

Fall 2021 (110-1)

控制系統
Control Systems

Unit 60
Bode Plot (w-Domain)

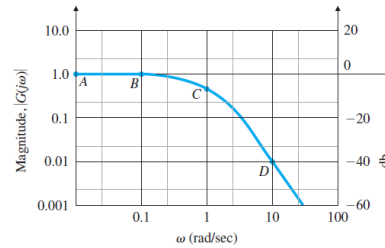
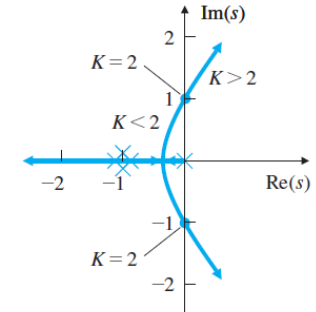
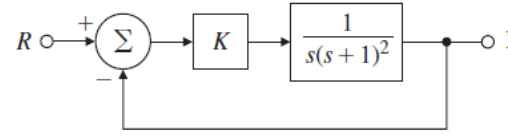
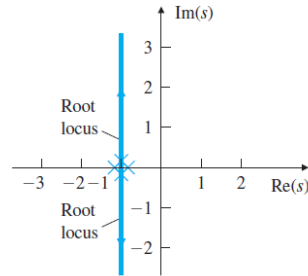
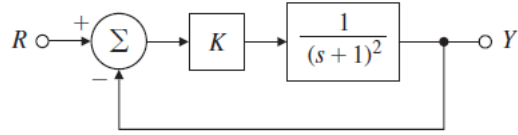
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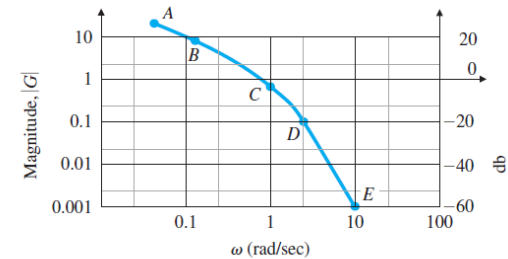
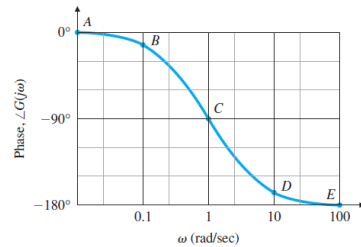
Sep 2021 – Jan 2022

Unit 6

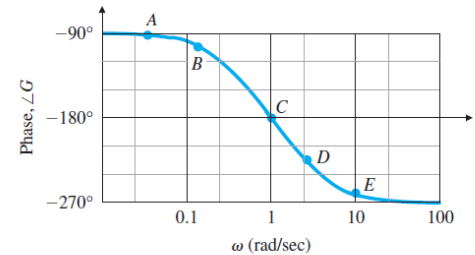
Bode Plot



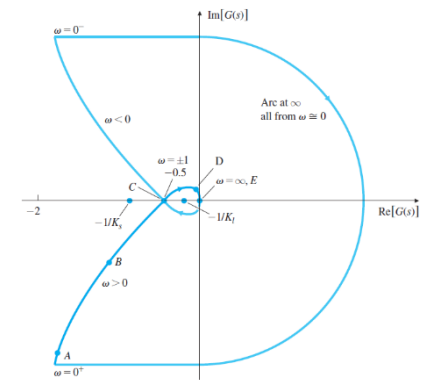
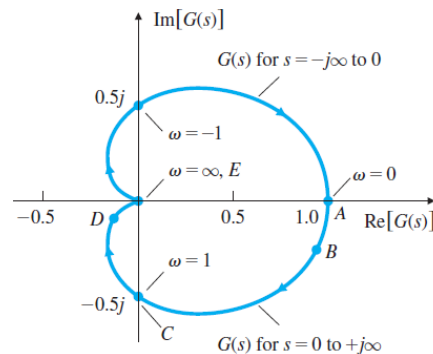
(a)



(b)



- Root Locus:
- Bode Plot:
- Nyquist Plot:



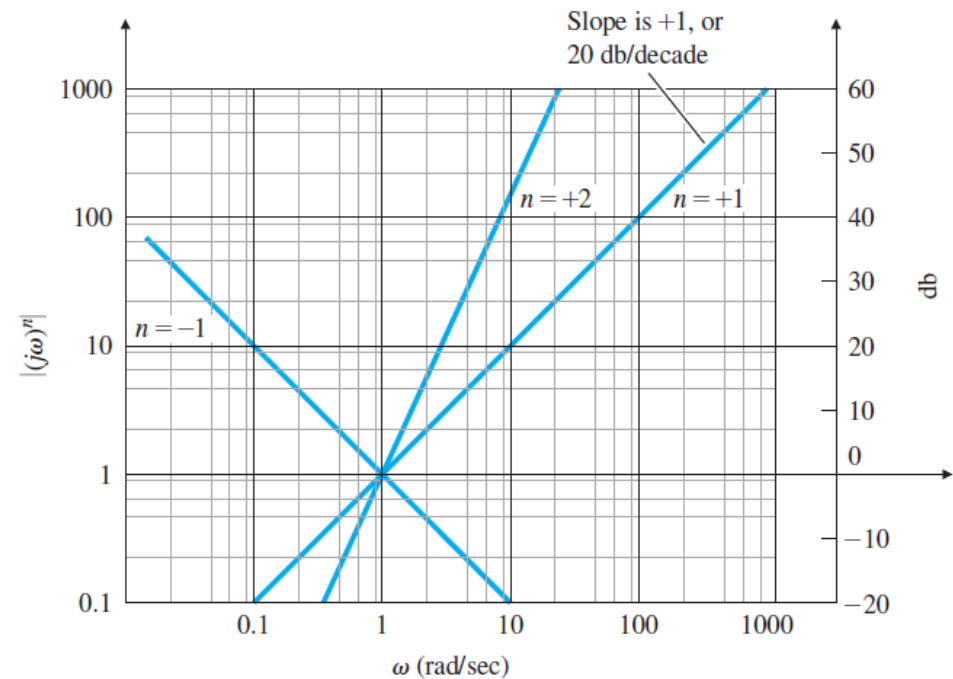
- **By Hand:**
 - **Hand Writing in Exam (40%)**
- Use the Bode Plot Techniques
 - to **roughly sketch** the **magnitude** and **phase** plots
 - of any transfer function
 - by **identifying** these **critical frequency** locations
- **By Computer:**
 - **Multiple Choice in Exam (60%)**
- Use Matlab codes
 - to draw the **exact Bode Plot, Nyquist Plot**
 - of any transfer function
- **Design proper** transfer function and
 - select associated and reasonable** gain value

- Class 1: Singularities at the origin

$$K_0 (j\omega)^n$$

$$\begin{aligned} \log K_0 |(j\omega)^n| \\ = \log K_0 + n \log |j\omega| \end{aligned}$$

$$\angle K_0 (j\omega)^n = n \times 90^\circ$$

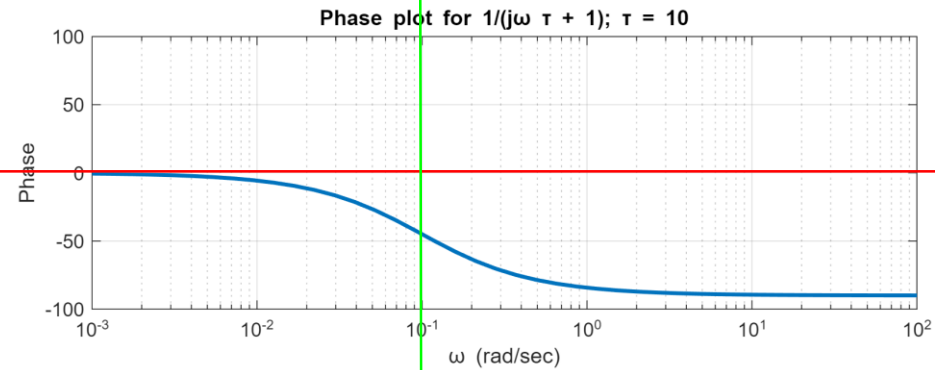
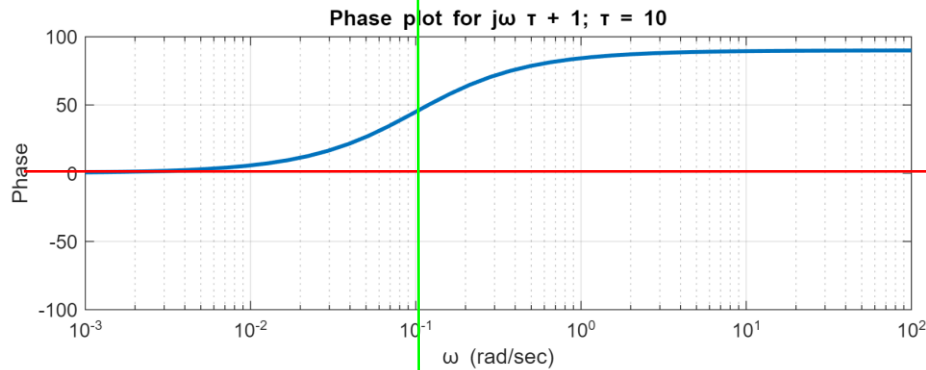
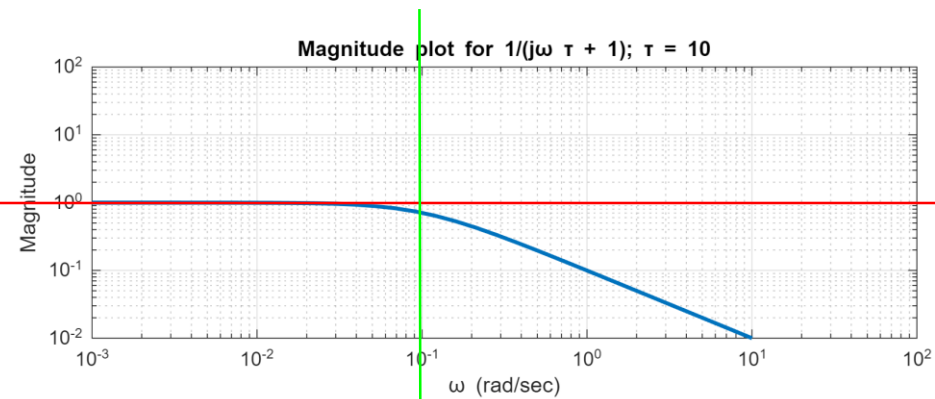
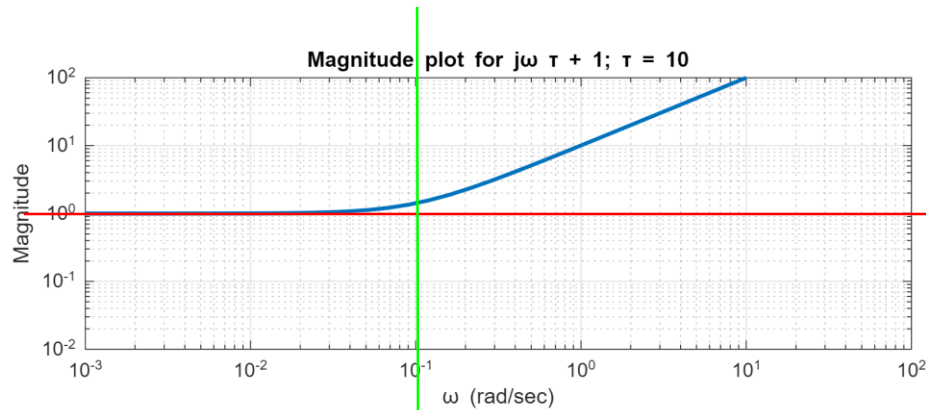


Class 2: First-order term

$$G(s) = 10s + 1$$

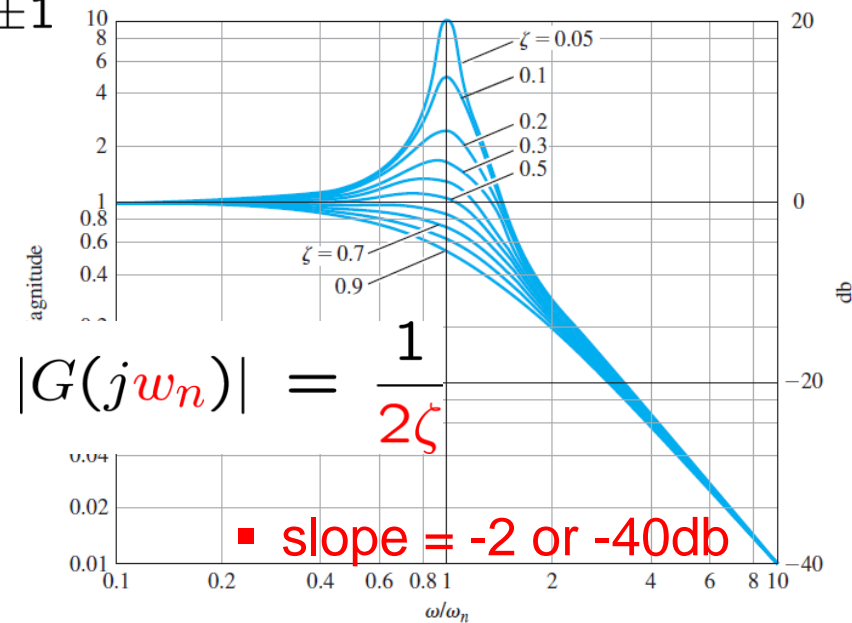
$$(j\omega\tau + 1)^{\pm 1}$$

$$G(s) = \frac{1}{10s + 1}$$



■ **Class 3:** $\left[\left(\frac{jw}{w_n} \right)^2 + 2\zeta \frac{jw}{w_n} + 1 \right]^{\pm 1}$

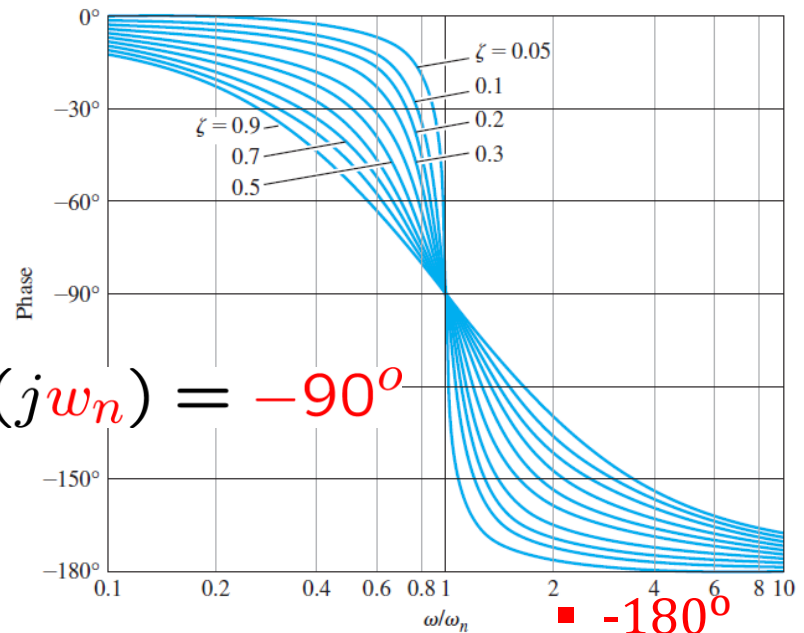
$$|G(s = jw)|$$



$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$

$$\angle G(jw_n) = -90^\circ$$

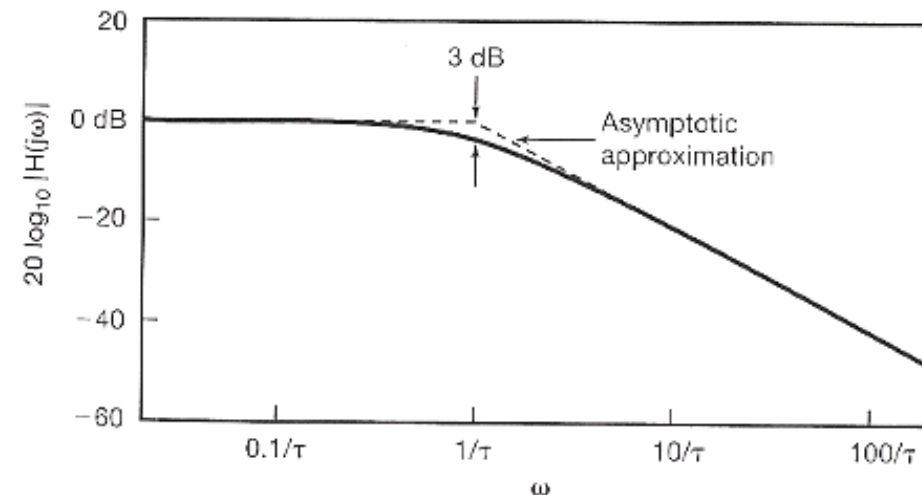


First-Order CT Systems:

$$H(j\omega) = \frac{1}{j\omega\tau + 1}$$

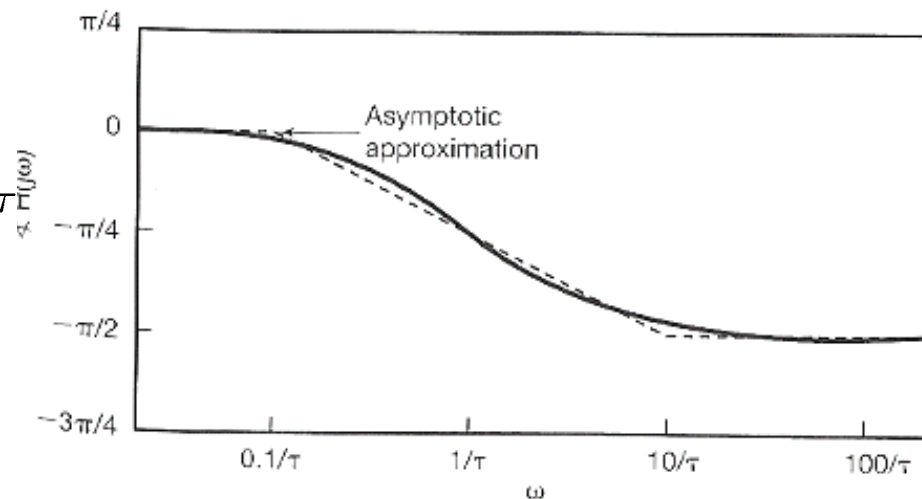
$$20 \log_{10} |H(j\omega)| =$$

$$\begin{cases} 0 & \omega \ll 1/\tau \\ -10 \log_{10}(2) \approx -3dB & \omega = 1/\tau \\ -20 \log_{10}(\omega\tau) & \omega \gg 1/\tau \\ = -20 \log_{10}(\omega) - 20 \log_{10}(\tau) \end{cases}$$



$$\angle H(j\omega) =$$

$$\begin{cases} 0 & \omega \leq 0.1/\tau \\ -(\pi/4)[\log_{10}(\omega\tau) + 1] & 0.1/\tau \leq \omega \leq 10/\tau \\ = -(\pi/4) [\log_{10}(\omega) + \log_{10}(\tau) + 1] \\ -\pi/4 & \omega = 1/\tau \\ -\pi/2 & \omega \geq 10/\tau \end{cases}$$

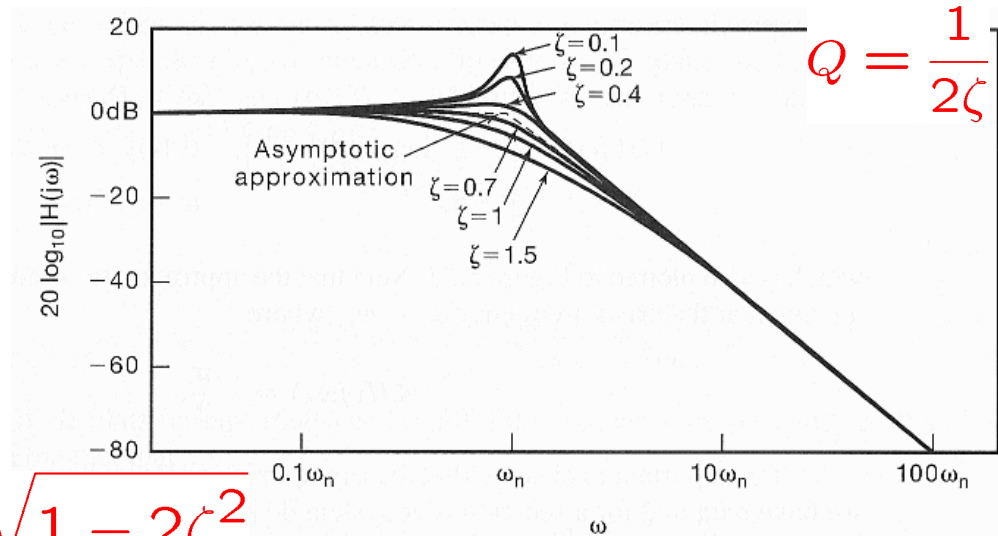


Second-Order CT Systems:

$$H(j\omega) = \frac{1}{(j\frac{\omega}{\omega_n})^2 + 2\zeta(j\frac{\omega}{\omega_n}) + 1}$$

$$20 \log_{10} |H(j\omega)| =$$

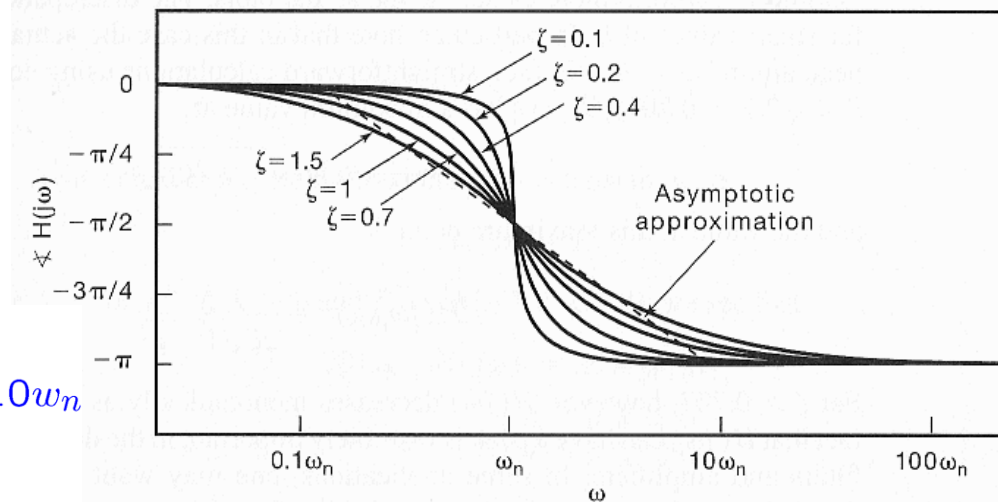
$$\begin{cases} 0 & \omega \ll \omega_n \\ -20 \log_{10}(2\zeta) & \omega = \omega_n \\ -40 \log_{10}(\omega) + 40 \log_{10}(\omega_n) & \omega \gg \omega_n \end{cases}$$



• For $\zeta < \frac{\sqrt{2}}{2}$ $\omega_{\max} = \omega_n \sqrt{1 - 2\zeta^2}$

$$\angle H(j\omega) =$$

$$\begin{cases} 0 & \omega \leq 0.1\omega_n \\ -(\pi/2)[\log_{10}(\omega/\omega_n) + 1] & 0.1\omega_n \leq \omega \leq 10\omega_n \\ -\pi/2 & \omega = \omega_n \\ -\pi & \omega \geq 10\omega_n \end{cases}$$



Examples

- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50

- (2) Asymptotes

- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

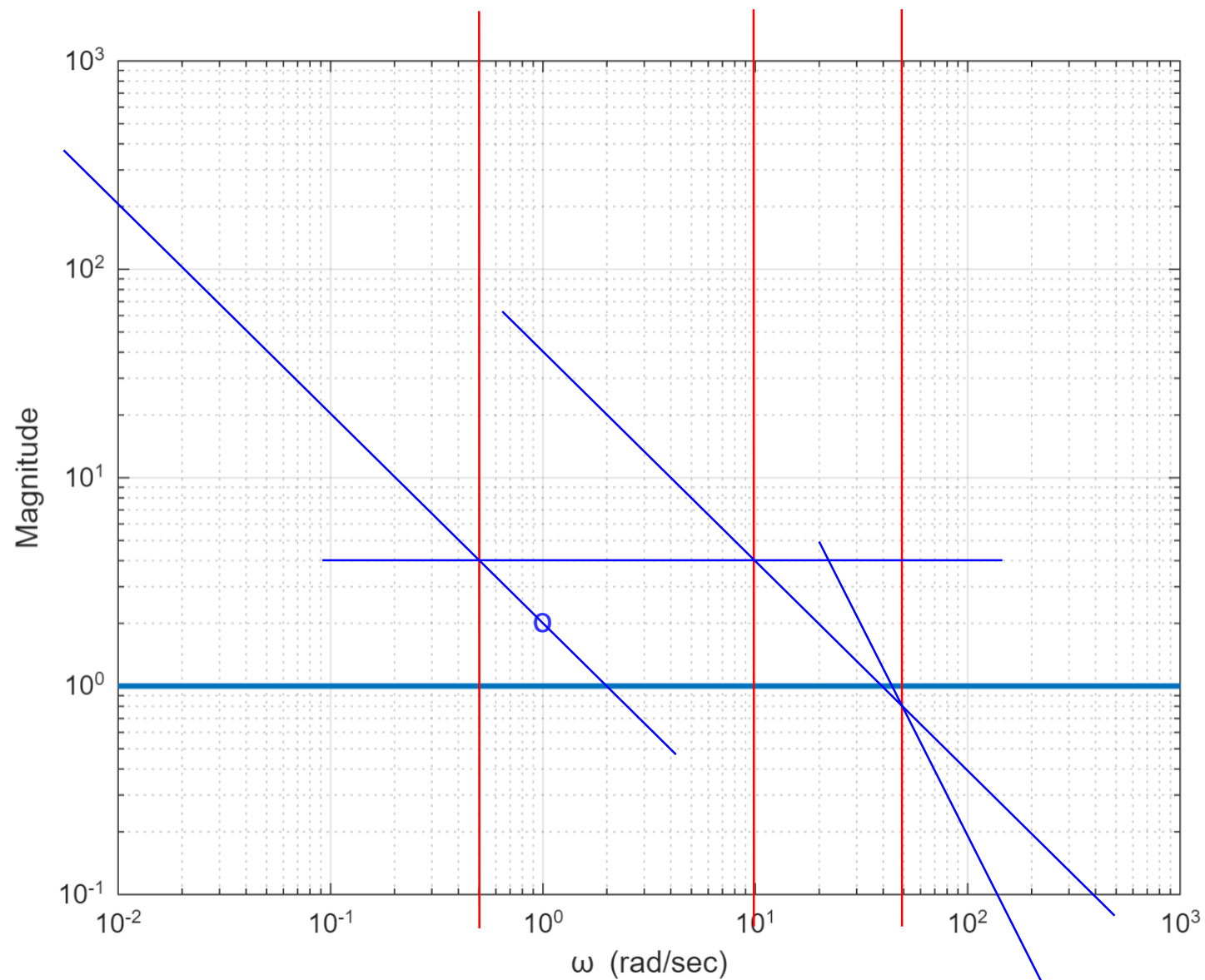
- $\omega \ll 0.5$: slope = -1 (or -20 db/decade)

- $0.5 < \omega < 10$: slope = 0 (or 0 db/decade)

- $10 < \omega < 50$: slope = -1 (or -20 db/decade)

- $50 < \omega$: slope = -2 (or -40 db/decade)

- Example 6.3: Bode Plot for Real Poles and Zeros

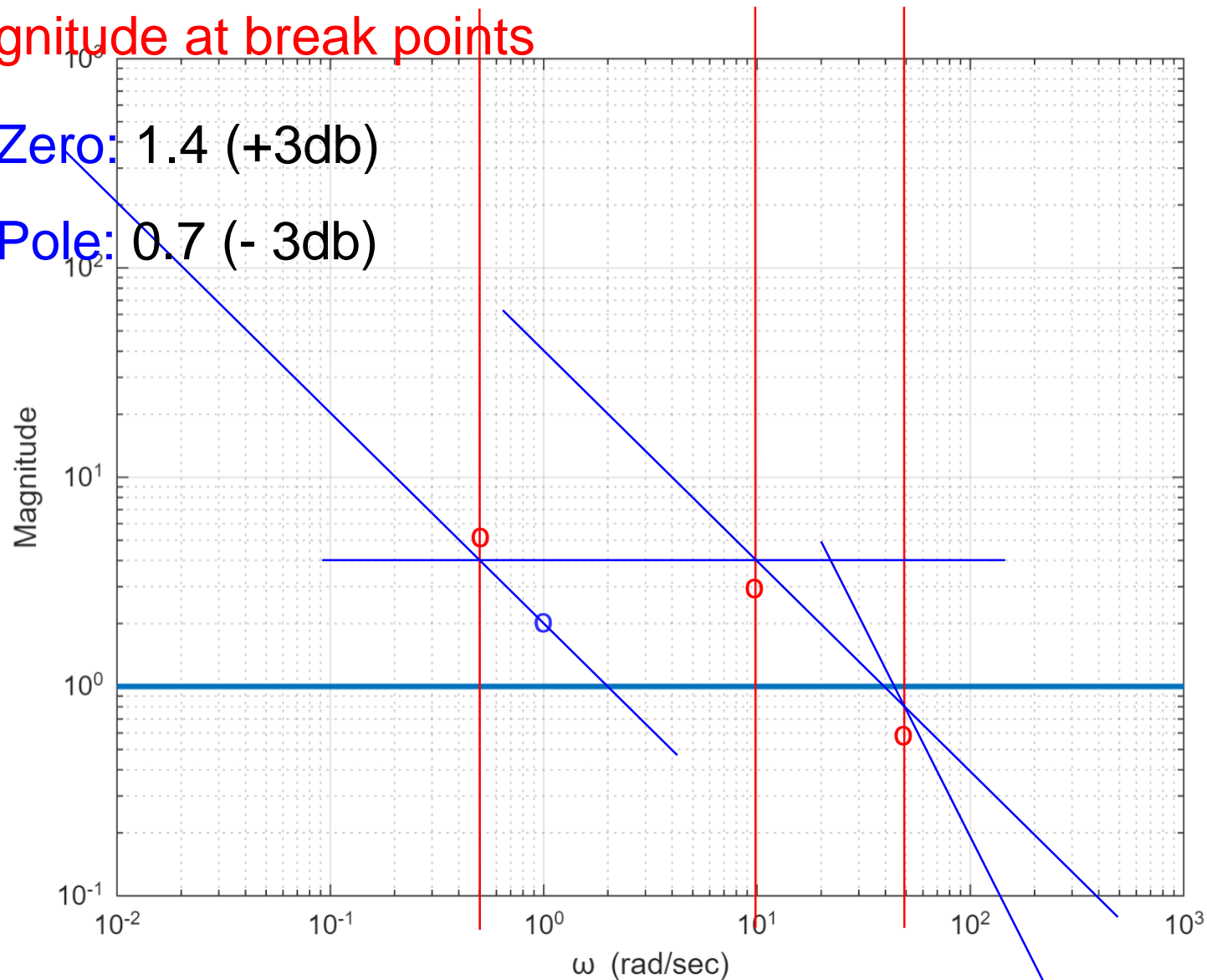


- Example 6.3: Bode Plot for Real Poles and Zeros

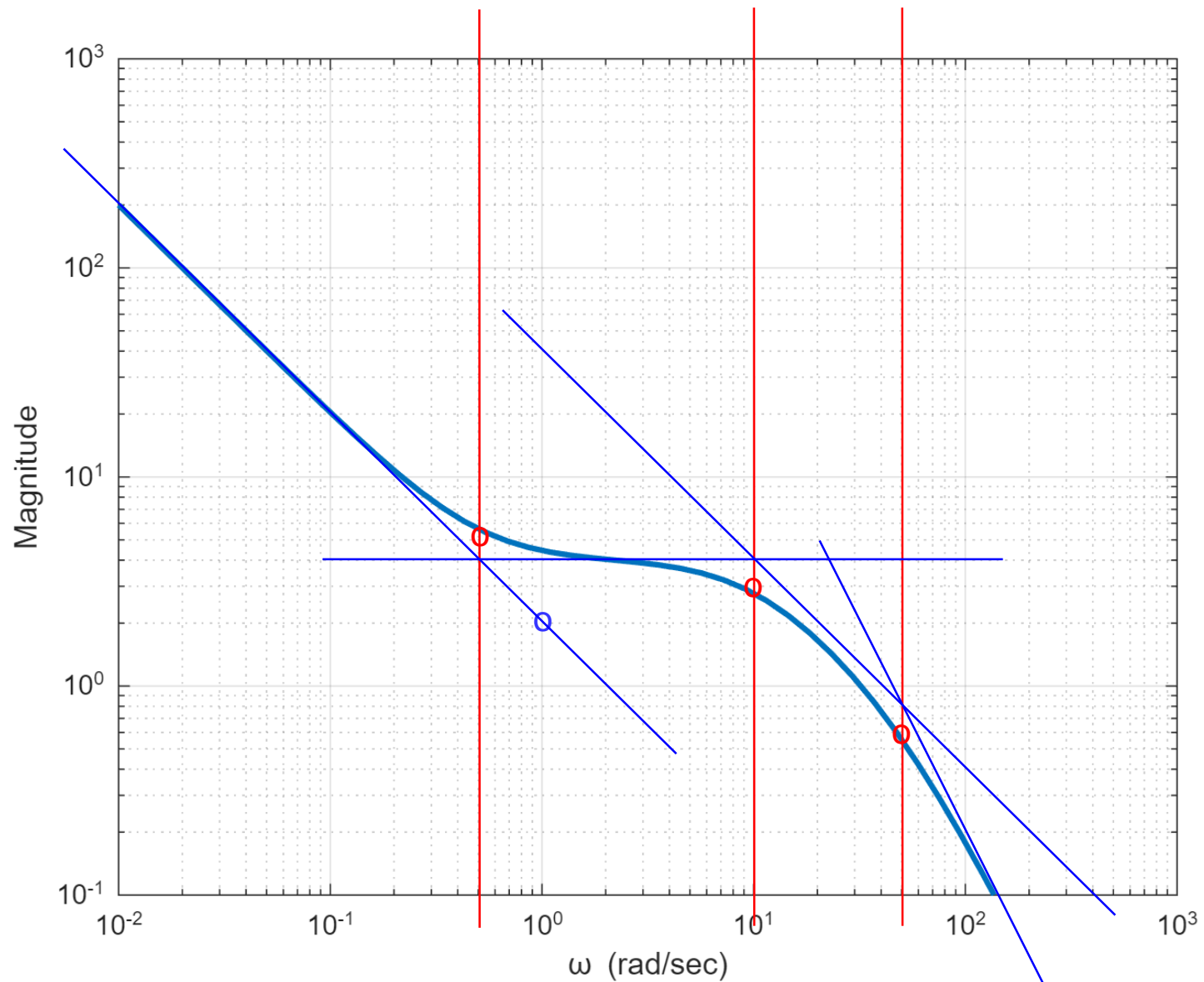
- (3) Magnitude at break points

- By Zero: 1.4 (+3db)

- By Pole: 0.7 (-3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



Examples

- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

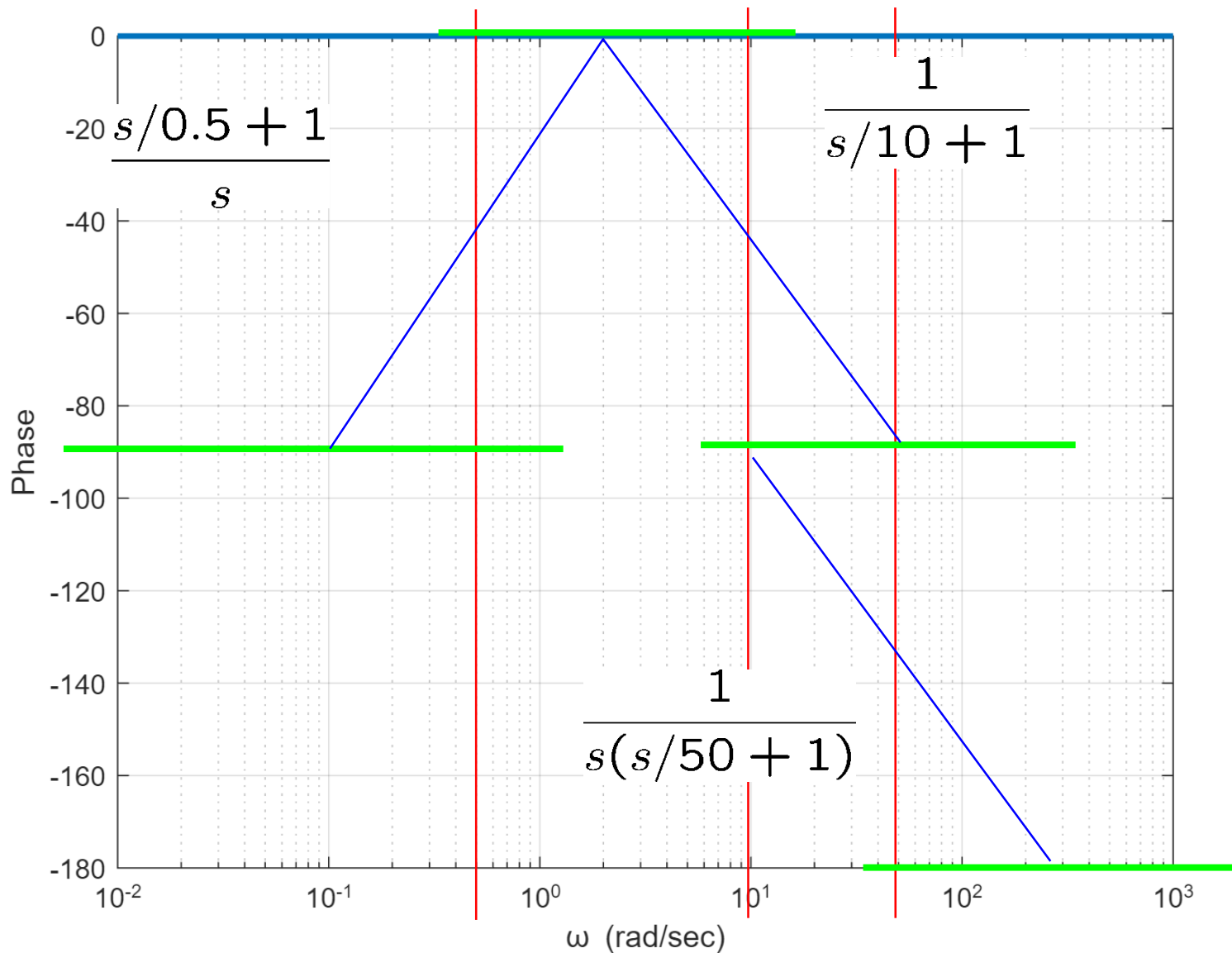
- Break points: 0.5, 10, 50

- (4) Phase

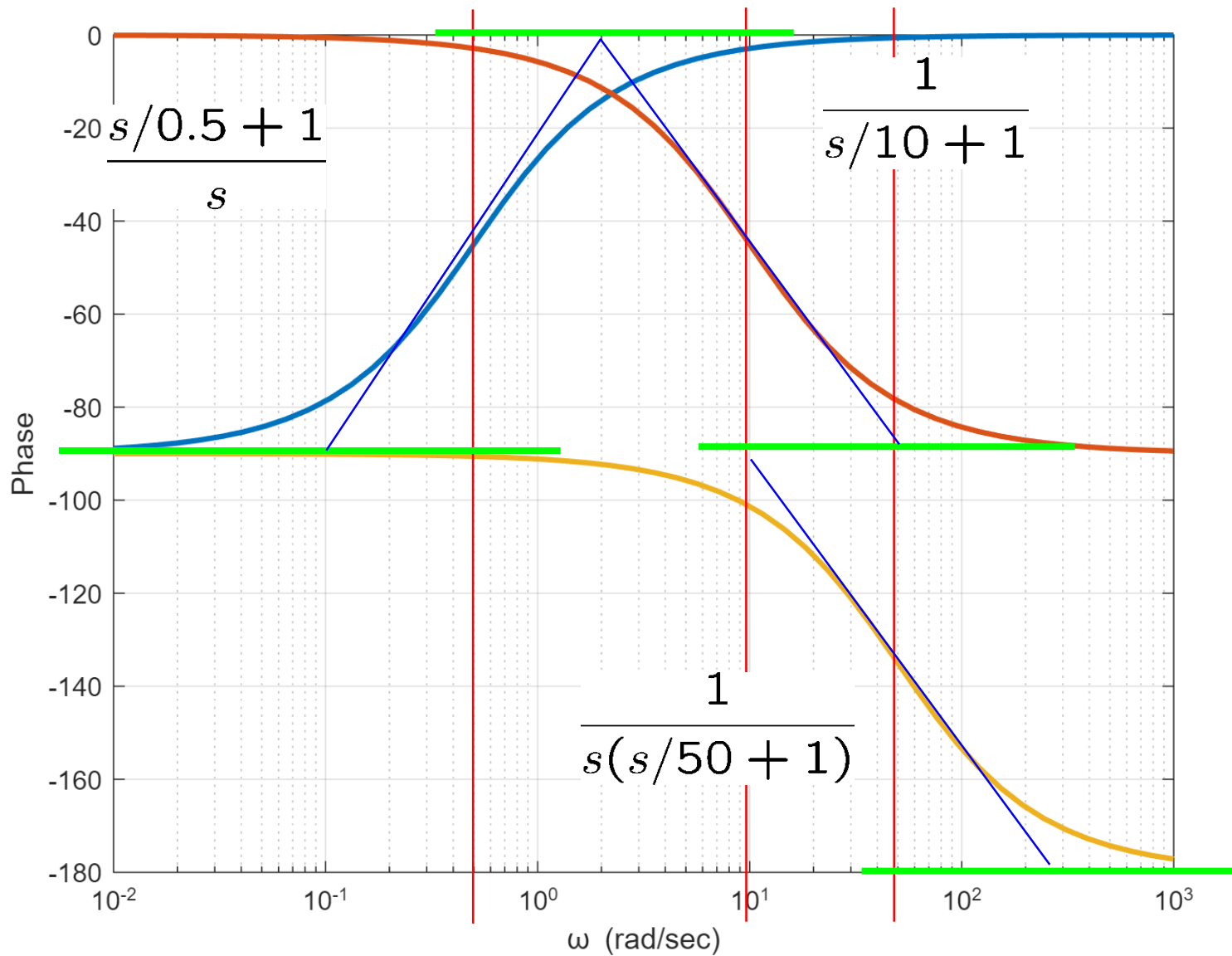
- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

- $\omega \ll 0.5$: phase = -90°
- $0.5 < \omega < 10$: phase = 0°
- $10 < \omega < 50$: phase = -90°
- $50 < \omega$: phase = -180°

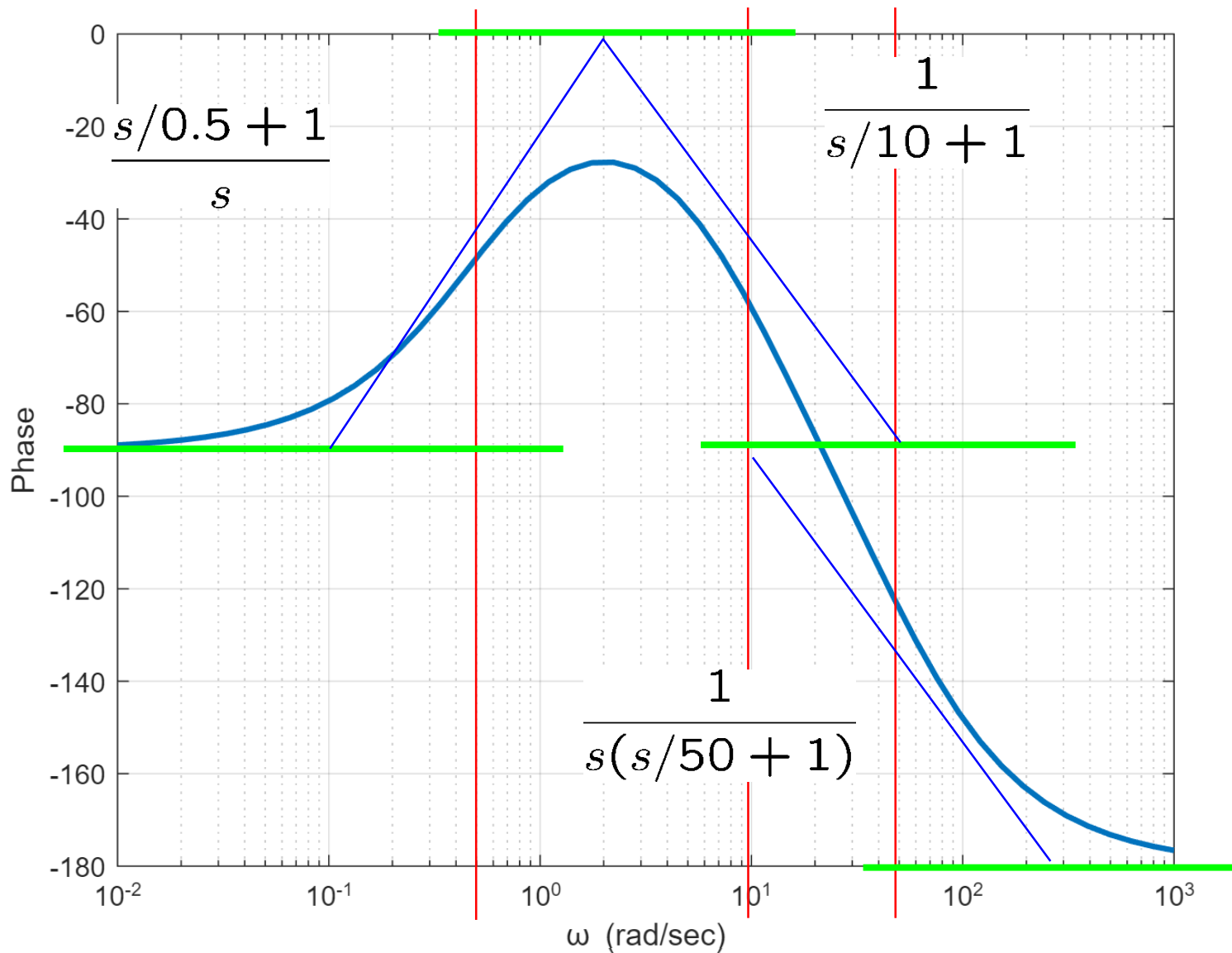
■ Example 6.3: Bode Plot for Real Poles and Zeros



■ Example 6.3: Bode Plot for Real Poles and Zeros



■ Example 6.3: Bode Plot for Real Poles and Zeros

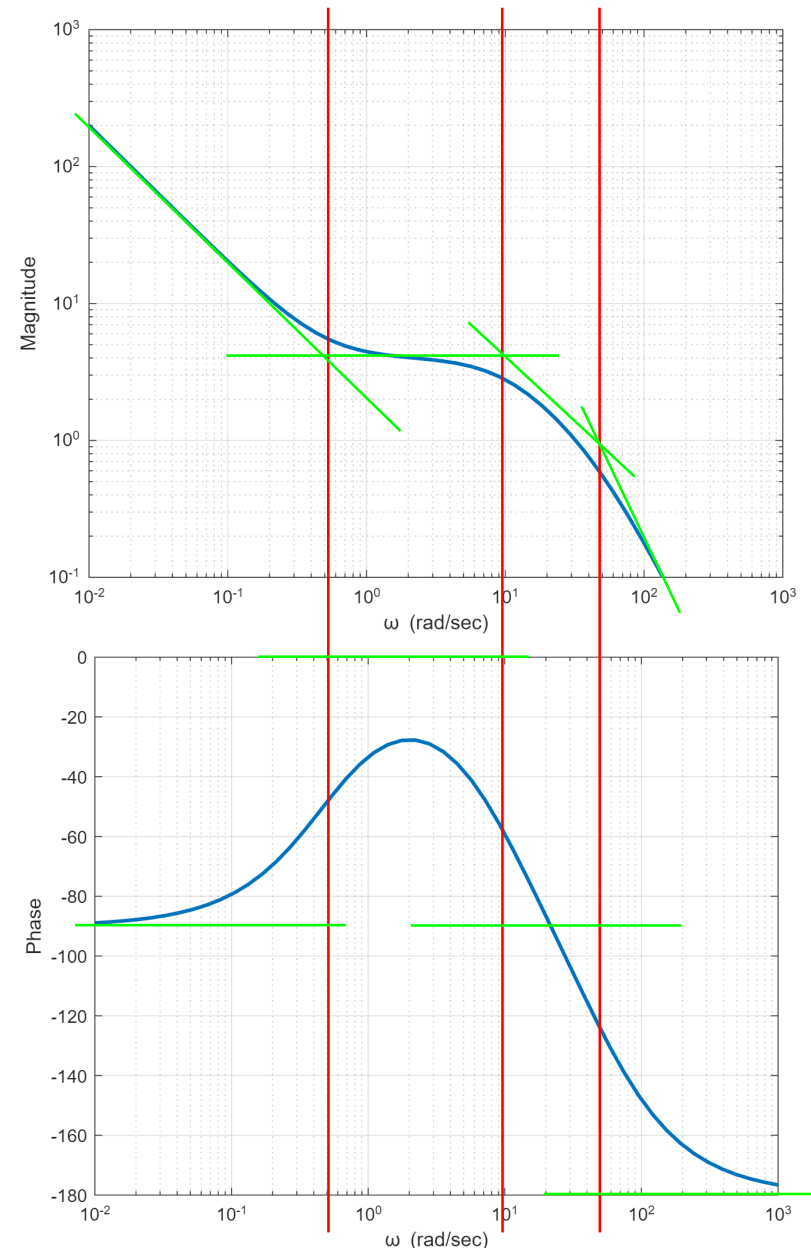


- Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

$$\frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50



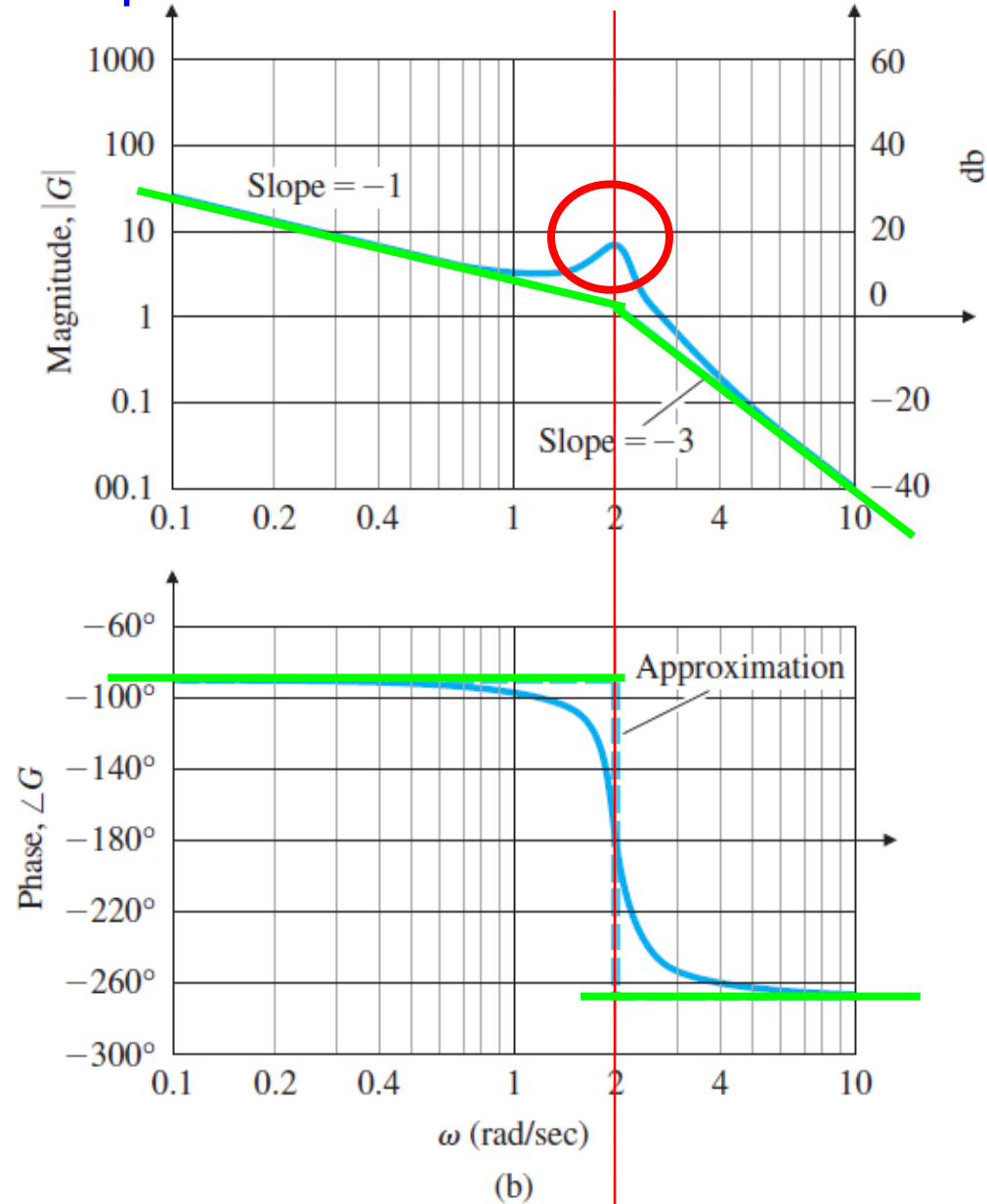
Examples

- Example 6.4: Bode Plot for Complex Poles

$$K G(s) = \frac{10}{s [s^2 + 0.4s + 4]}$$

$$= \frac{10}{4} \frac{1}{s \left[\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1 \right]}$$

- Break points: 2

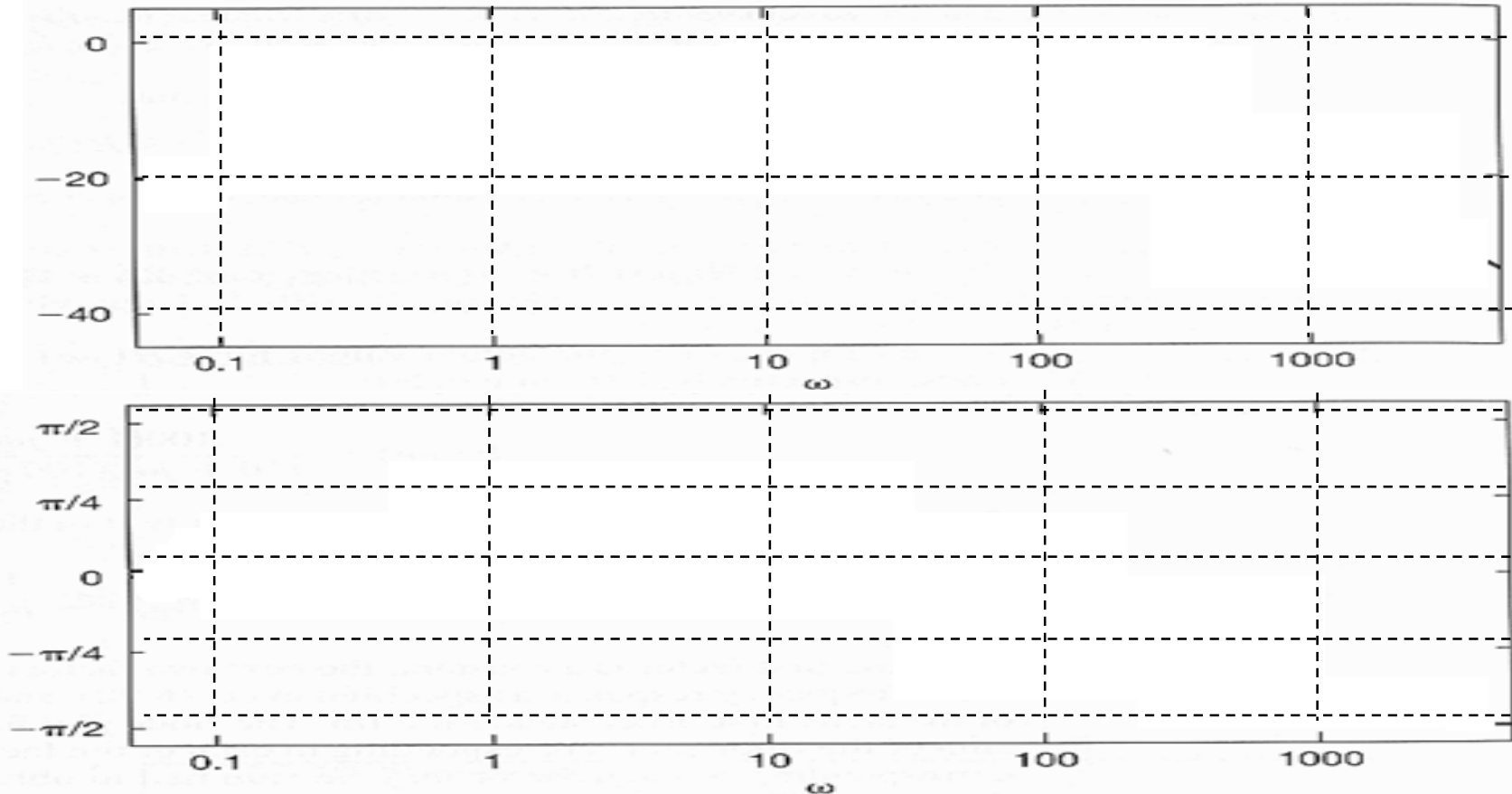


Examples

- Example 6.5:

$$H(j\omega) = \frac{100(1 + j\omega)}{(10 + j\omega)(100 + j\omega)}$$

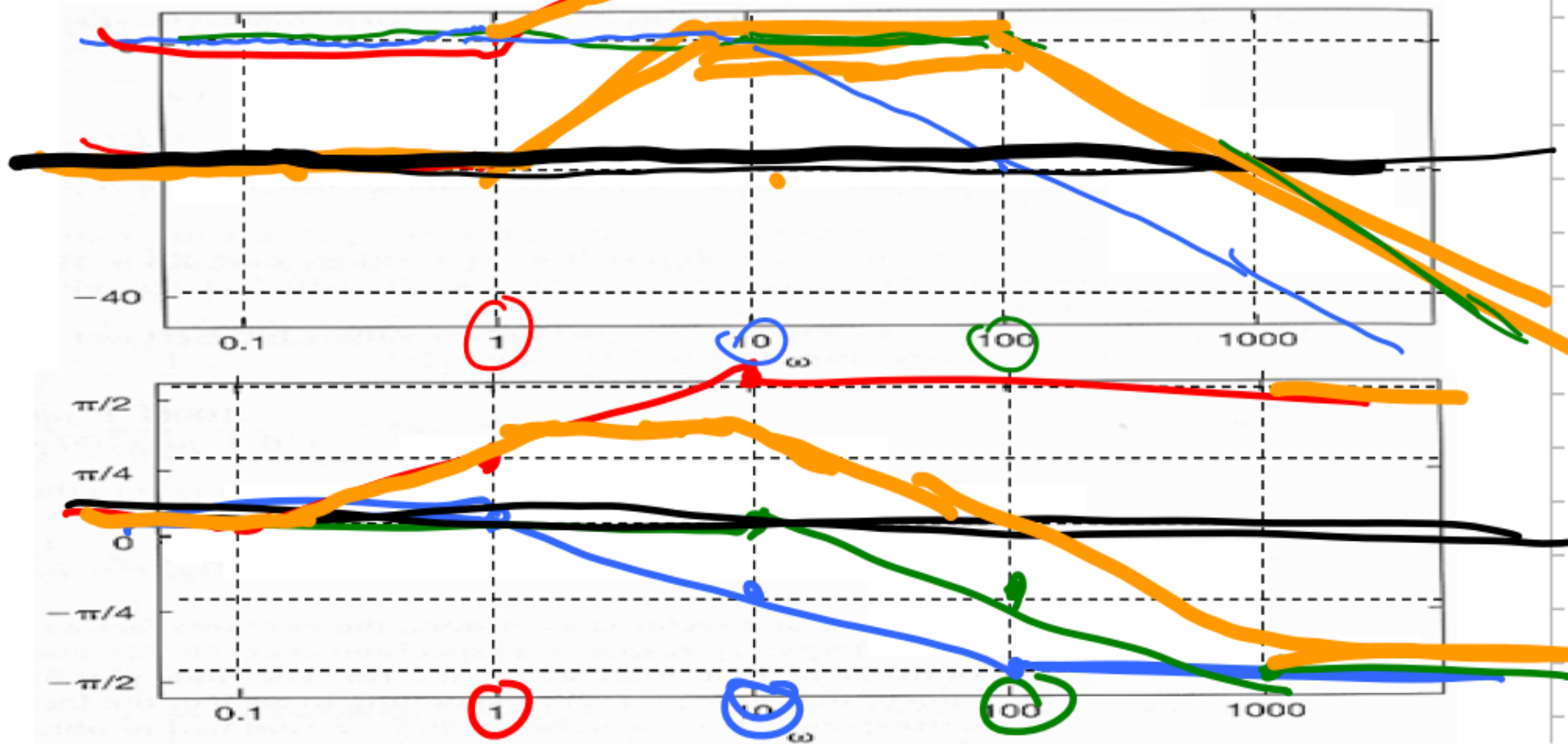
$$= \left(\frac{1}{10}\right) (1 + j\omega) \left(\frac{1}{1 + j\frac{\omega}{10}}\right) \left(\frac{1}{1 + j\frac{\omega}{100}}\right)$$

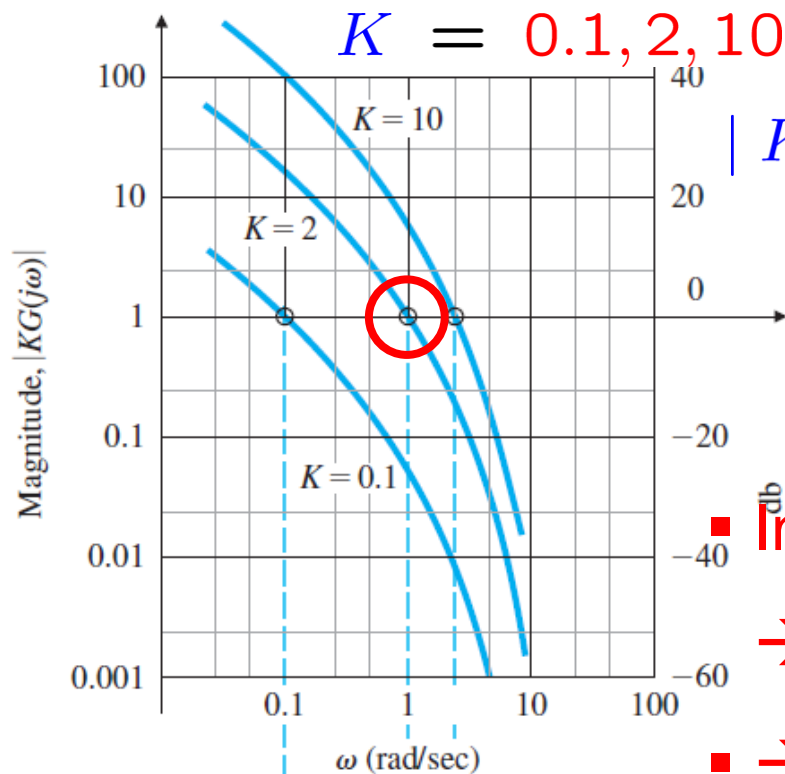


■ Example 6.5:

$$H(j\omega) = \frac{100(1 + j\omega)}{(10 + j\omega)(100 + j\omega)}$$

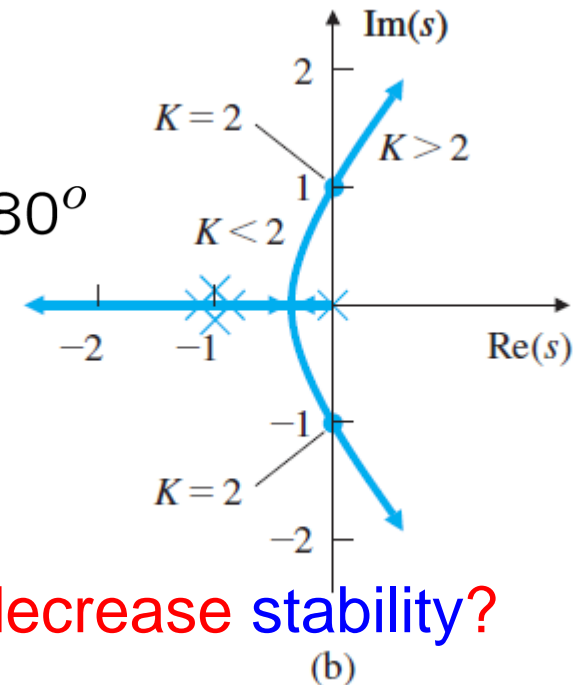
$$= \left(\frac{1}{10}\right) (1 + j\omega) \left(\frac{1}{1 + j\frac{\omega}{10}}\right) \left(\frac{1}{1 + j\frac{\omega}{100}}\right)$$





$$|KG(j\omega)| = 1$$

$$\angle G(j\omega) = 180^\circ$$



- Increasing gain
- increase OR decrease stability?

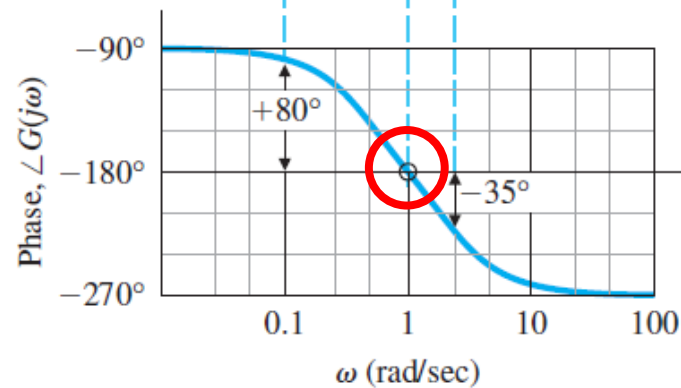
- $|KG(j\omega)| < 1$!!!

- However, for some systems,

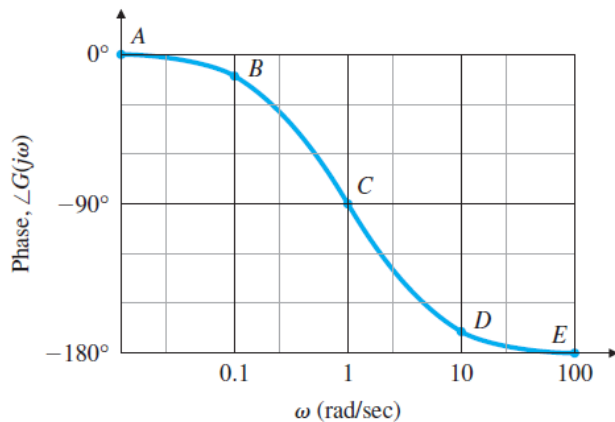
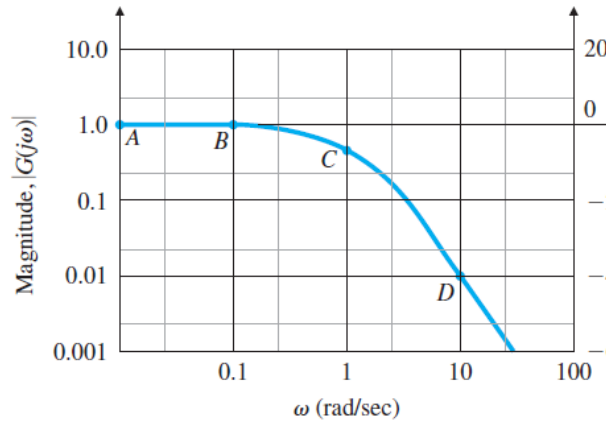
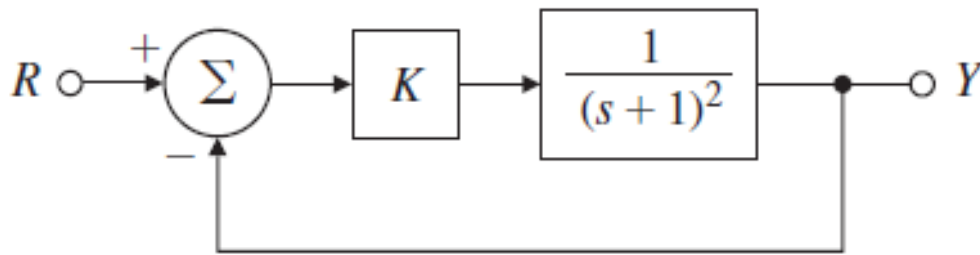
- $|KG(j\omega)| > 1$!!!

- Need to check their root locus

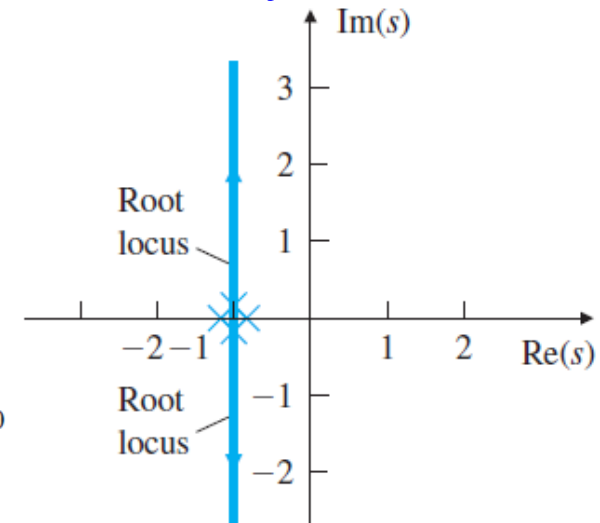
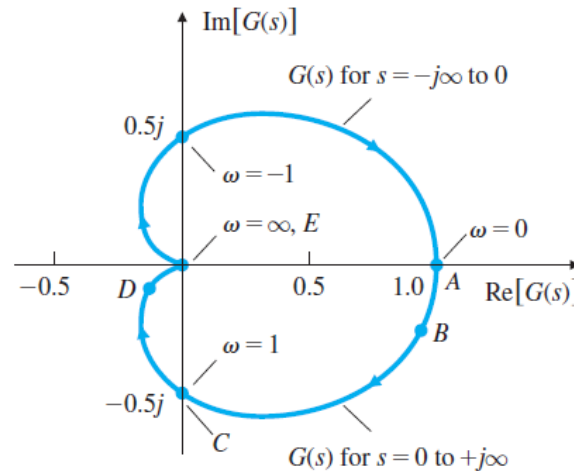
- OR, by Nyquist Stability Criterion



Example 6.8: Nyquist Plot for a Second-Order System

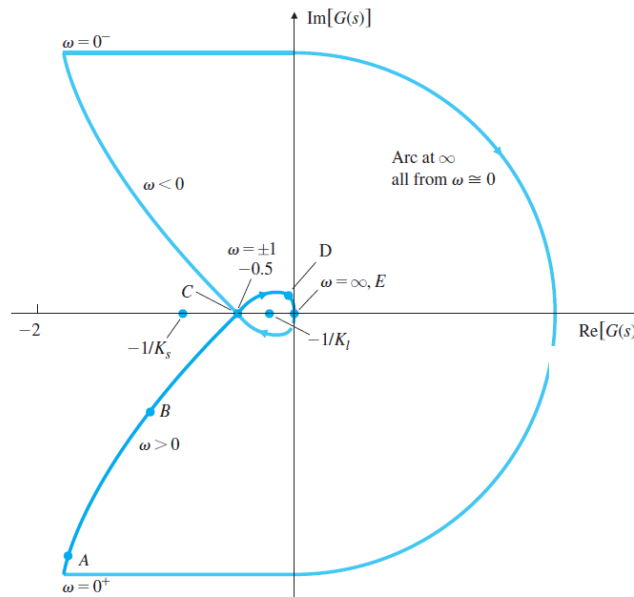
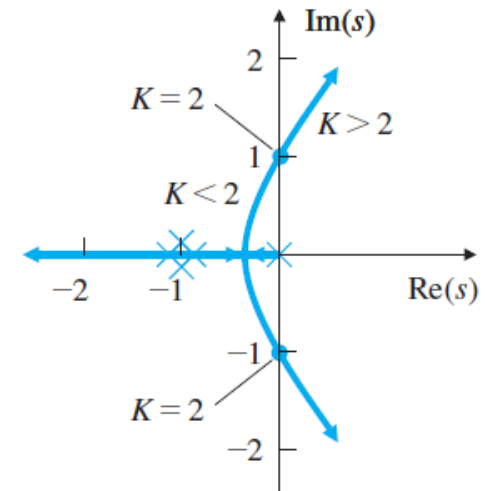
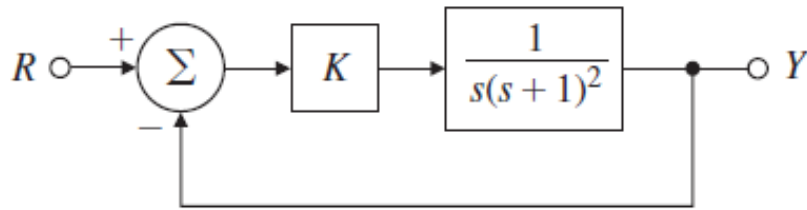


(b)

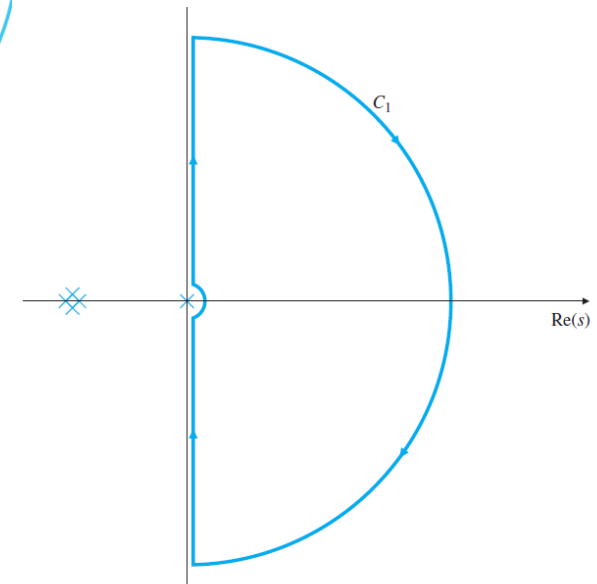


- $N = 0$: not encircle -1
- $P = 0$: no poles of $G(s)$ in RHP
- $Z = N + P \rightarrow Z = 0$,
no unstable roots for $K = 1$
- $K > 0$ also holds

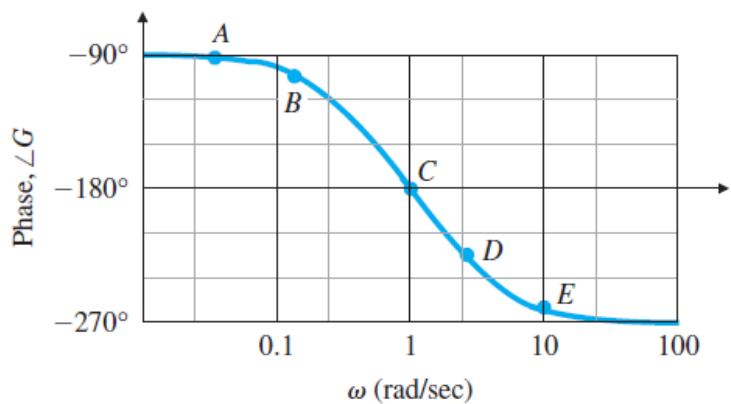
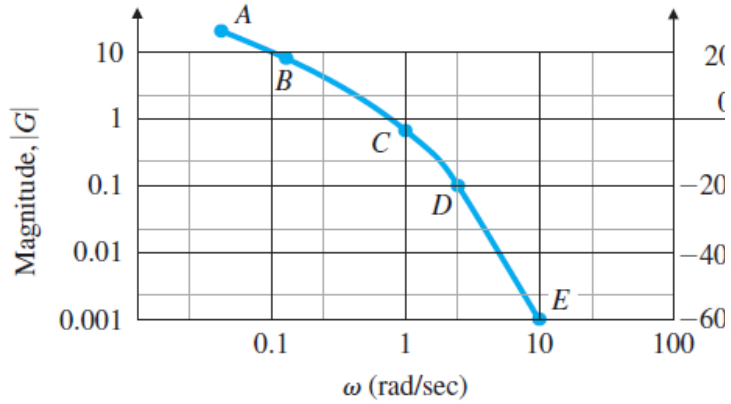
Example 6.9: Nyquist Plot for a Third-Order System



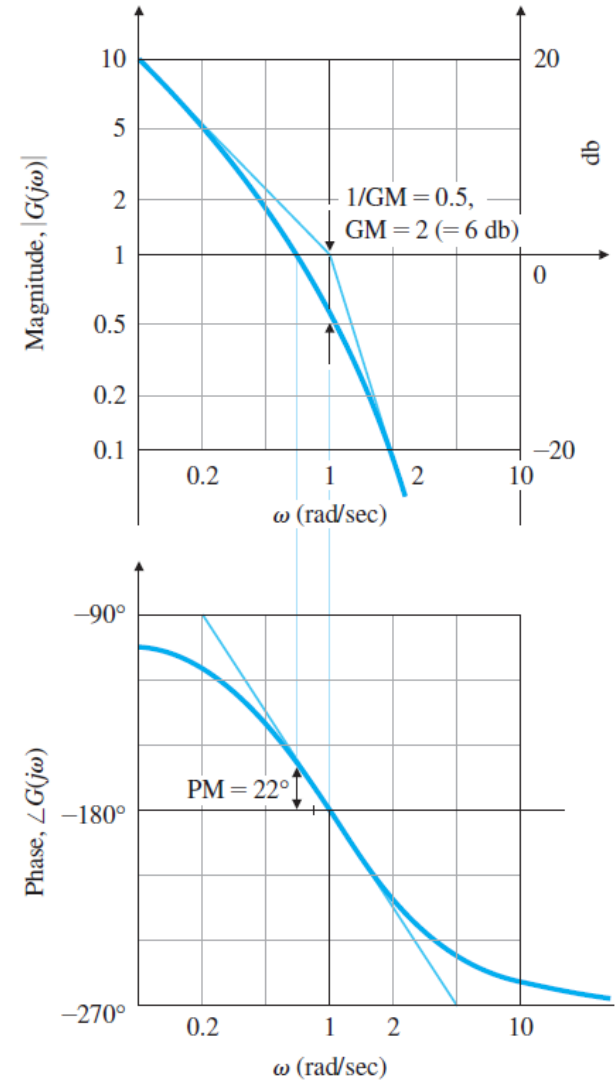
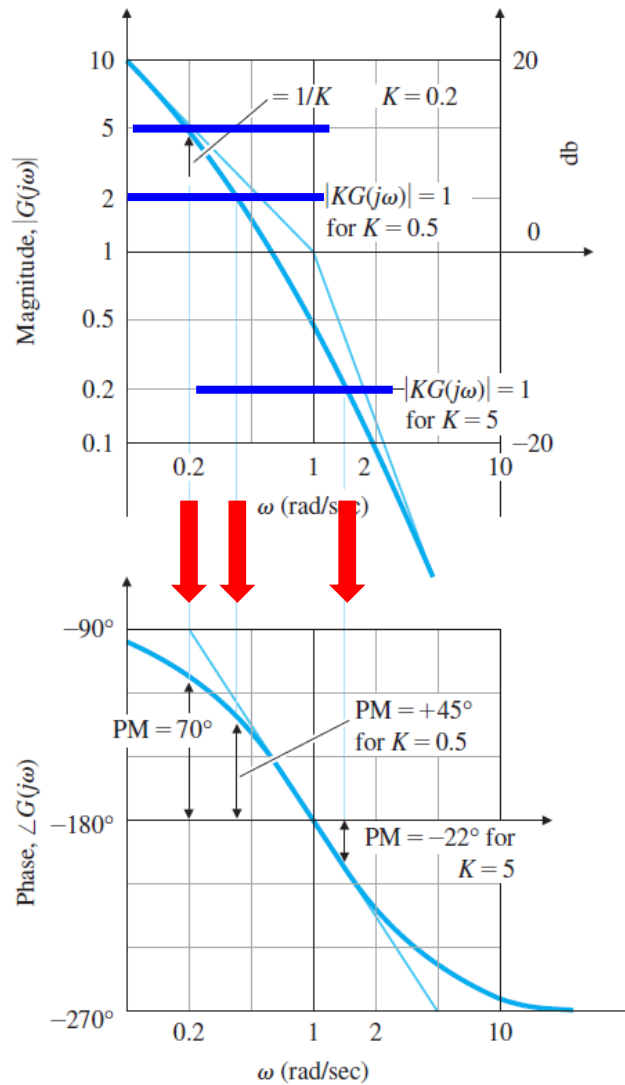
(b)



(b)



- PM vs K
- $K = 5$
- $|KG(j\omega)| = 1$
- $PM = -22^\circ$
- $K = 0.5$
- $|KG(j\omega)| = 1$
- $PM = +45^\circ$
- $K = 0.2$
- $|KG(j\omega)| = 1$
- $PM = +70^\circ$

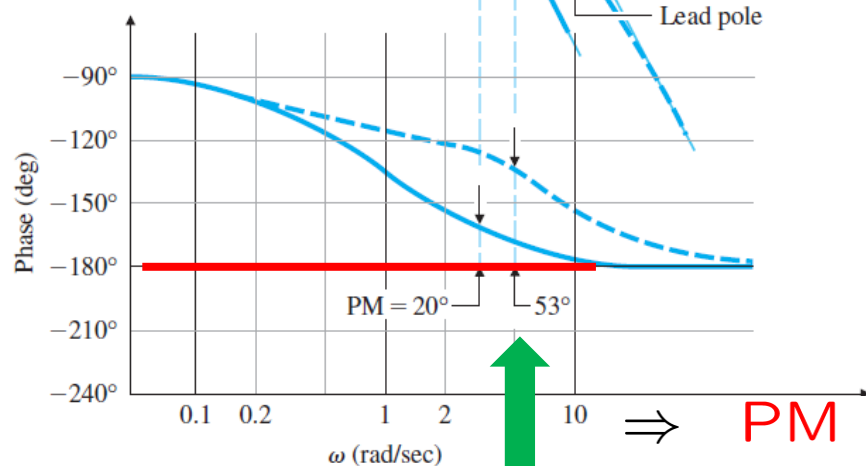
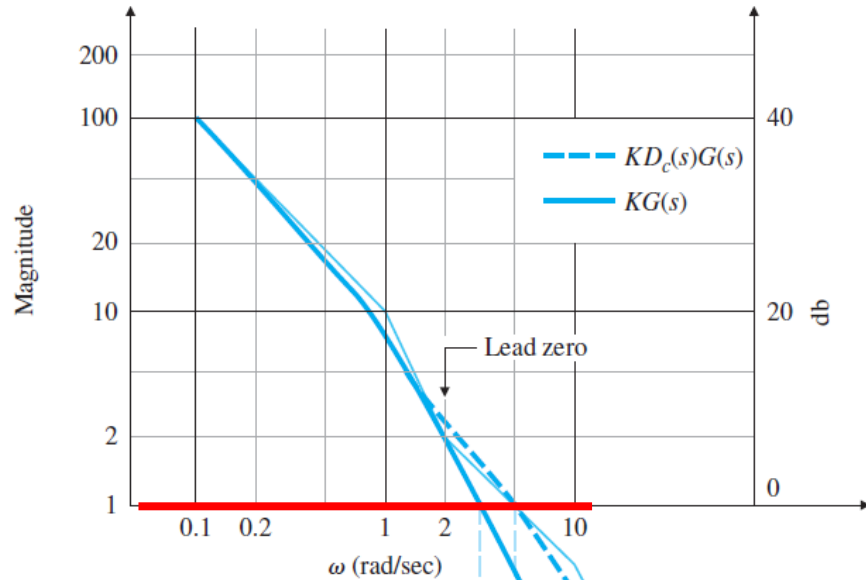


Examples

Example 6.15: Lead Compensation for a DC Motor

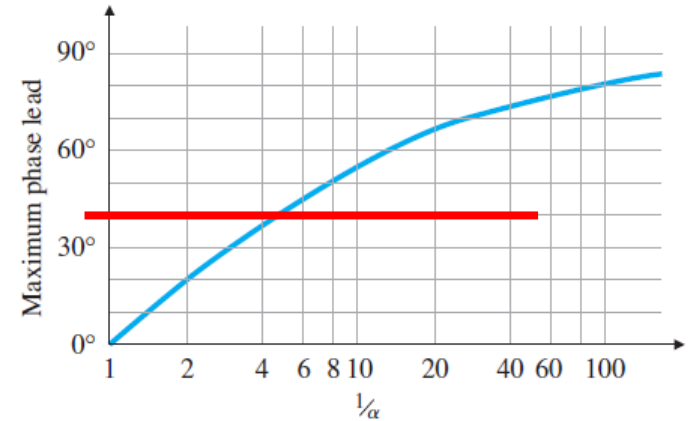
$$K G(s)$$

$$K D_c(s) G(s)$$



⇒ $PM \geq 25^\circ$, at $\omega_c = 3$

⇒ Phase lead = 40° ,



$$\Rightarrow \frac{1}{\alpha} = 5$$

a zero at $\omega = 2$ rad/sec

a pole at $\omega = 10$ rad/sec

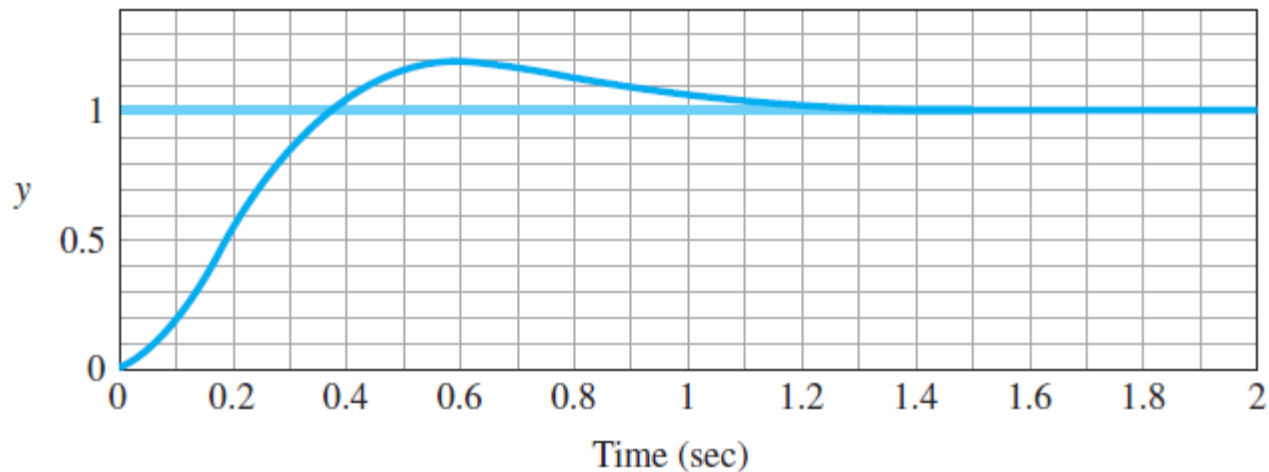
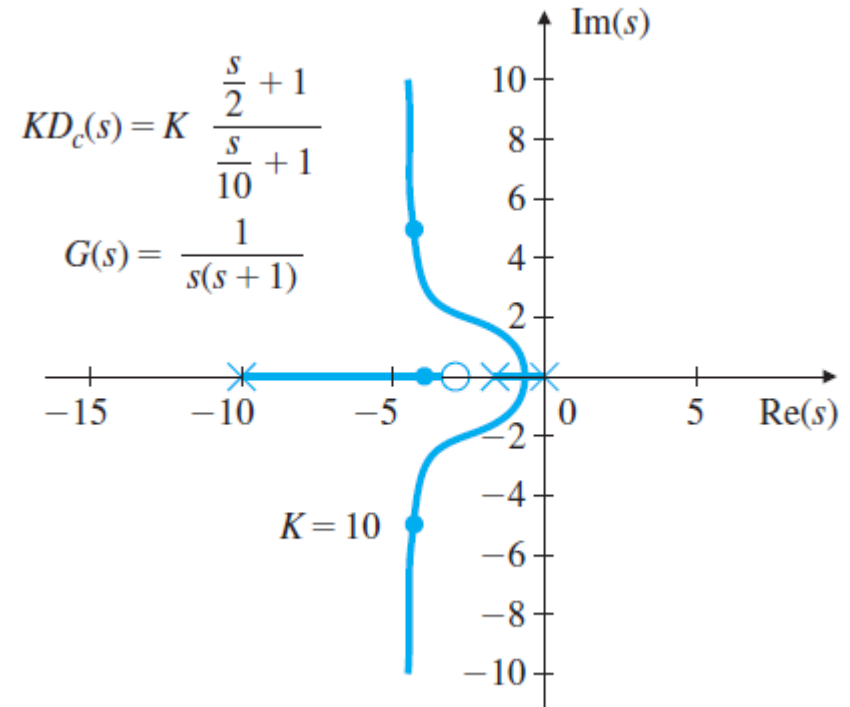
$$K D_c(s) = 10 \frac{\frac{s}{2} + 1}{\frac{s}{10} + 1}$$

⇒ $PM = 53^\circ$, at $\omega_c = 5$

Examples

- Example 6.15: Lead Compensation for a DC Motor

$$K D_c(s) = 10 \frac{\frac{s}{2} + 1}{\frac{s}{10} + 1}$$



Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

$$K G(s) = K \frac{10}{s \left(\frac{s}{2.5} + 1\right) \left(\frac{s}{6} + 1\right)}$$

- $K_v = 10$

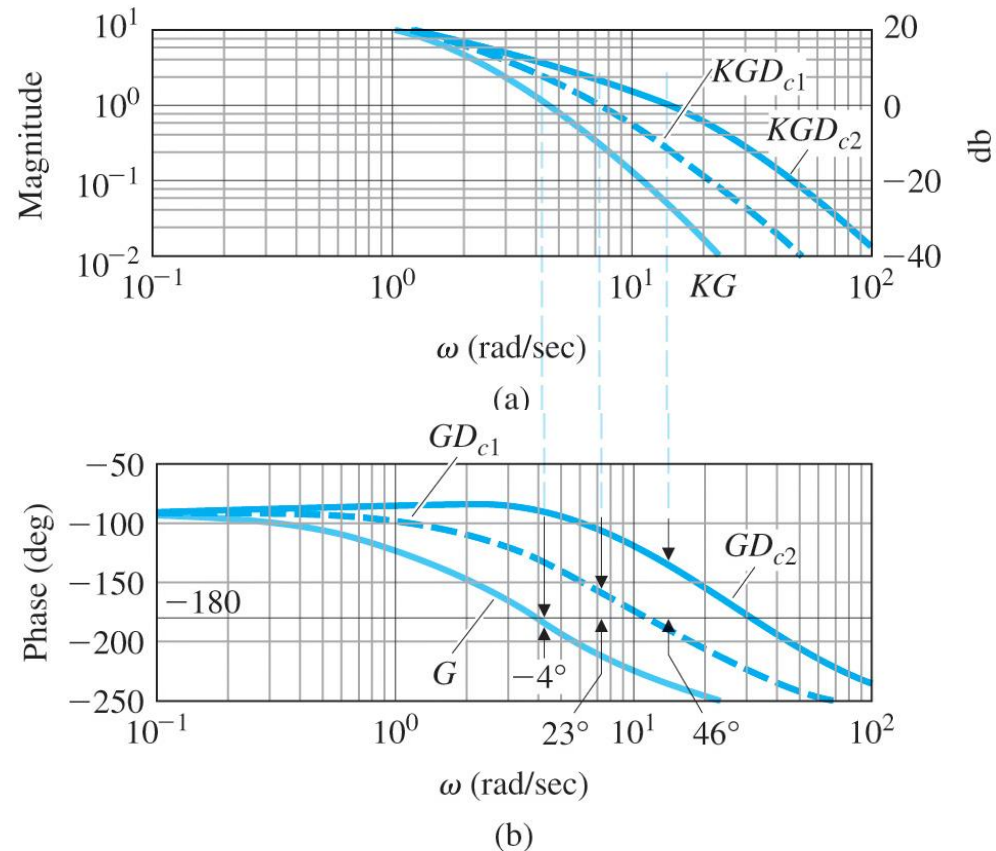
- $PM = 45^\circ$

1. Determine gain K:

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} s K G(s) \\ &= K \times 10 = 10 \\ \Rightarrow K &= 1 \end{aligned}$$

2. Bode plot of $KG(s)$, $K = 1$

$$\rightarrow PM \approx -4, W_{cp} \approx 4$$



Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

3. Allow for 5° of extra margin

$$\rightarrow 45^\circ + 5^\circ - (-4^\circ) = 54^\circ$$

4. Pick $\alpha \rightarrow 1/\alpha = 10$

5. Zero & Pole

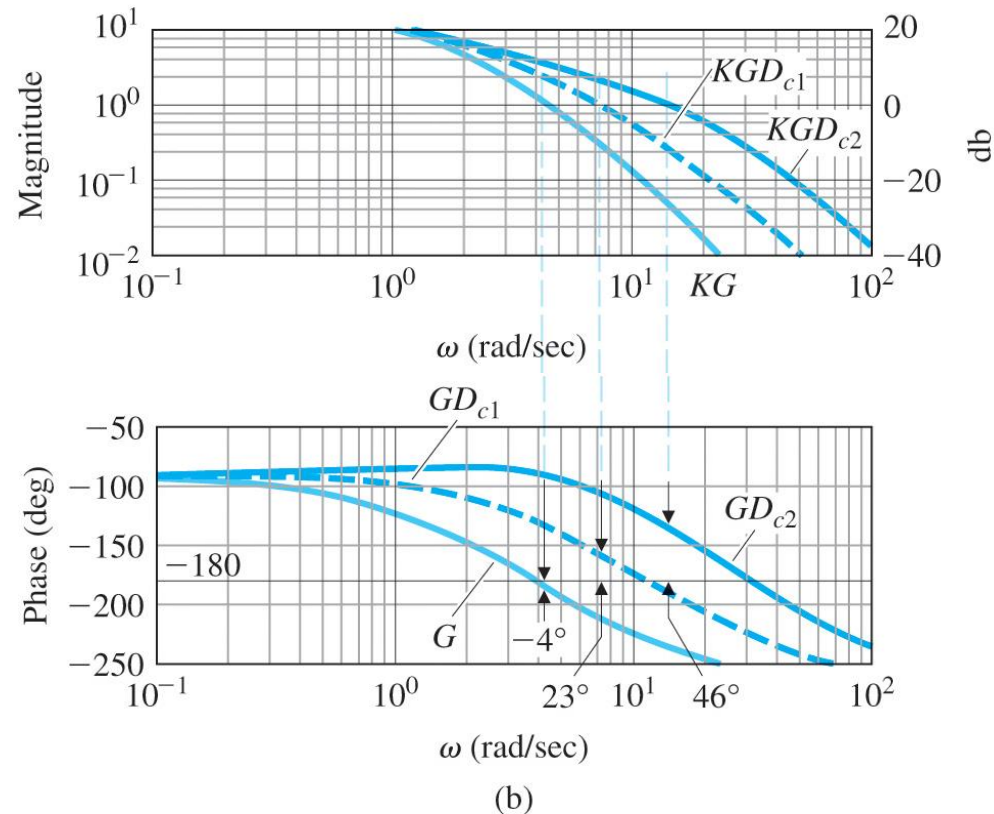
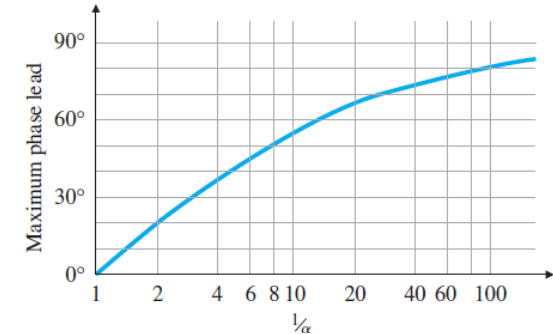
a zero at 2

a pole at 20

$$D_1(s) = \frac{\left(\frac{s}{2} + 1\right)}{\left(\frac{s}{20} + 1\right)}$$

$$= \frac{1}{0.1} \left(\frac{s + 2}{s + 20}\right)$$

\rightarrow PM ≈ 23 , $W_{cp} \approx 7$

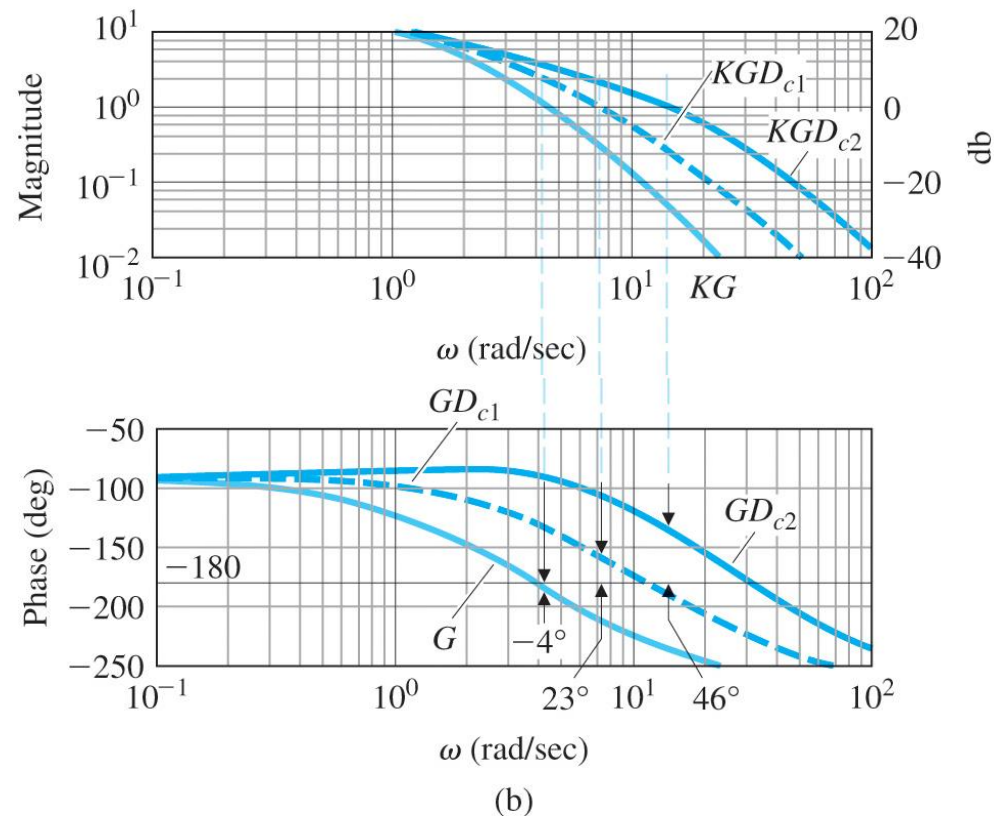


Example 6.17: Lead-Compensation Design for Type 1 Servomechanism System

7. A double-lead compensator:

$$D_2(s) = \frac{\left(\frac{s}{2} + 1\right) \left(\frac{s}{4} + 1\right)}{\left(\frac{s}{20} + 1\right) \left(\frac{s}{40} + 1\right)} = \frac{1}{(0.1)^2} \frac{(s + 2)(s + 4)}{(s + 20)(s + 40)}$$

■ $PM = 46^\circ$



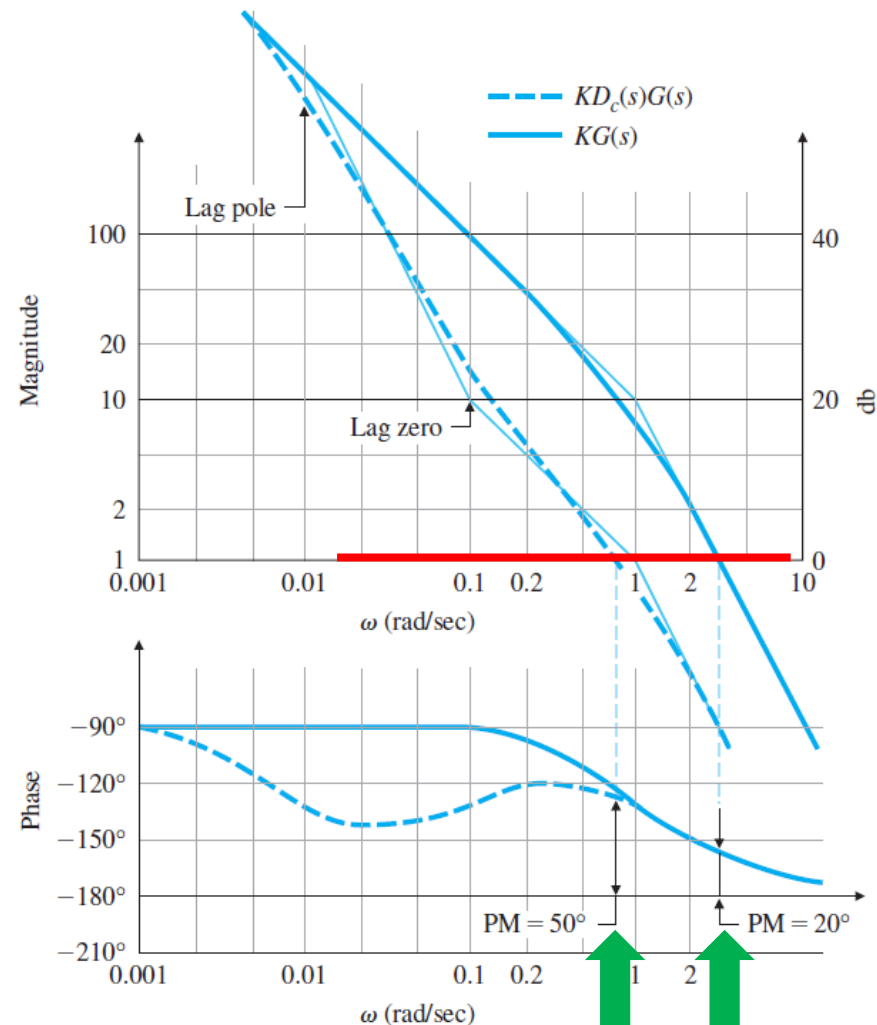
Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- $K = 10$
- $PM = 20^\circ$ at $\omega_c \approx 3$
- Select break points
 - ✓ ω_c is lowered
 - ✓ more favorable PM results
- Lag zero = 0.10
- Lag pole = 0.01
- ✓ $PM = 50^\circ$

▪ Error constant: $K_v = 10$

▪ $PM = 45^\circ$

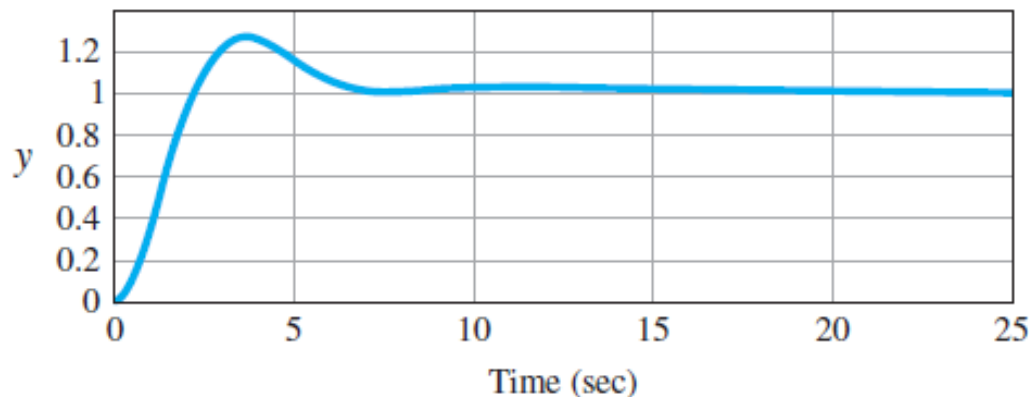


- Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- Error constant: $K_v = 10$

- PM = 45°



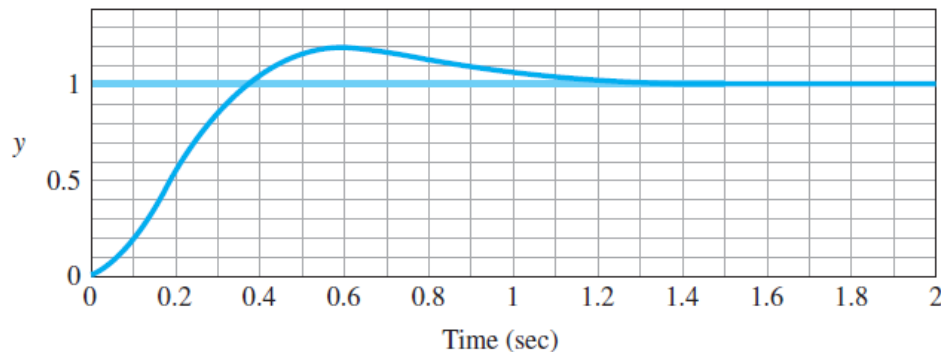
- No steady-state error

- ✓ a Type 1 system

- Settling time ≈ 25 sec

- Rise time ≈ 2 sec

- Example 6.15: Lead Compensation



Example

Example 6.20: PID Compensation

$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s + 2}$$

$\frac{1}{T_I} = 0.5$	$\frac{1}{T_D} = 10$
$\frac{1}{T_I} = 0.05$	$\frac{1}{T_D} = 1$
$\frac{1}{T_I} = 0.005$	$\frac{1}{T_D} = 0.1$

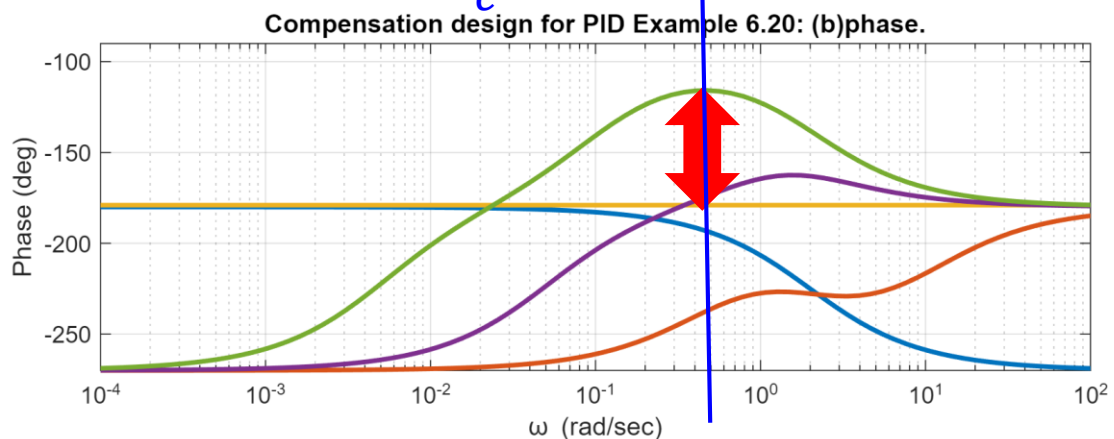
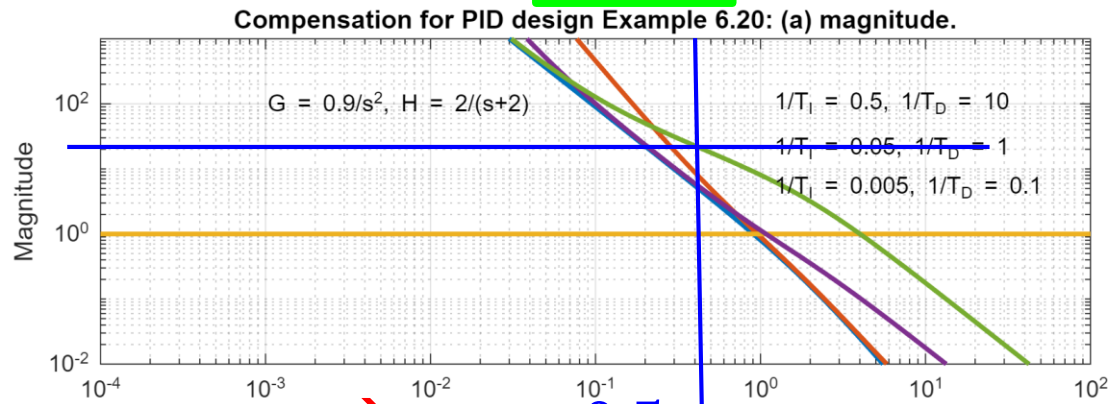
PM = 65°

→ Find K

| D_c(s) G(s) | = 20

1/K = 20

K = 0.05

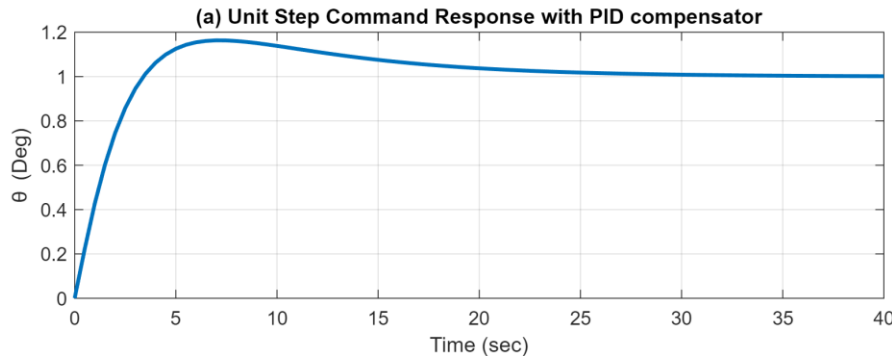


Example

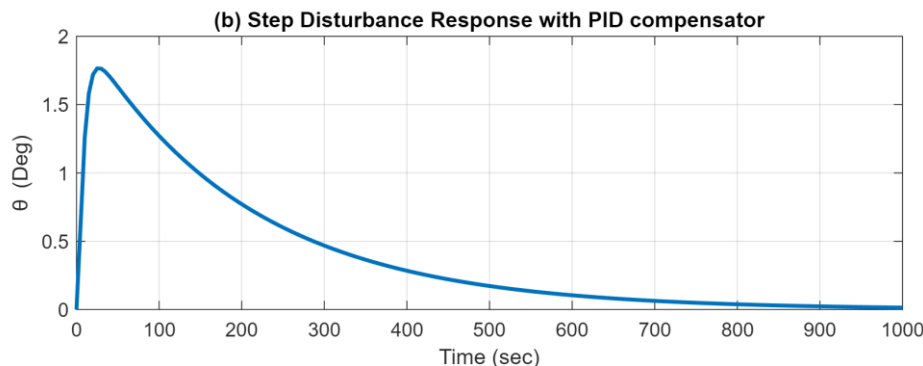
Example 6.20: PID Compensation

for Spacecraft Attitude Control

$$T(s) = \frac{\Theta}{\Theta_{com}} = \frac{D_c G}{1 + D_c G H}$$



$$\frac{\Theta}{T_d} = \frac{G}{1 + D_c G H}$$



Frequency Response of $T(s)$ and $S(s)$ are shown:

