

Fall 2021 (110-1)

控制系統
Control Systems

Unit 6B
Bode Plot Techniques

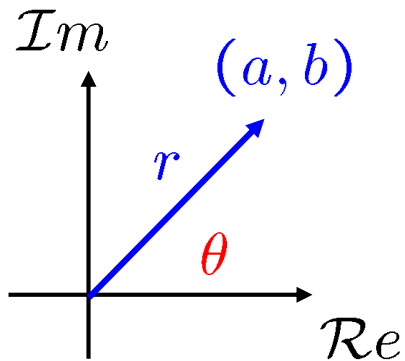
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NTU-EE

Sep 2021 – Jan 2022

- The hand plotting was developed by **H.W. Bode** at **Bell Laboratories** between **1932-1942**.
- Now, most control system designers **use computer programs** to illustrate the Bode plot.
- However, it is still important to **develop good intuition** so that you can **quickly identify erroneous** computer result and **perform sanity check** and determine **approximate result** by hand
- The idea in Bode's method is **to plot magnitude curves** using a **logarithmic scale** and **phase curves** using a **linear scale**

■ Magnitude & Phase Representation:



$$a + jb \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = r e^{j\theta}$$

$$|a + jb| = \sqrt{a^2 + b^2} \quad \left| \frac{1}{a + jb} \right| = \frac{1}{\sqrt{a^2 + b^2}}$$

$$\angle a + jb = \tan^{-1}\left(\frac{b}{a}\right) \quad \angle \frac{1}{a + jb} = -\tan^{-1}\left(\frac{b}{a}\right)$$

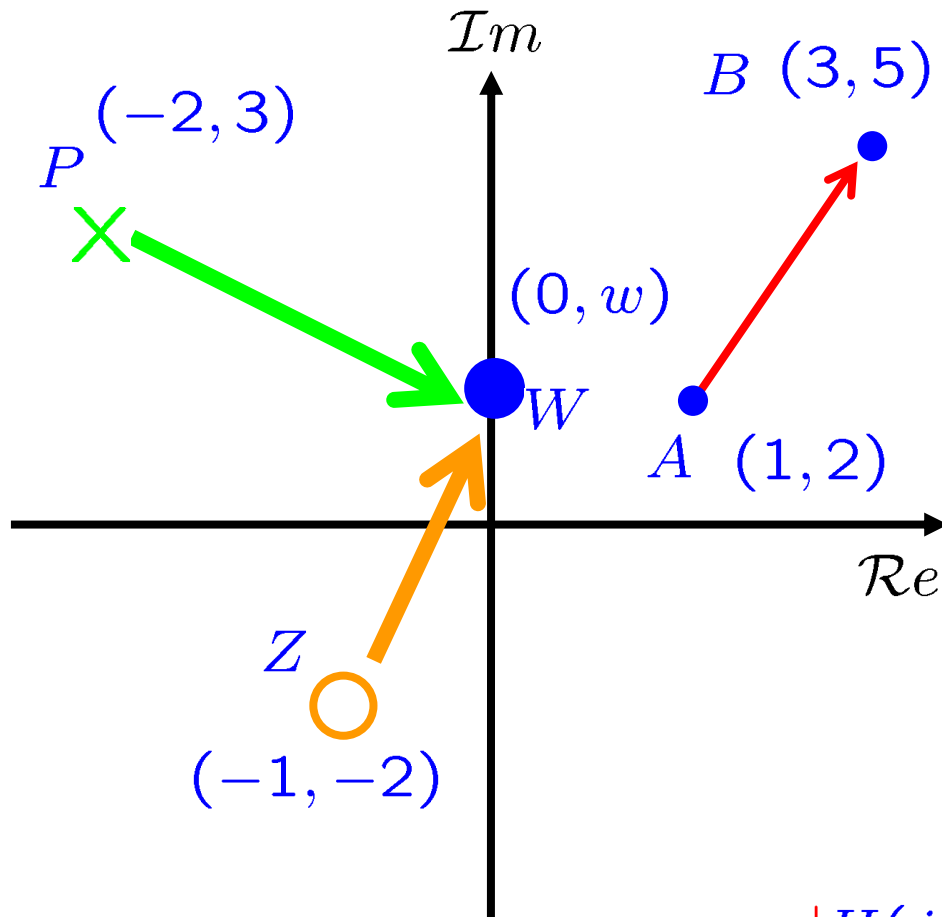
$$X(j\omega) = \text{Re}\{X(j\omega)\} + j \text{Im}\{X(j\omega)\} = |X(j\omega)| e^{j\angle X(j\omega)}$$

$$X(e^{j\omega}) = \text{Re}\{X(e^{j\omega})\} + j \text{Im}\{X(e^{j\omega})\} = |X(e^{j\omega})| e^{j\angle X(e^{j\omega})}$$

$|X(j\omega)|$ or $|X(e^{j\omega})|$: magnitude

$\angle X(j\omega)$ or $\angle X(e^{j\omega})$: phase angle

In s-plane:



$$\begin{aligned}\overrightarrow{AB} &= (3 + 5j) - (1 + 2j) \\ &= 2 + 3j\end{aligned}$$

$$\overrightarrow{AB} = (3, 5) - (1, 2) = (2, 3)$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2}$$

$$\angle \overrightarrow{AB} = \tan^{-1} \frac{(5 - 2)}{(3 - 1)}$$

$$H(s) = \frac{s - (-1 - 2j)}{s - (-2 + 3j)}$$

$$|H(jw)| = \frac{|jw - (-1 - 2j)|}{|jw - (-2 + 3j)|} = \frac{|\overrightarrow{ZW}|}{|\overrightarrow{PW}|}$$

$$\angle H(jw) = \angle \overrightarrow{ZW} - \angle \overrightarrow{PW}$$

- For example,

$$G(j\omega) = \frac{\overrightarrow{s_1} \overrightarrow{s_2}}{\overrightarrow{s_3} \overrightarrow{s_4} \overrightarrow{s_5}} = \frac{(r_1 e^{j\theta_1}) (r_2 e^{j\theta_2})}{(r_3 e^{j\theta_3}) (r_4 e^{j\theta_4}) (r_5 e^{j\theta_5})}$$

$$= \frac{r_1 r_2}{r_3 r_4 r_5} e^{j(\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)}$$

$$|G(j\omega)| = \frac{r_1 r_2}{r_3 r_4 r_5} \quad \angle G(j\omega) = (\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$\log_{10} |G(j\omega)| = \log_{10} r_1 + \log_{10} r_2 - \log_{10} r_3 - \log_{10} r_4 - \log_{10} r_5$$

- Power db: $|G(j\omega)|_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$

- Voltage db: $|G(j\omega)|_{\text{db}} = 20 \log_{10} \frac{V_2}{V_1}$

■ Advantages of working with **Frequency Response**

in terms of **Bode Plots**

1. **Dynamic compensator design**

can be based **entirely** on Bode plots.

2. Bode plots can be **determined experimentally**.

3. Bode plots of systems in series (or tandem) **simply add**, which is quite convenient.

4. The use of a **log scale** permits a much **wider range** of frequencies to be displayed on **a single plot** than is possible with **linear scales**.

- The open-loop transfer function:

$$K G(s) = K \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

$$K G(j\omega) = K_0 (j\omega)^n \frac{(j\omega\tau_1 + 1)(j\omega\tau_2 + 1) \cdots}{(j\omega\tau_a + 1)(j\omega\tau_b + 1) \cdots}$$

- For example,

$$K G(j\omega) = K_0 \frac{(j\omega\tau_1 + 1)}{(j\omega)^2 (j\omega\tau_a + 1)}$$

$$\angle K G(j\omega) = \angle K_0 + \angle (j\omega\tau_1 + 1) - \angle (j\omega)^2 - \angle (j\omega\tau_a + 1)$$

$$\begin{aligned} \log |K G(j\omega)| &= \log |K_0| + \log |(j\omega\tau_1 + 1)| \\ &\quad - \log |(j\omega)^2| - \log |(j\omega\tau_a + 1)| \\ |K G(j\omega)|_{\text{db}} &= 20 \log |K_0| + 20 \log |(j\omega\tau_1 + 1)| \\ &\quad - 20 \log |(j\omega)^2| - 20 \log |(j\omega\tau_a + 1)| \end{aligned}$$

▪ Class 1: Singularities at the origin

$$K_0 (j\omega)^n$$

▪ Class 2: First-order term

$$(j\omega\tau + 1)^{\pm 1}$$

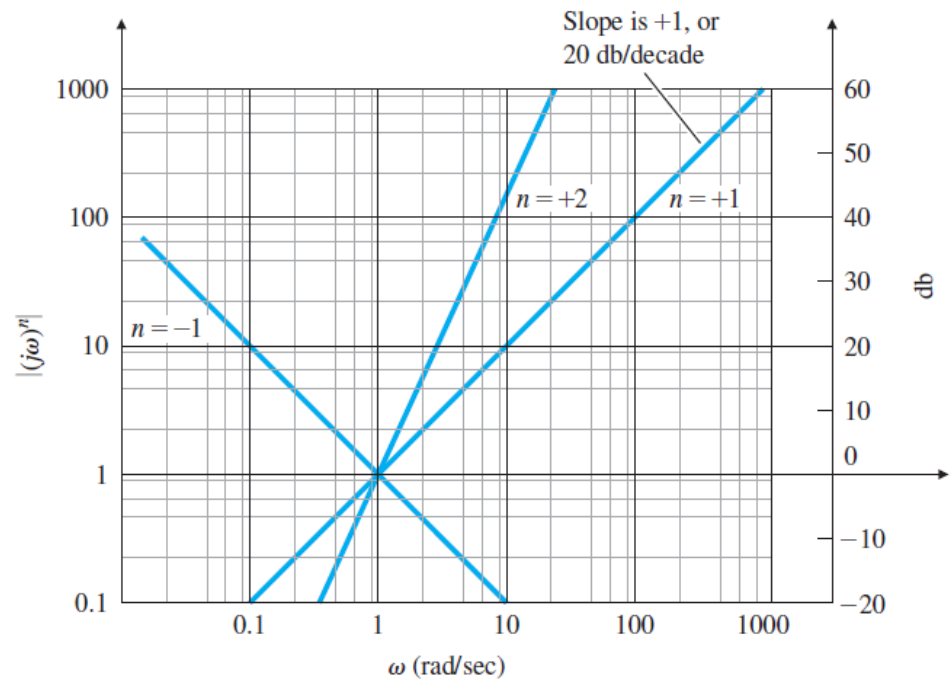
▪ Class 3: Second-order term

$$\left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1 \right]^{\pm 1}$$

▪ Class 1: $K_0 (j\omega)^n$

$$\begin{aligned} \log K_0 |(j\omega)^n| \\ = \log K_0 + n \log |j\omega| \end{aligned}$$

$$\angle K_0 (j\omega)^n = n \times 90^\circ$$



Class 2: Magnitude

$$(j\omega\tau + 1)^{\pm 1}$$

a) For $\omega\tau \ll 1$, $j\omega\tau + 1 \cong 1$

b) For $\omega\tau \gg 1$, $j\omega\tau + 1 \cong j\omega\tau$

▪ $\omega = 1/\tau$: Break Point

$$? G(s) = \frac{1}{10s + 1}$$

▪ For example,

$$G(s) = 10s + 1$$

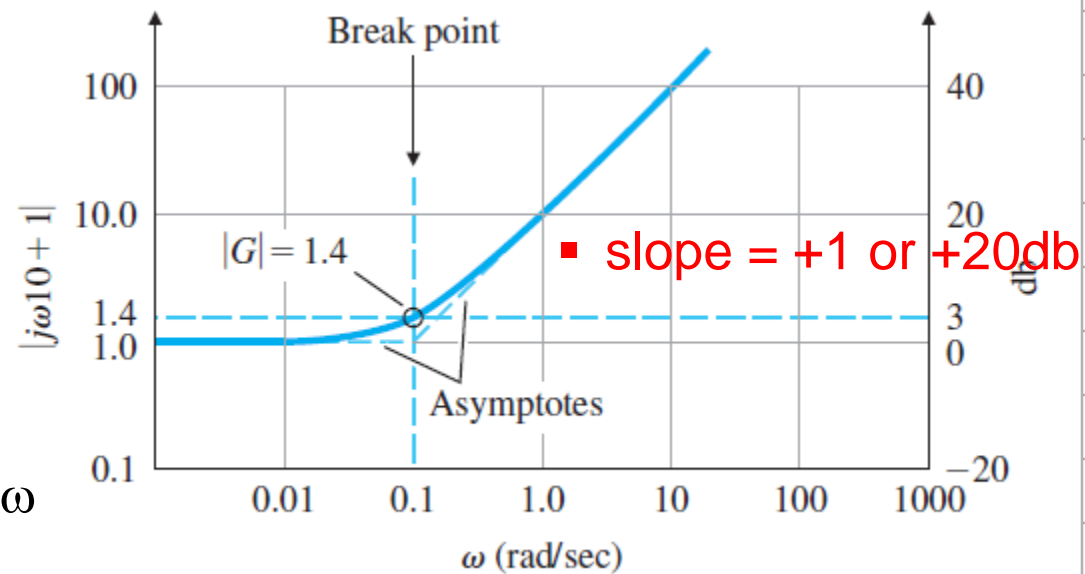
$$G(j\omega) = j(10\omega) + 1$$

a) For $10\omega \ll 1$, $j10\omega + 1 \cong 1$

b) For $10\omega \gg 1$, $j10\omega + 1 \cong j10\omega$

▪ $\omega = 1/10$: Break Point

$$|G(j0.1)| = |j(1) + 1| = 1.414 = +3 \text{ db}$$



Class 2: Phase

$$(j\omega\tau + 1)^{\pm 1}$$

- a) For $\omega\tau \ll 1$, $\angle 1 = 0^\circ$
- b) For $\omega\tau \gg 1$, $\angle j\omega\tau = 90^\circ$
- c) For $\omega\tau \cong 1$, $\angle (j\omega\tau + 1) \cong 45^\circ$

- a) For $\omega\tau \ll 1$, $\angle 1 = 0^\circ$
- b) For $\omega\tau \gg 1$, $\angle j\omega\tau = 90^\circ$
- c) For $\omega\tau \cong 1$, $\angle (j\omega\tau + 1) \cong 45^\circ$
- $\omega = 1/\tau$: Break Point
- For example,

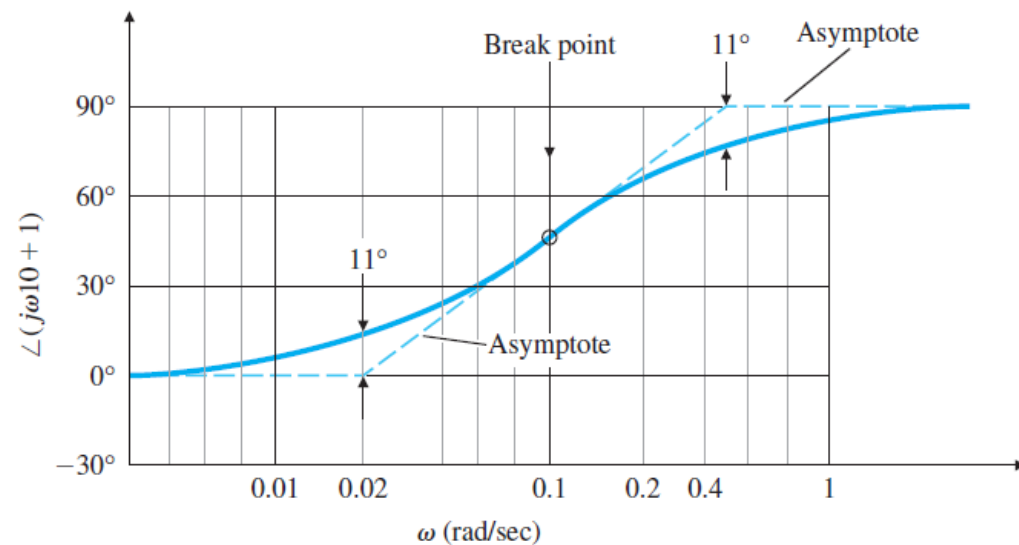
▪ $\omega = 1/\tau$: Break Point

▪ For example,

$$G(s) = 10s + 1$$

$$G(j\omega) = j(10\omega) + 1$$

▪ $\omega = 1/10$: Break Point

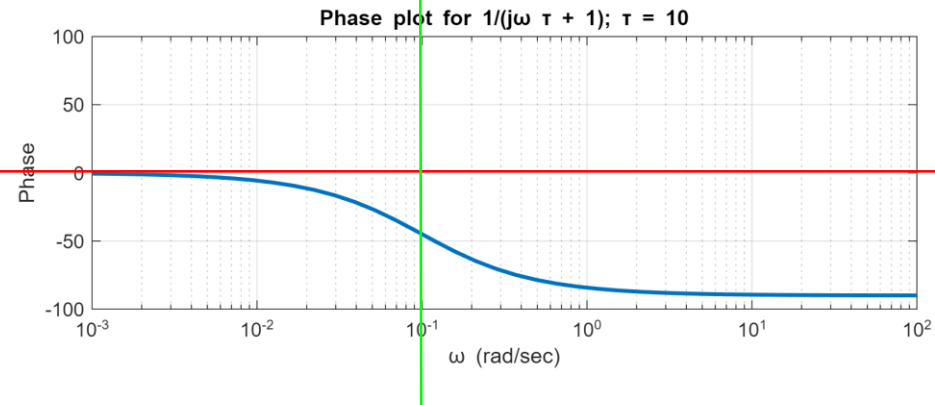
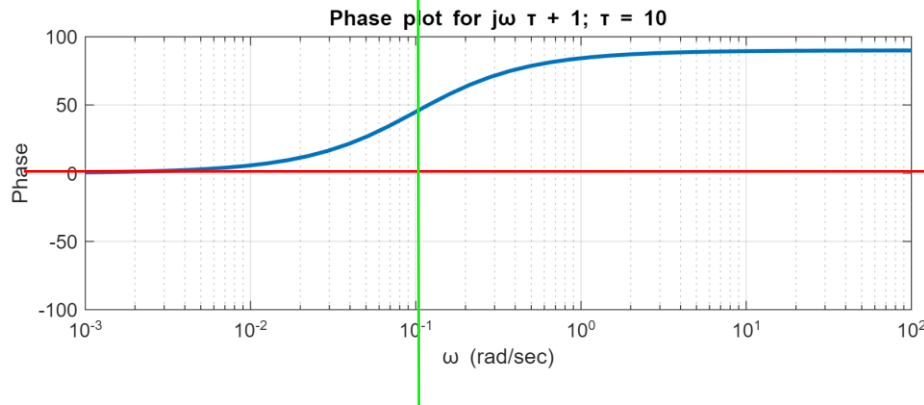
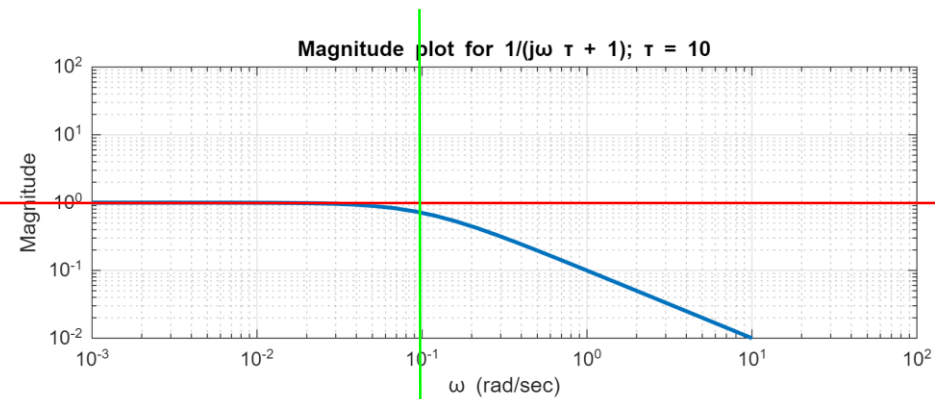
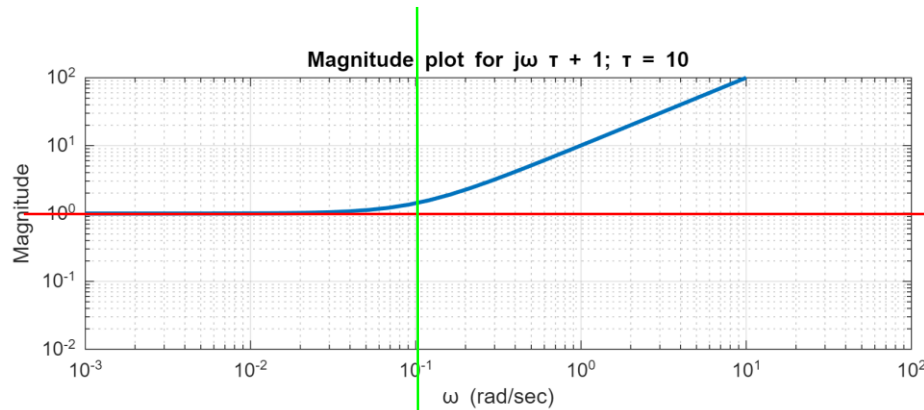


Class 2:

$$(j\omega\tau + 1)^{\pm 1}$$

$$G(s) = 10s + 1$$

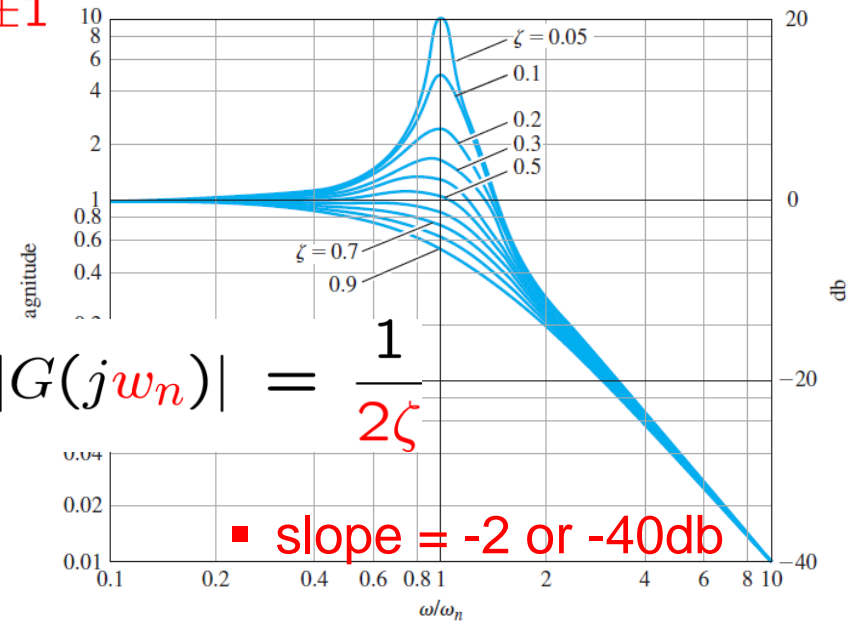
$$G(s) = \frac{1}{10s + 1}$$



■ **Class 3:** $\left[\left(\frac{jw}{w_n} \right)^2 + 2\zeta \frac{jw}{w_n} + 1 \right]^{\pm 1}$

$$|G(s = jw)|$$

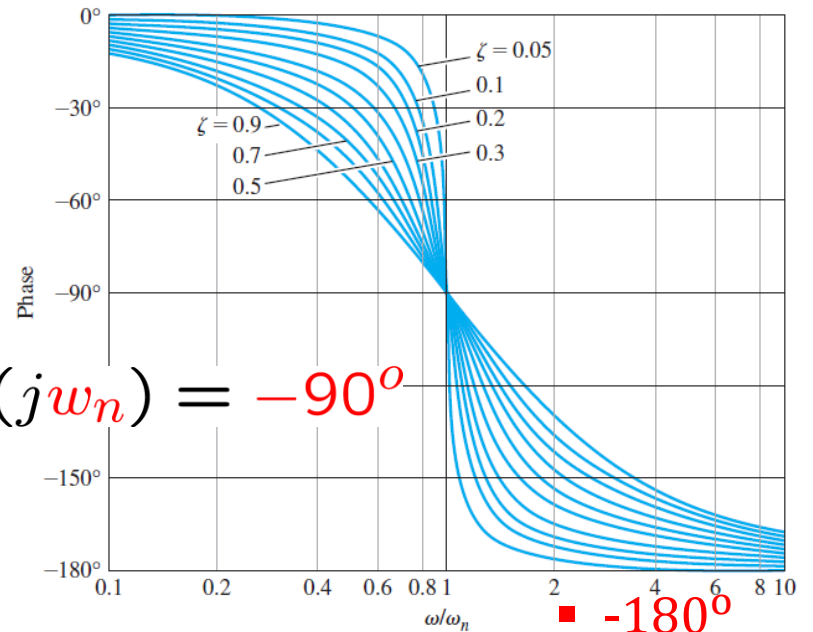
$$|G(jw_n)| = \frac{1}{2\zeta}$$



$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$

$$\angle G(jw_n) = -90^\circ$$



Examples

- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50

- (2) Asymptotes

- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

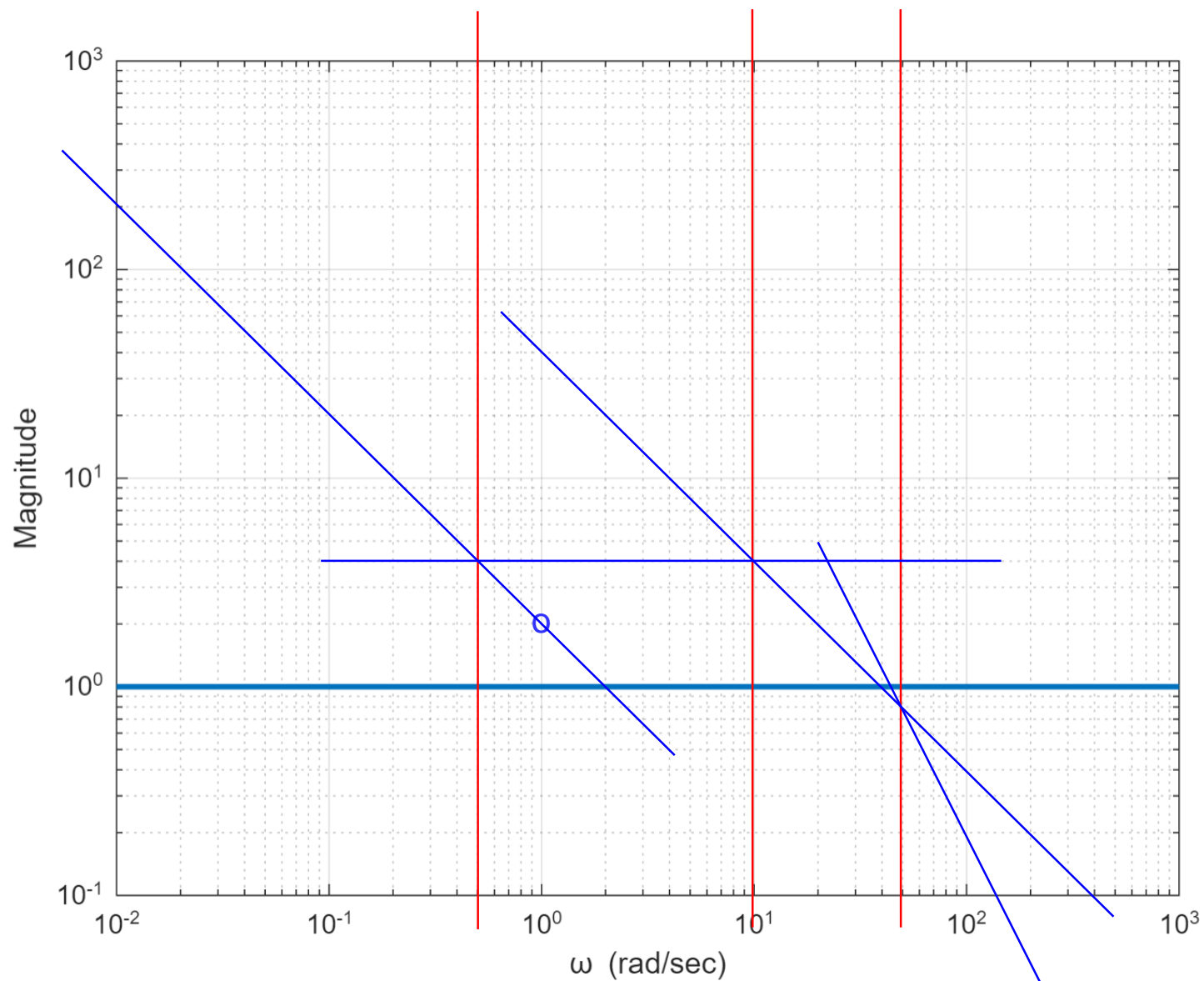
- $\omega \ll 0.5$: slope = -1 (or -20 db/decade)

- $0.5 < \omega < 10$: slope = 0 (or 0 db/decade)

- $10 < \omega < 50$: slope = -1 (or -20 db/decade)

- $50 < \omega$: slope = -2 (or -40 db/decade)

- Example 6.3: Bode Plot for Real Poles and Zeros

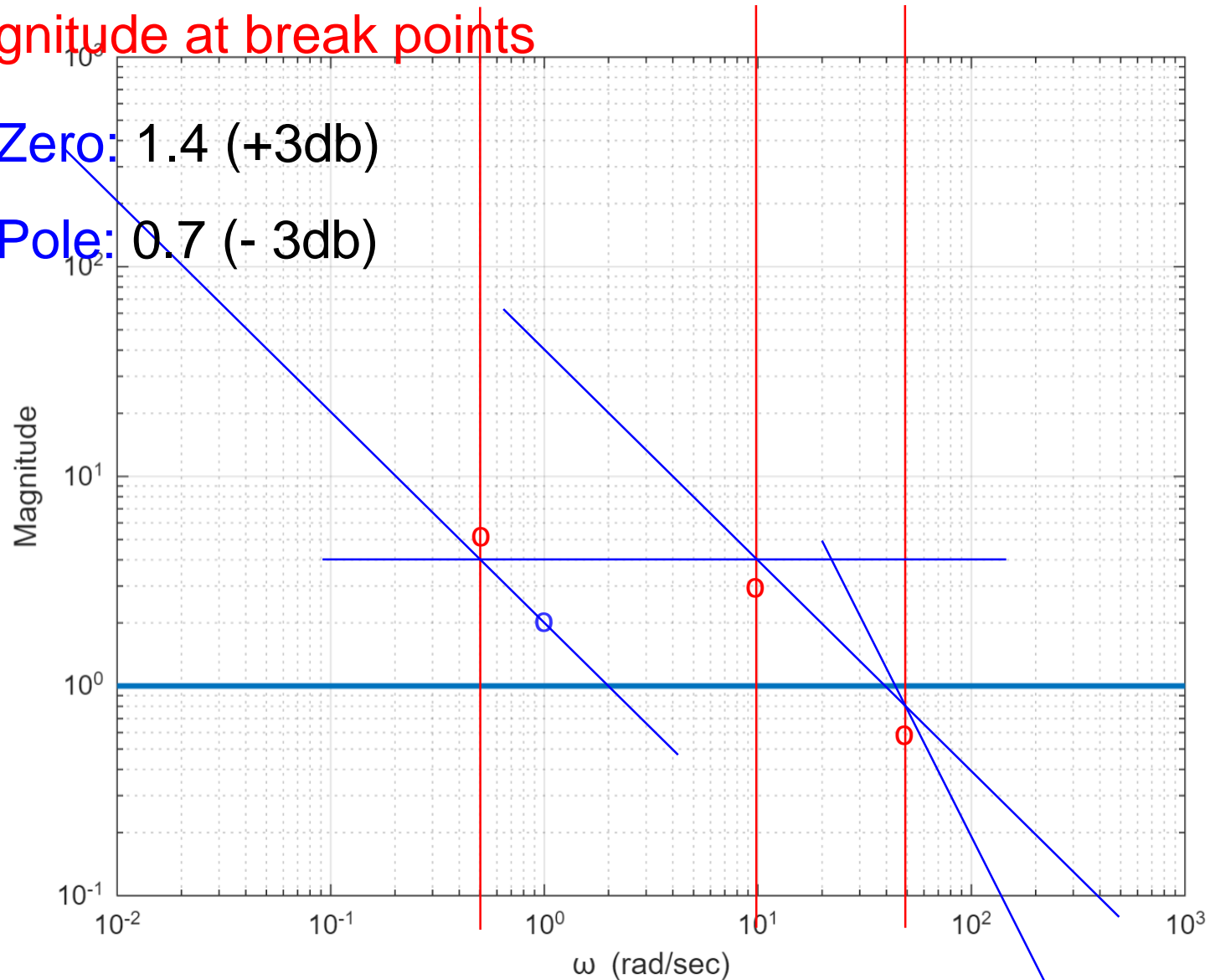


- Example 6.3: Bode Plot for Real Poles and Zeros

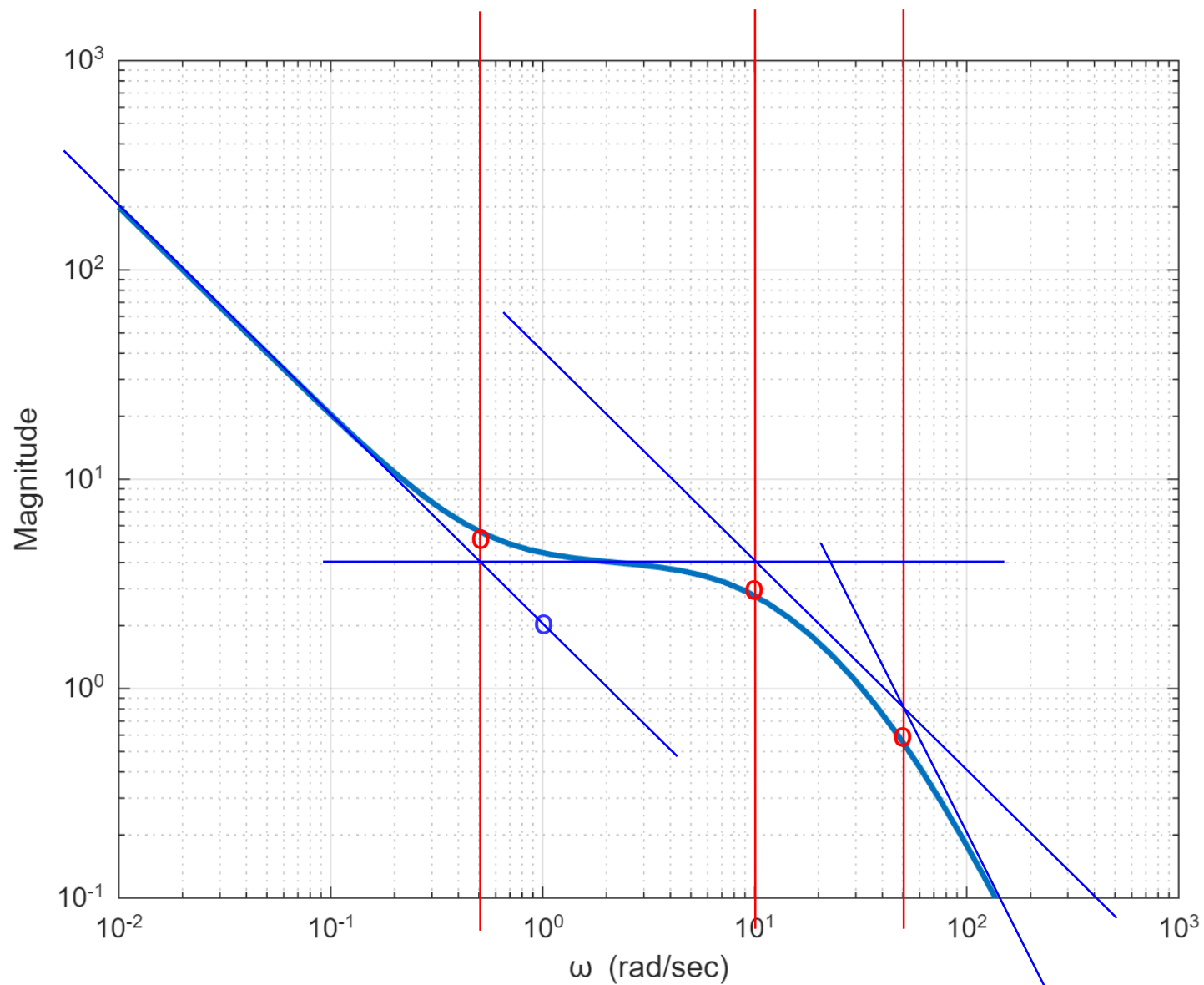
- (3) Magnitude at break points

- By Zero: 1.4 (+3db)

- By Pole: 0.7 (- 3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

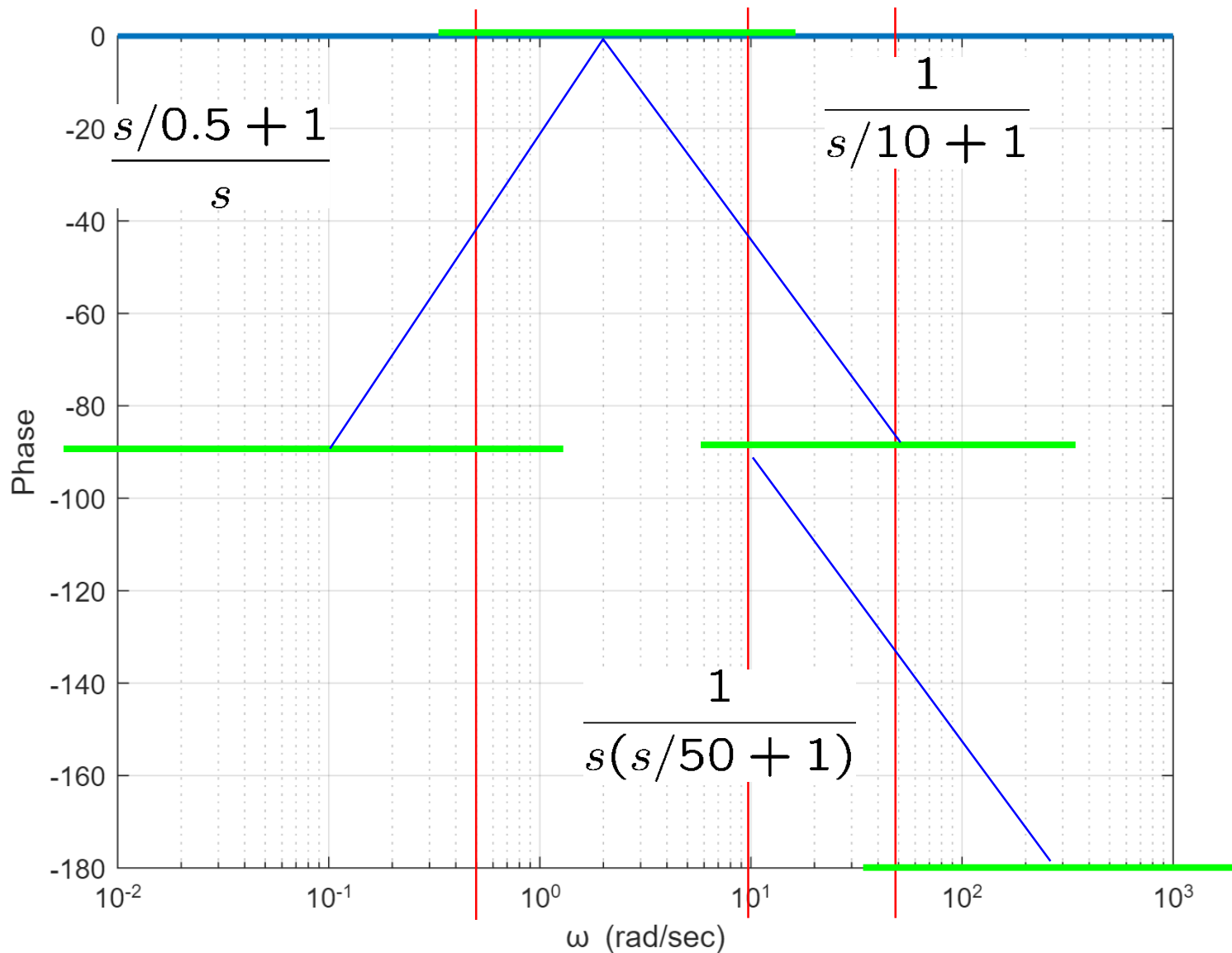
- Break points: 0.5, 10, 50

- (4) Phase

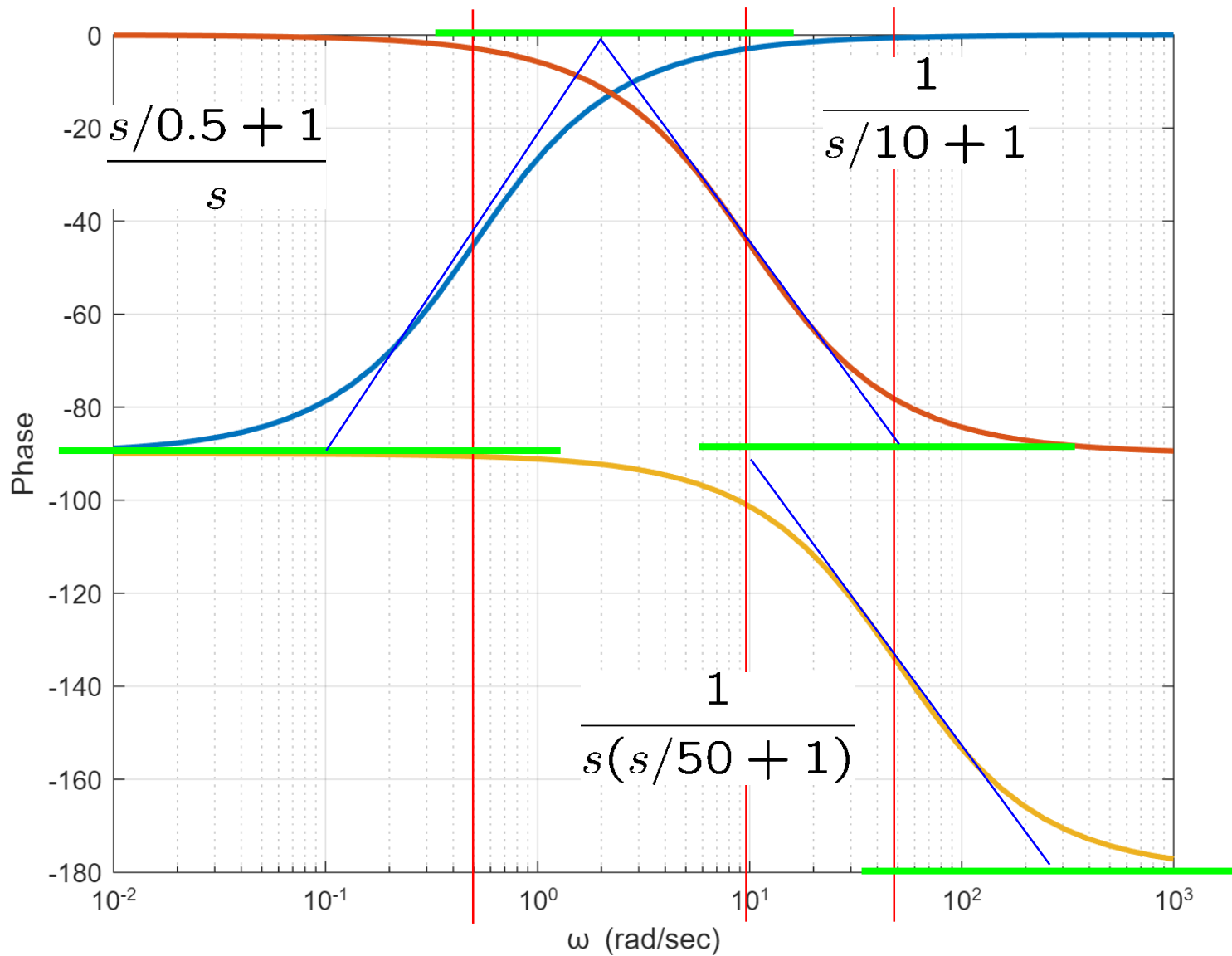
- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

- $\omega \ll 0.5$: phase = -90°
- $0.5 < \omega < 10$: phase = 0°
- $10 < \omega < 50$: phase = -90°
- $50 < \omega$: phase = -180°

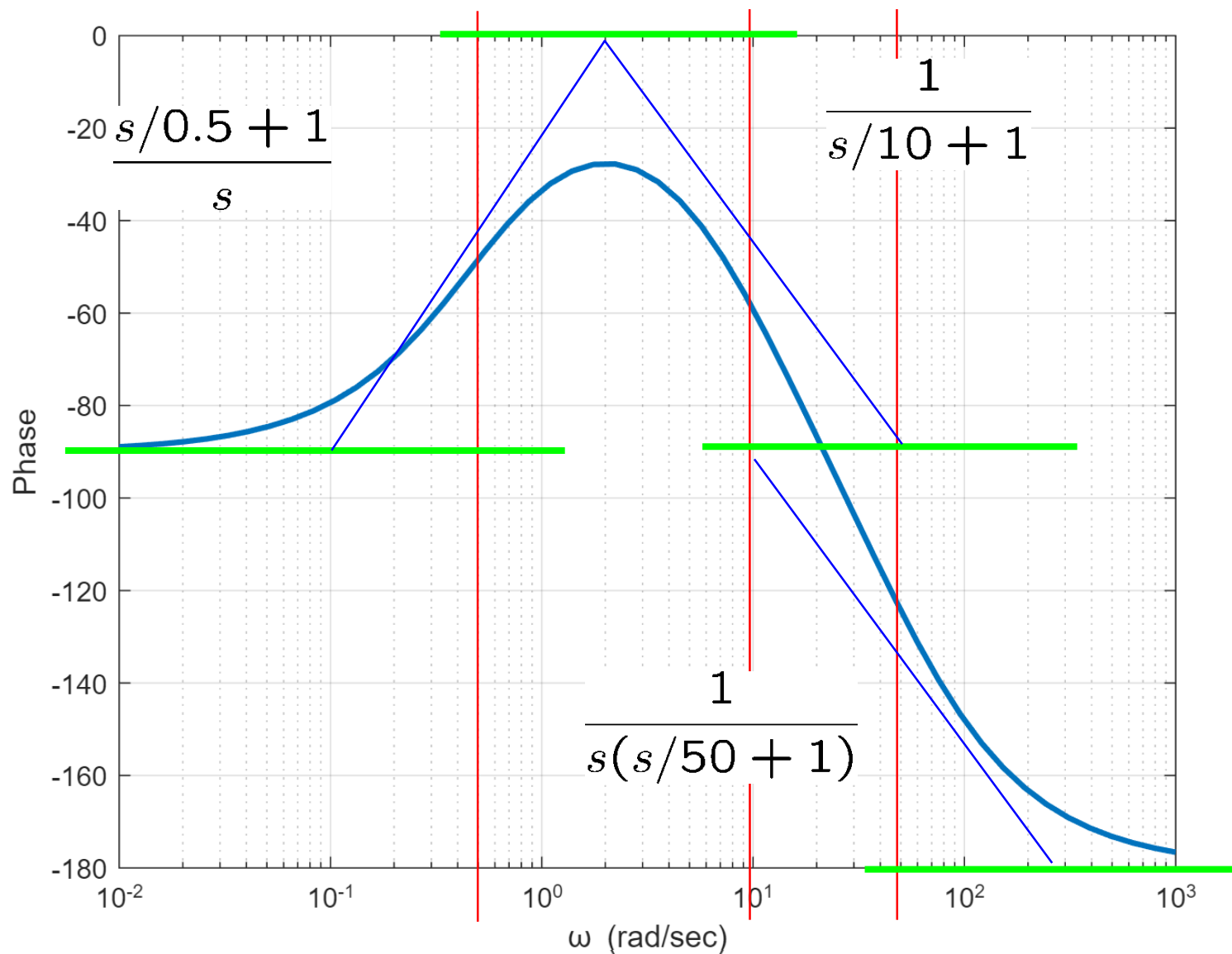
Example 6.3: Bode Plot for Real Poles and Zeros



Example 6.3: Bode Plot for Real Poles and Zeros



■ Example 6.3: Bode Plot for Real Poles and Zeros

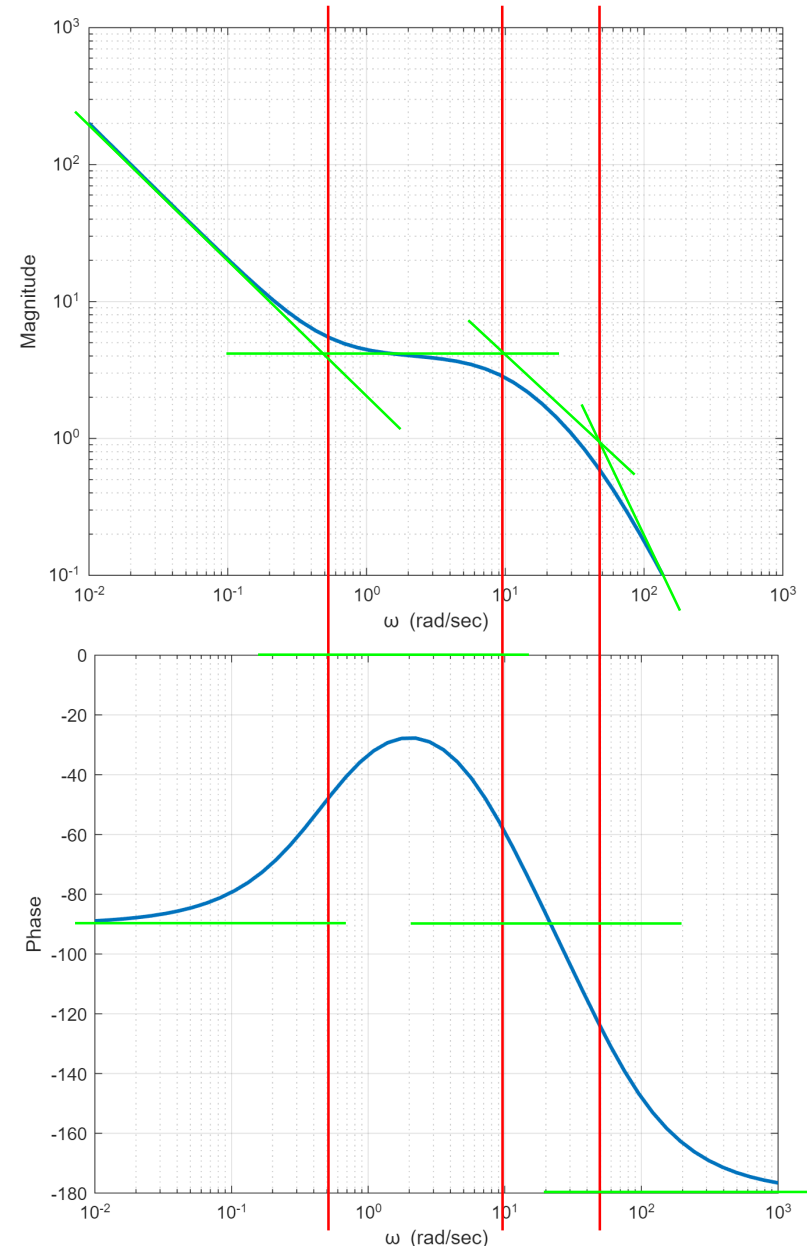


Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

$$\frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50

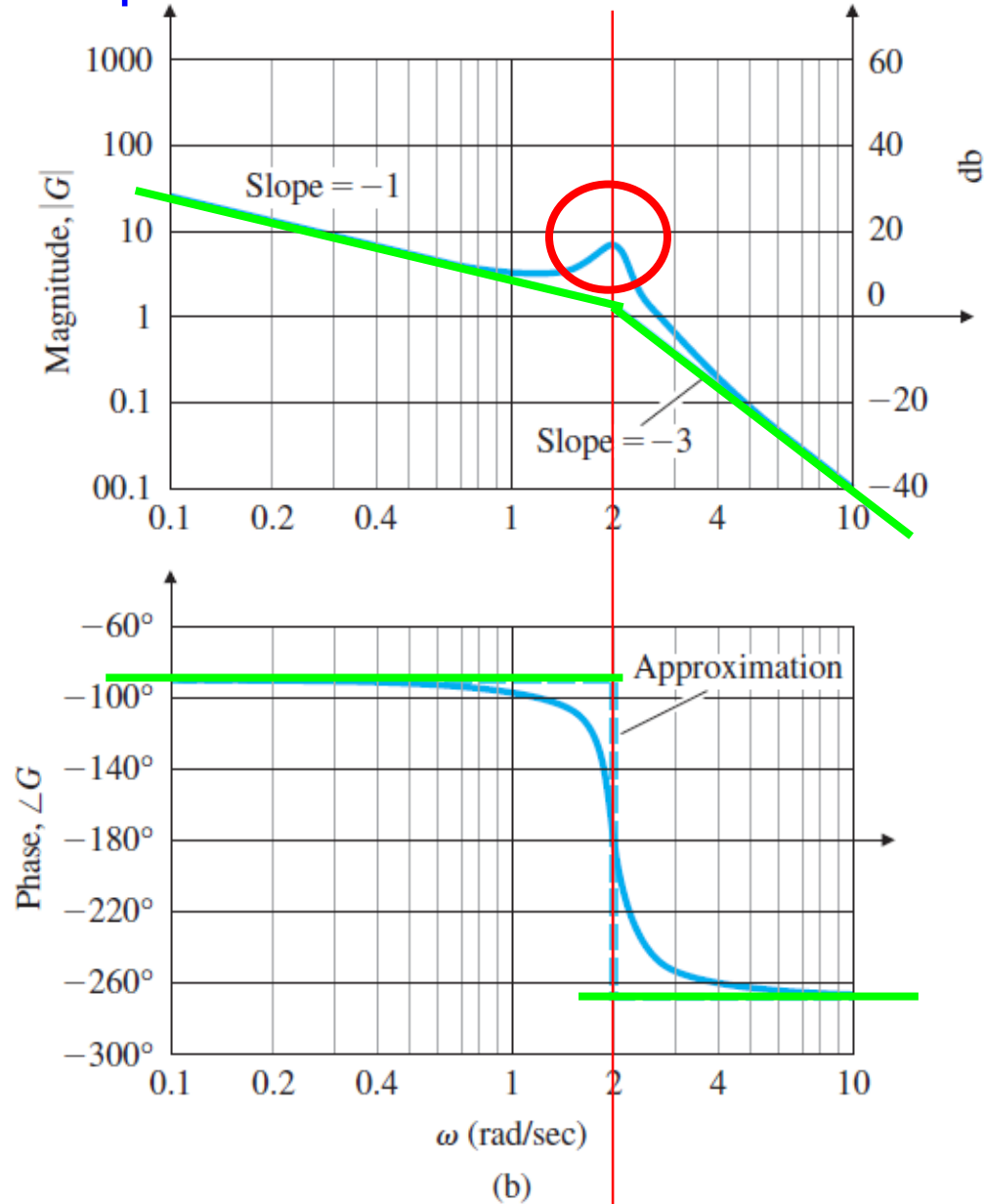


Example 6.4: Bode Plot for Complex Poles

$$K G(s) = \frac{10}{s [s^2 + 0.4s + 4]}$$

$$= \frac{10}{4} \frac{1}{s \left[\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1 \right]}$$

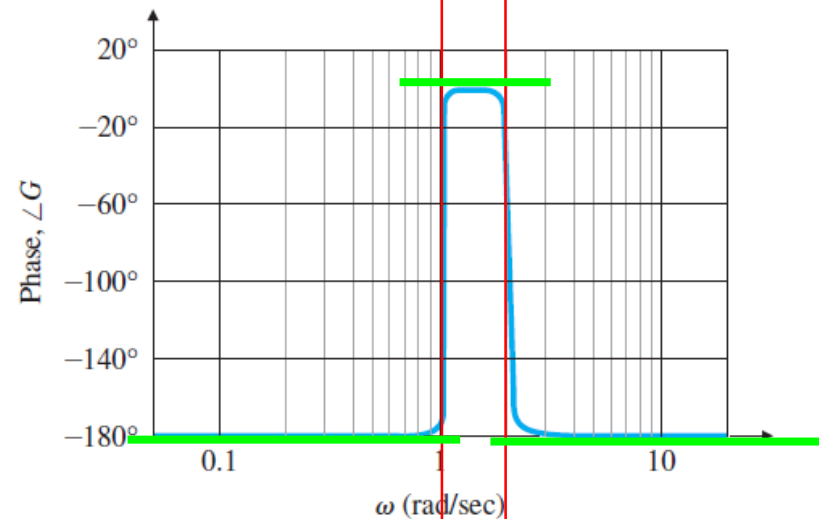
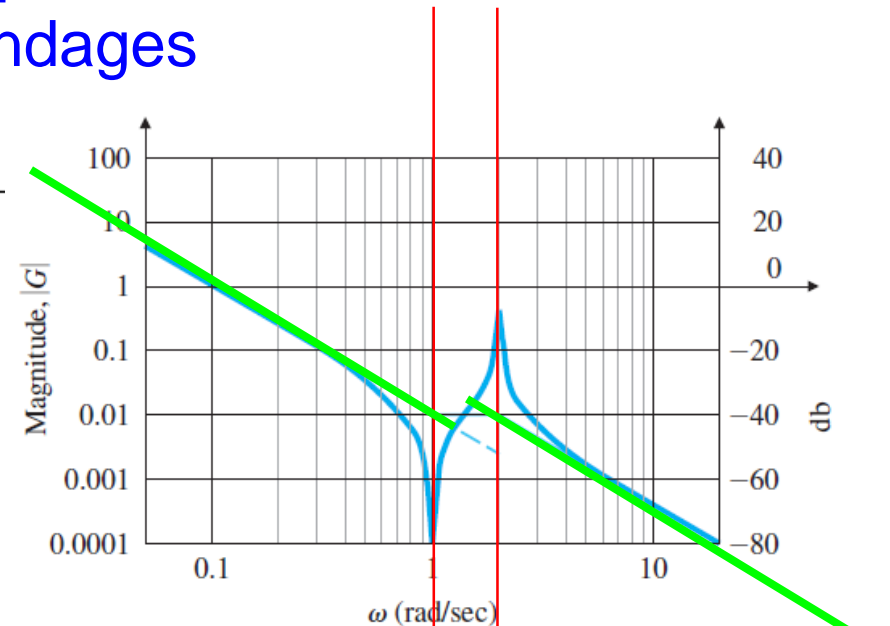
- Break points: 2



- Example 6.5: Bode Plot for Complex Poles and Zeros:
Satellite with Flexible Appendages

$$K G(s) = \frac{0.01 [s^2 + 0.01s + 1]}{s^2 [(s^2/4) + 0.02(s/2) + 1]}$$

- Break points: 1, 2



(b)

Example 6.6: Computer-Aided Bode Plot for Complex Poles and Zeros

```
num = 0.01*[1 0.01 1];  
den = conv([1 0 0],[.25 0.01 1]);
```

```
w = logspace(-2,2,1000);
```

```
[m,p] = bode(num, den, w);
```

```
subplot(2,1,1)  
loglog(w, m, 'LineWidth', 2);
```

```
subplot(2,1,2)  
semilogx(w, p, 'LineWidth', 2);
```

