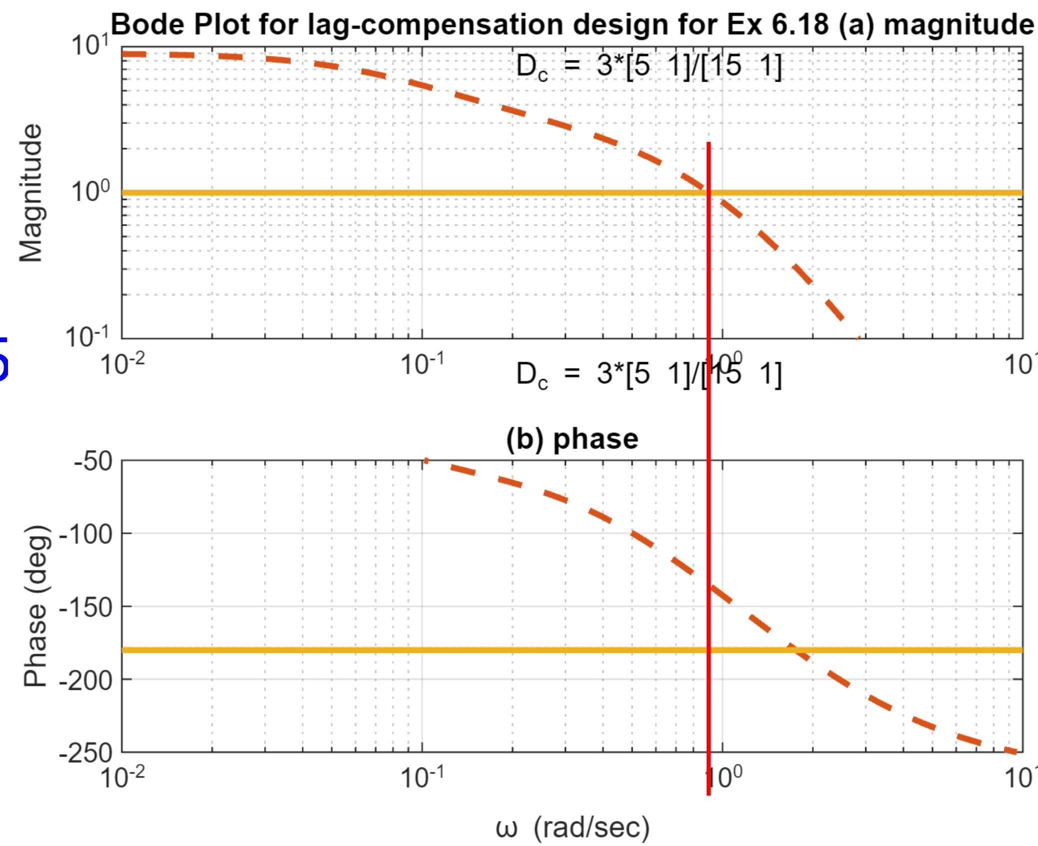
- Example 6.18: Lag-Compensation Design for Temperature Control System
- (2)
- K = 3
- PM ~= 50^o
- Low-frequency gain = 3
- (3)
- The low-frequency gain should be raised by a factor of 3,
- → the lag compensation needs to have $\alpha = 3$



- Example 6.18: Lag-Compensation Design for Temperature Control System
- (4)
- Zero: 1/5 = 0.2
- → $1/T_I$ = 0.2, or $T_I = 5$
- (5)
- $\omega = 1/\alpha T_I$ = 1 / (3x5) = 1/15

$$D_c(s) = 3\,\frac{5\,s + 1}{15\,s + 1}$$

- $K\,D_I(0)G(0) = 3K = 9$
- $K_p = 9,\ PM = 44^o$



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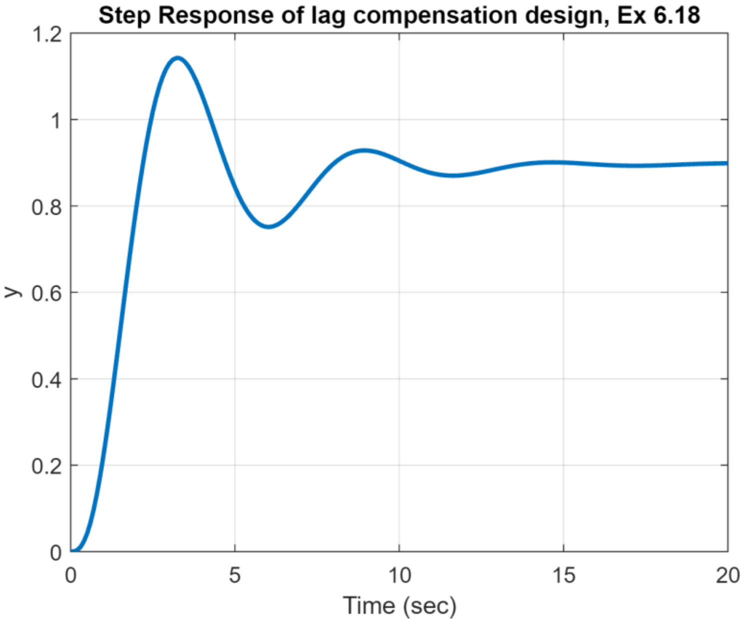
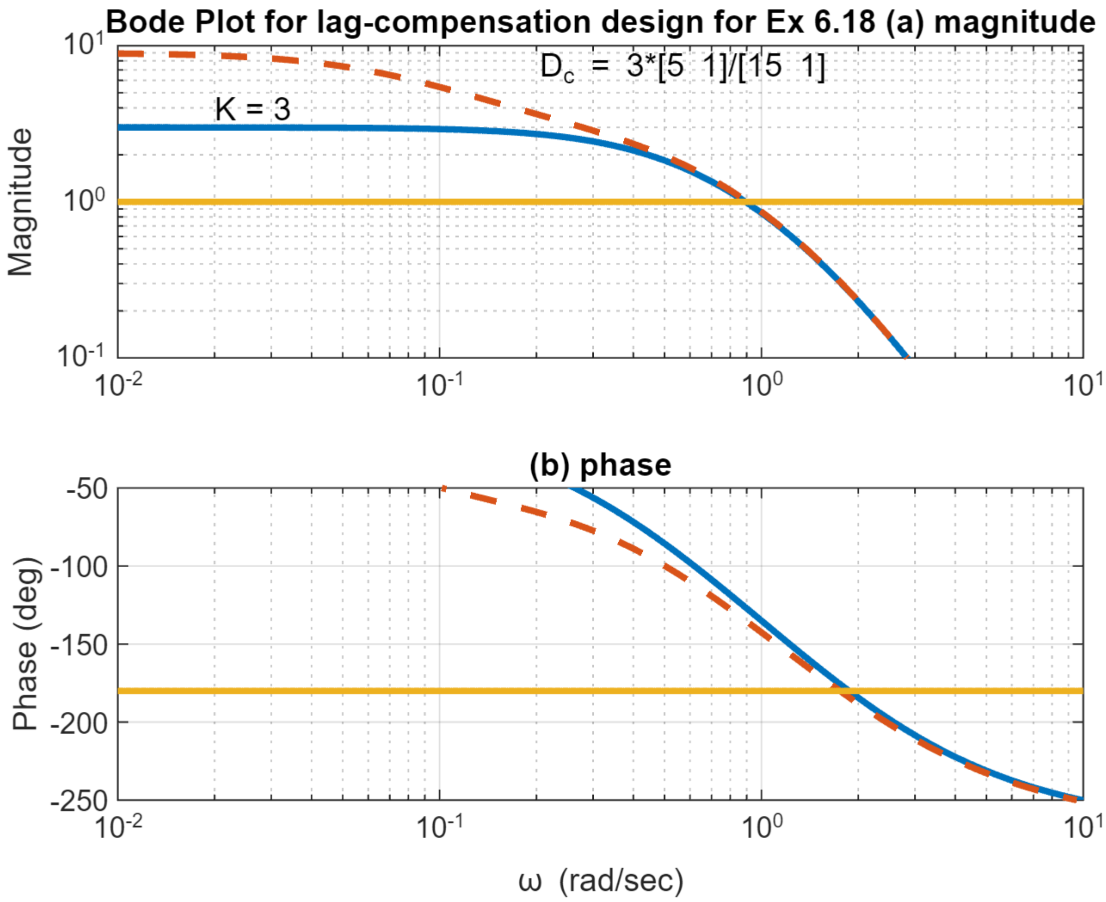
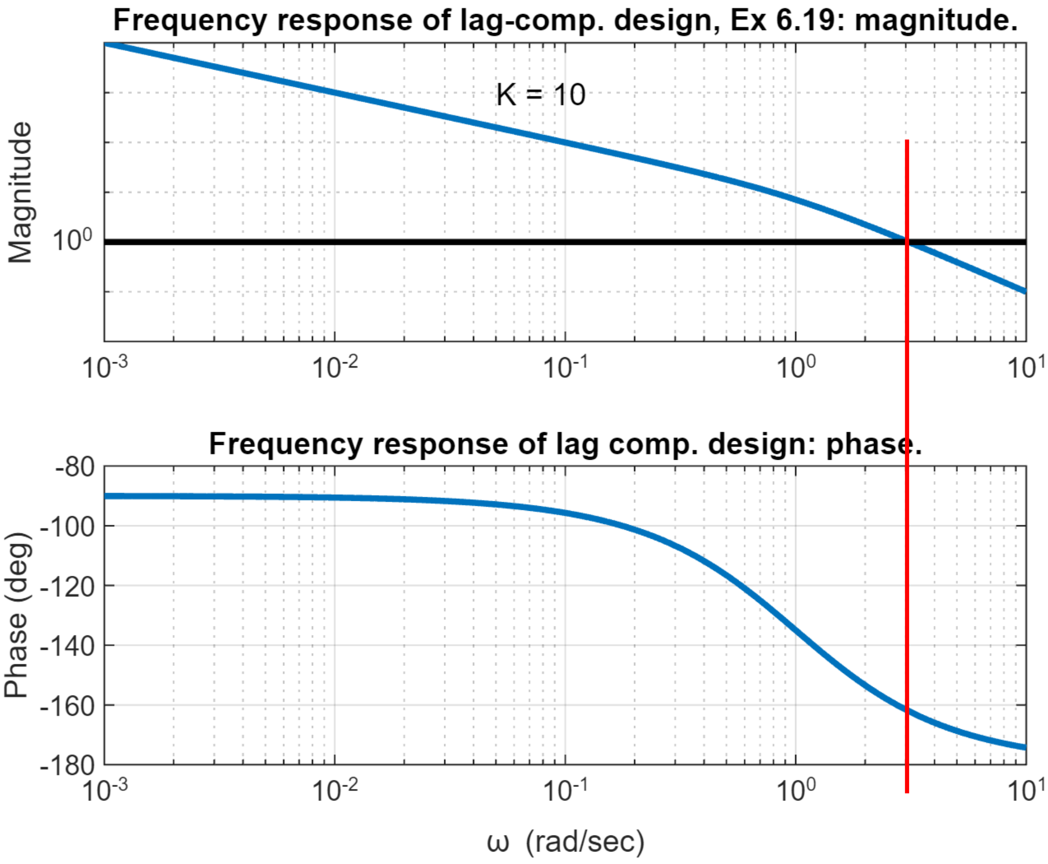




- Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- Error constant: $K_v = 10$
- PM = 45^o
- K = 10
- PM = 20^o at ω_c ~= 3
- Select break points
  - ✓ ω_c is lowered
  - ✓ more favorable PM



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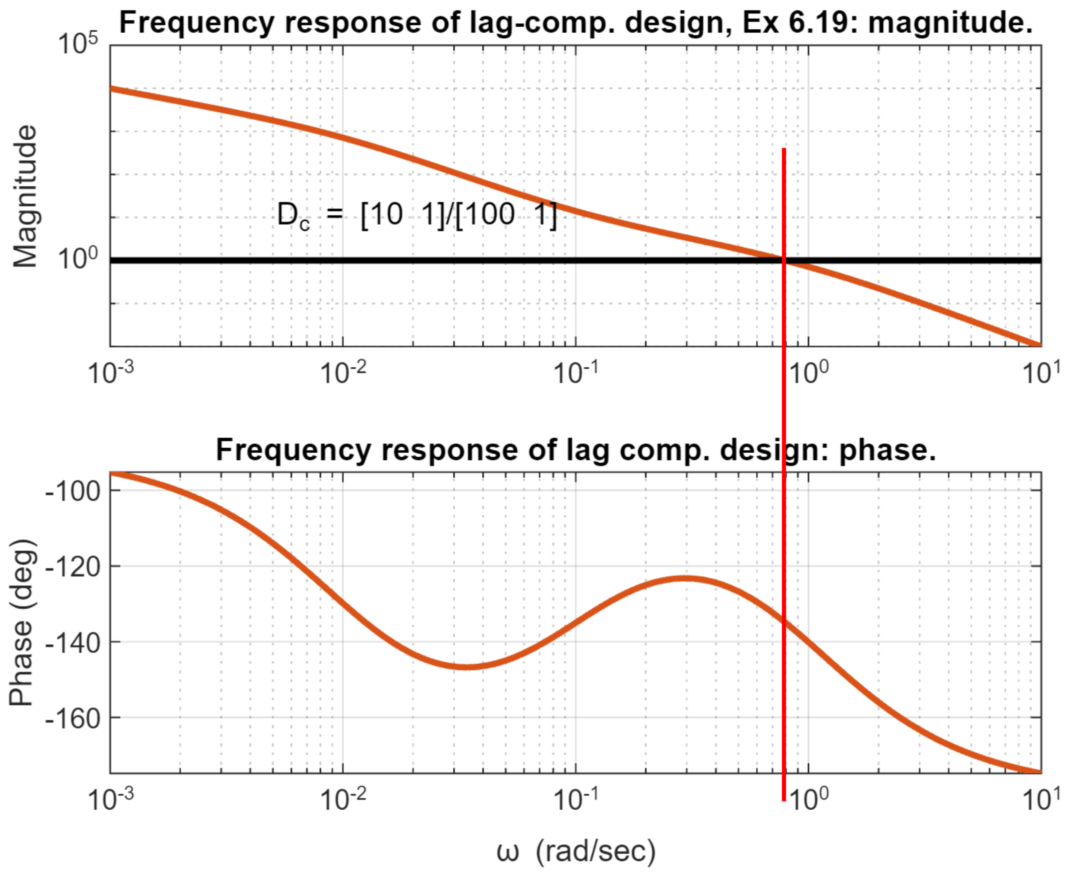
- ➢ Lag pole = 0.01
  - ✓ PM = 50^o



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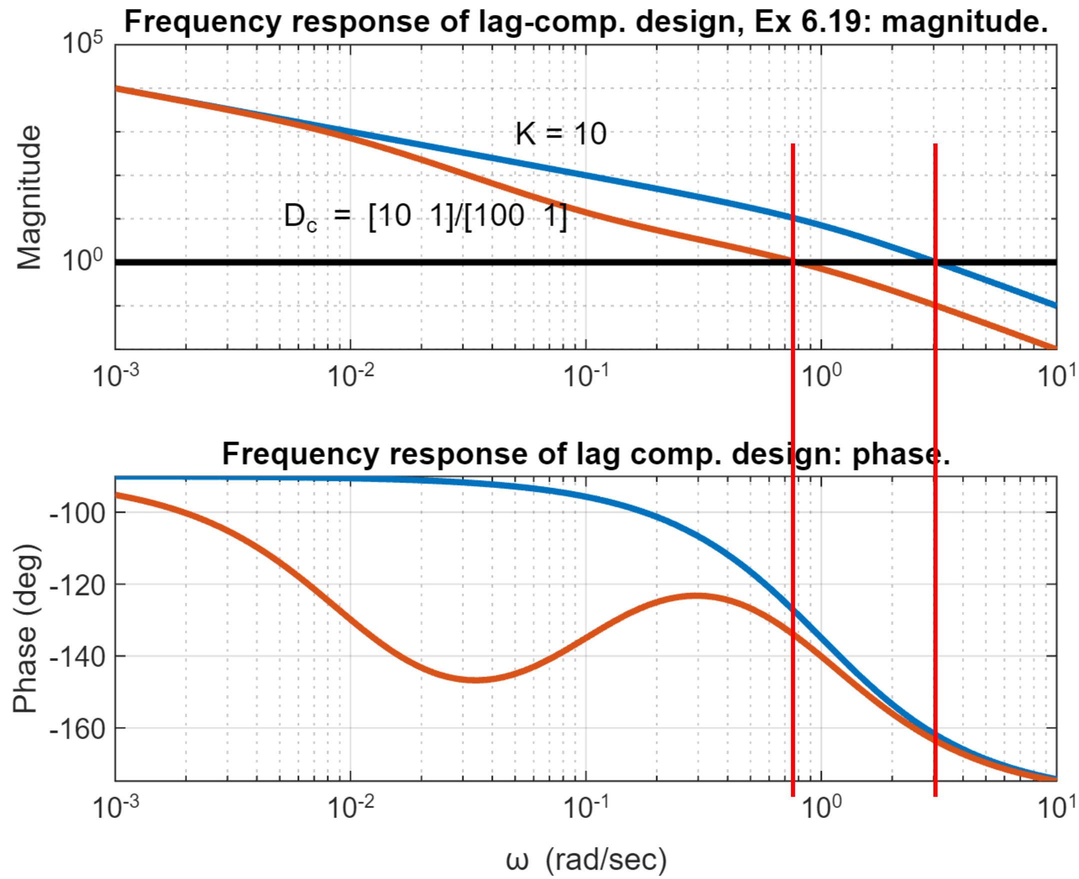

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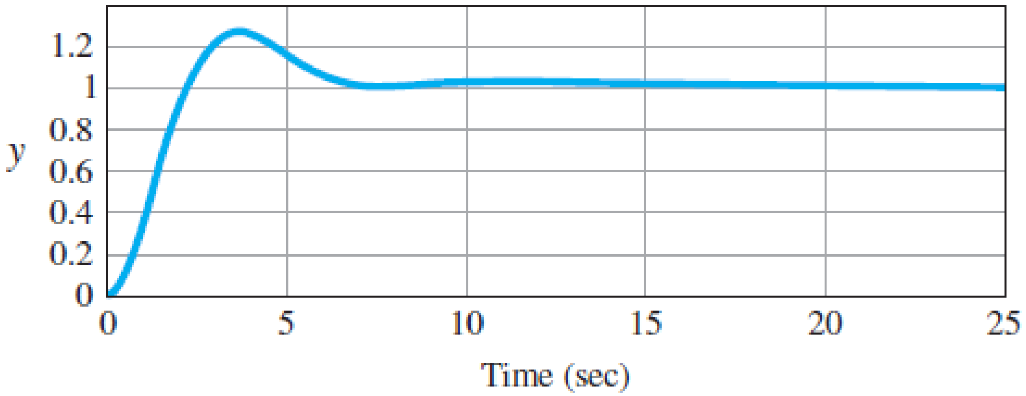
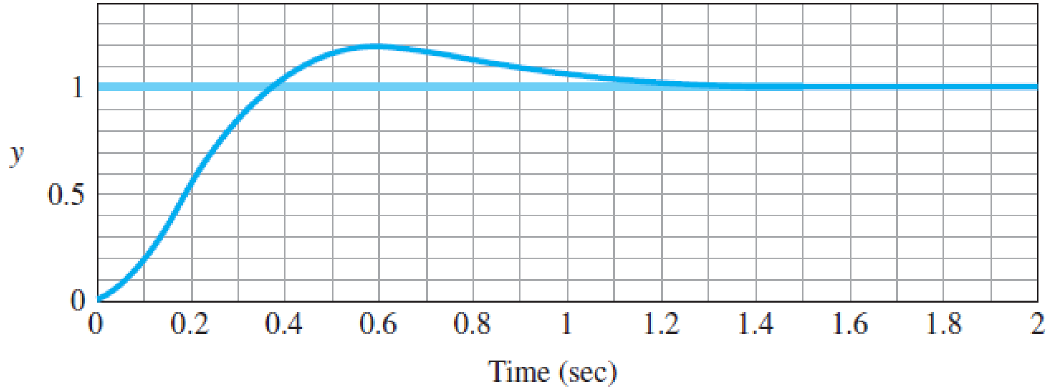


- No steady-state error
  - ✓ a Type 1 system
- Settling time ~= 25 sec
- Rise time ~= 2 sec

- Example 6.15: Lead Compensation



- Rise time ~= 0.33 sec