

Fall 2021 (110-1)

控制系統  
Control Systems

Unit 5D  
Design Using Dynamic Compensation

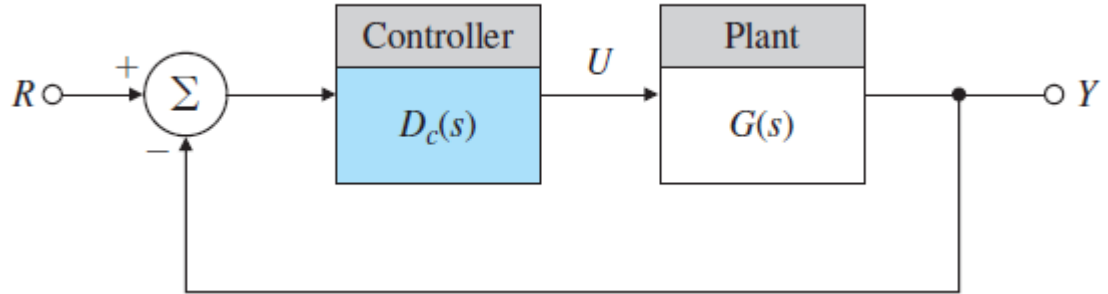
Feng-Li Lian

NTU-EE

Sep 2021 – Jan 2022

- Three categories of Particularly Simple and Effective Designs:
- Lead Compensation:
  - Approximates the function of PD control and
  - Acts mainly to speed up a response  
by lowering rise time and decreasing the transient overshoot
- Lag Compensation:
  - Approximates the function of PI control and
  - Is usually used to improve the steady-state accuracy
- Notch Compensation:
  - To achieve stability for systems with lightly damped flexible modes

## Model:



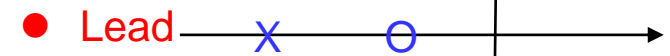
$$D_c(s) = K \frac{s + z}{s + p}$$

$$\Rightarrow 1 + D_c(s) G(s) = 0$$

$$\Rightarrow 1 + K L(s) = 0$$

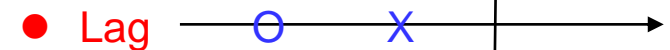
## Lead Compensation: if $z < p$

- Provides a positive phase shift



## Lag Compensation: if $z > p$

- Provides a negative phase shift



## 2nd-Order Position Control System

$$D_c(s) = K$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K \frac{1}{s(s+1)} = 0$$

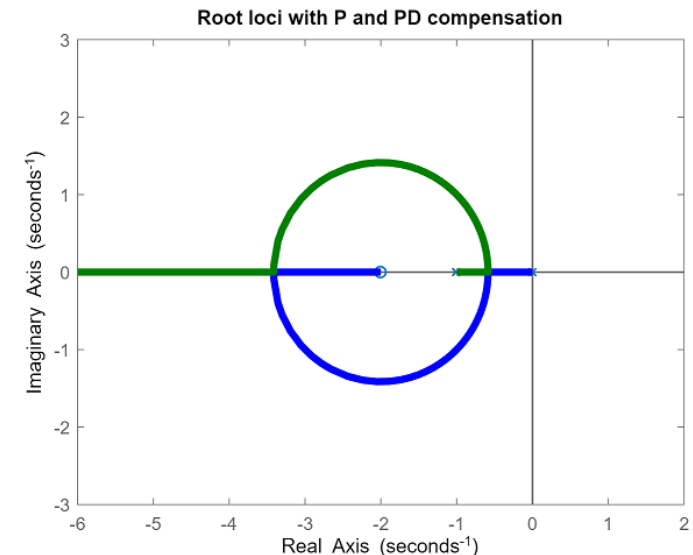
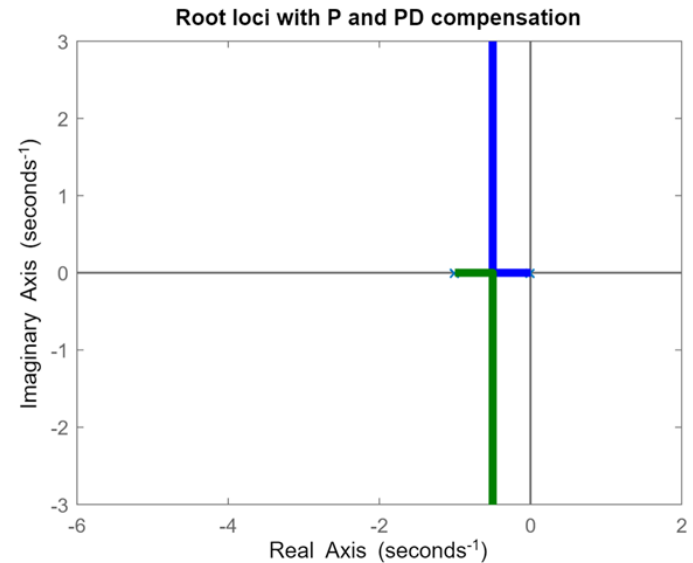
$$\Rightarrow s(s+1) + K = 0$$

$$D_c(s) = K(s+2)$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K(s+2) \frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K(s+2) = 0$$



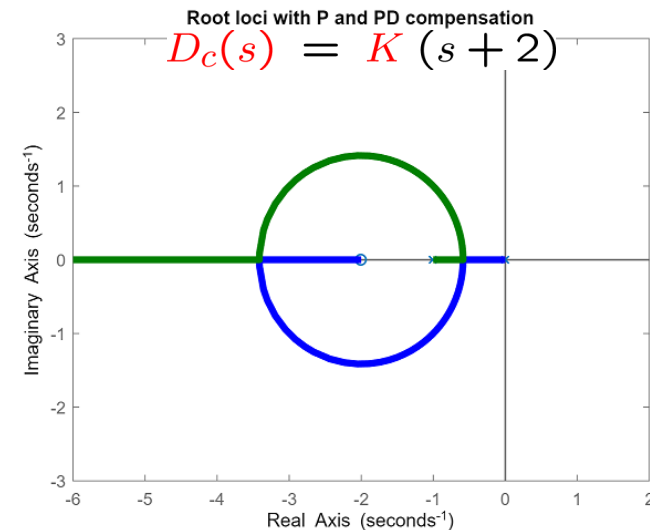
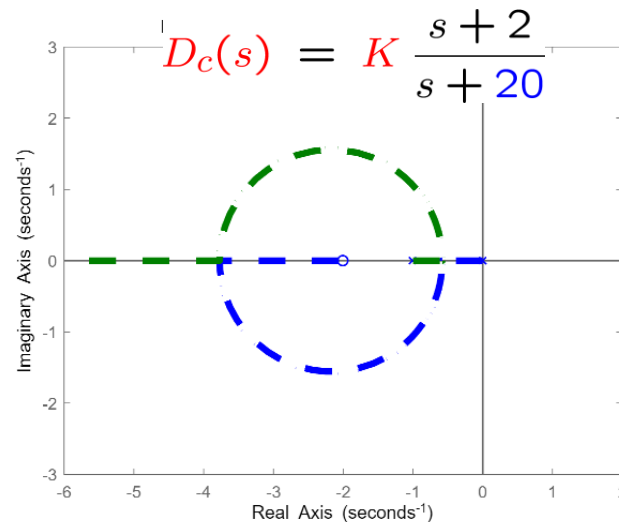
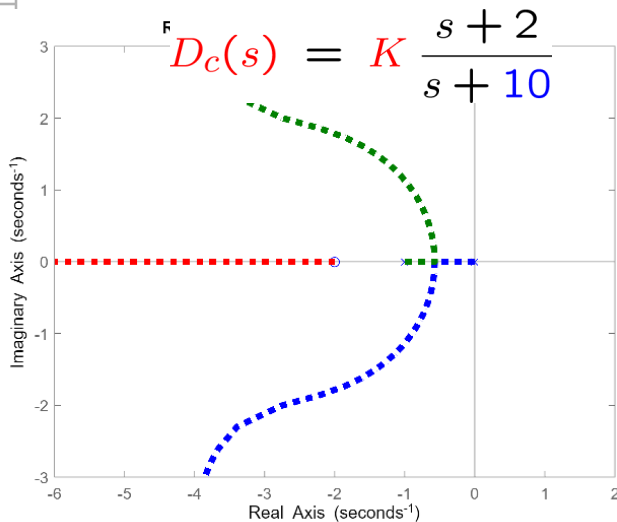
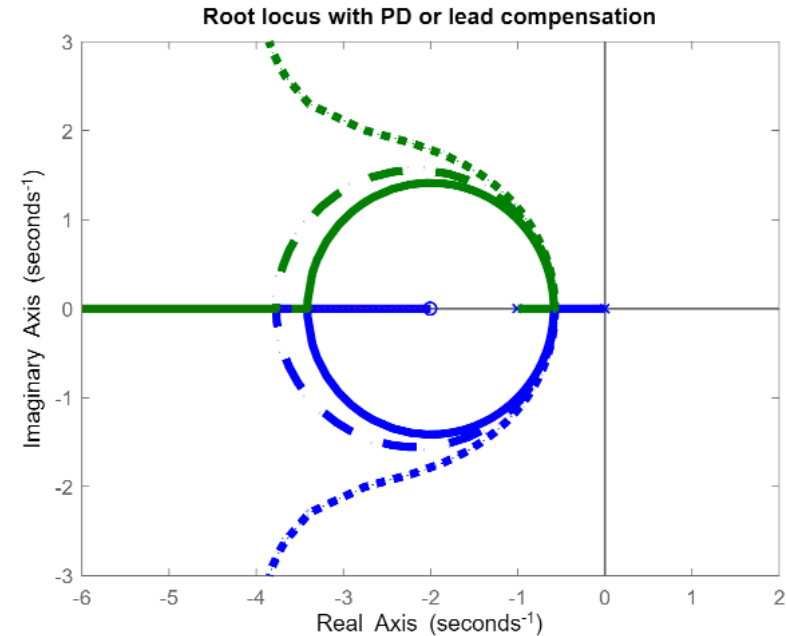
## 2nd-Order Position Control System

$$D_c(s) = K \frac{s + 2}{s + p}$$

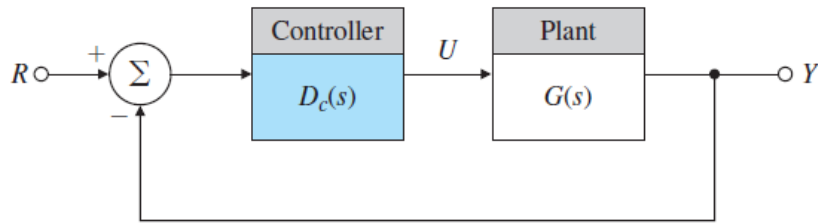
$$G(s) = \frac{1}{s(s + 1)}$$

$$\Rightarrow 1 + K \frac{s + 2}{s + p} \frac{1}{s(s + 1)} = 0$$

$$\Rightarrow 1 + K \frac{(s + 2)}{s(s + 1)(s + p)} = 0 \quad \blacksquare \quad p = 10, 20$$



## Example 5.11: Design Using Lead Compensation



$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = K \frac{s+2}{s+10}$$

$$1 + K \frac{s+2}{s+10} \frac{1}{s(s+1)} = 0$$

$$(s+10)s(s+1) + K(s+2) = 0$$

$$s^3 + 11s^2 + 10s + K(s+2) = 0$$

$$\Rightarrow K = 70$$

$$\Rightarrow s^3 + 11s^2 + 80s + 140 = 0$$

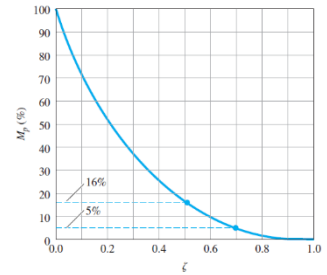
$$\Rightarrow s = -2.34, -4.33 \pm 6.40i$$

■ Overshoot  $\leq 20\%$

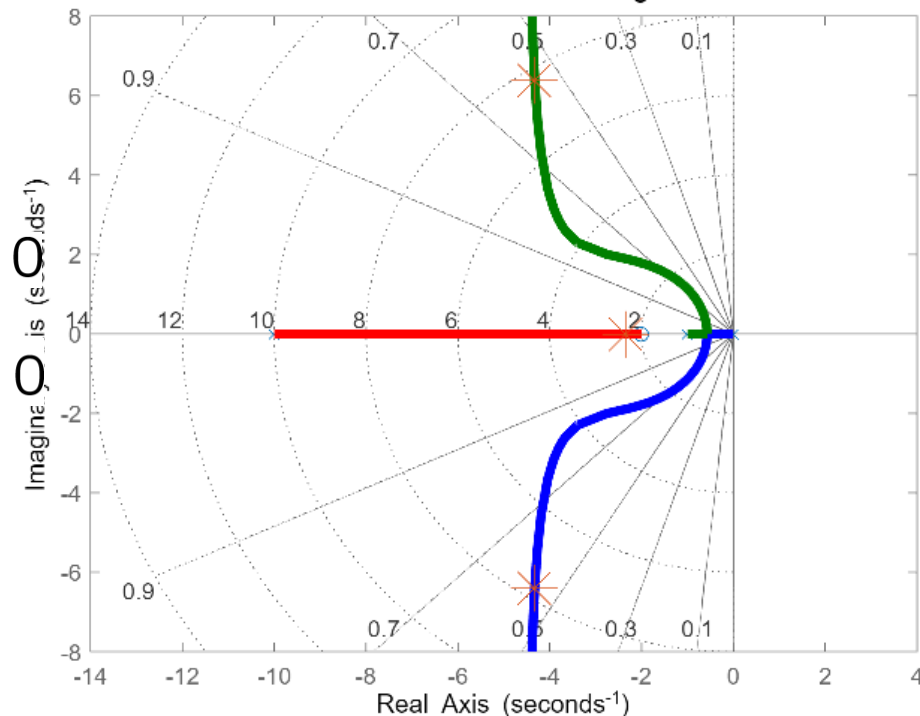
■ Rise Time  $\leq 0.3$  sec

$$\Rightarrow \zeta \geq 0.5$$

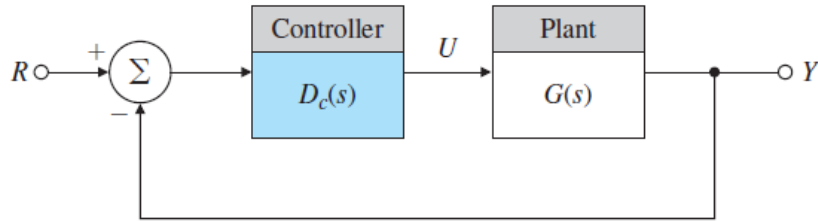
$$\Rightarrow \omega_n \approx \frac{1.8}{0.3} \approx 6 \Rightarrow \omega_n \geq 7$$



Root locus for lead design



## Example 5.11: Design Using Lead Compensation



- Overshoot  $\leq 20\%$
- Rise Time  $\leq 0.3$  sec

$$G(s) = \frac{1}{s(s+1)}$$

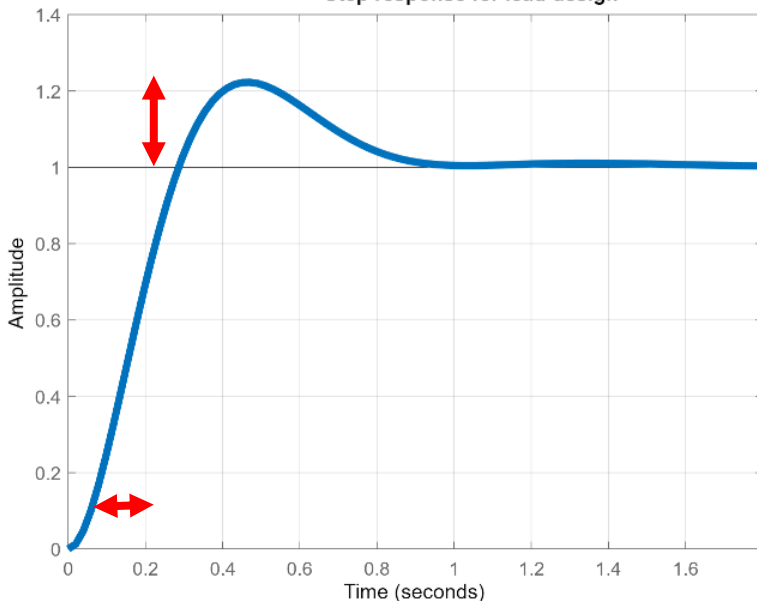
$$D_c(s) = 70 \frac{s+2}{s+10}$$

$$\Rightarrow \zeta \geq 0.5$$

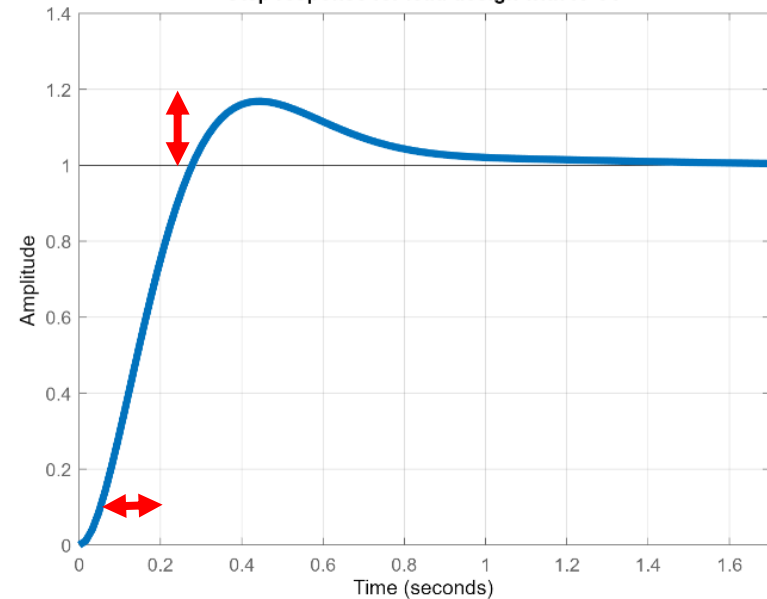
$$\Rightarrow \omega_n \approx \frac{1.8}{0.3} \approx 6 \Rightarrow \omega_n \geq 7$$

$$D_c(s) = 91 \frac{s+2}{s+13}$$

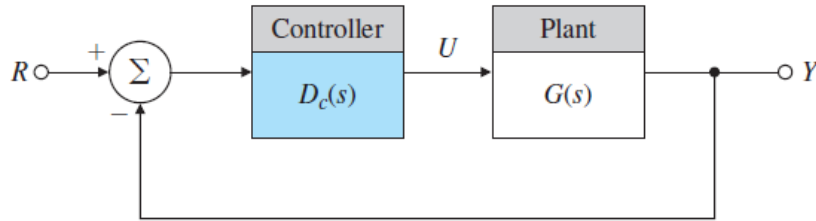
Step response for lead design



Step response for lead design with K=91



## Example 5.11: Design Using Lead Compensation



- Overshoot  $\leq 20\%$
- Rise Time  $\leq 0.3$  sec

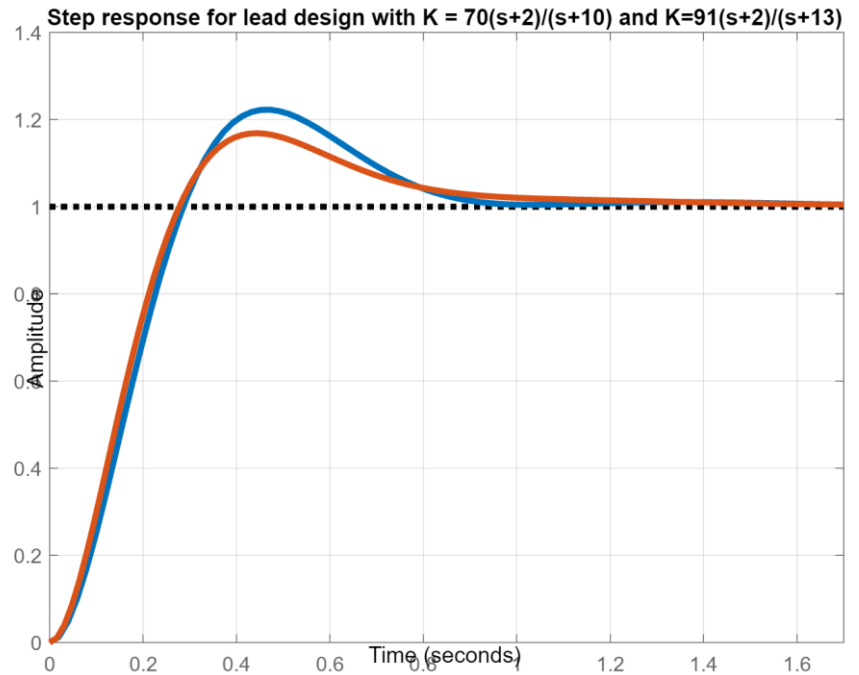
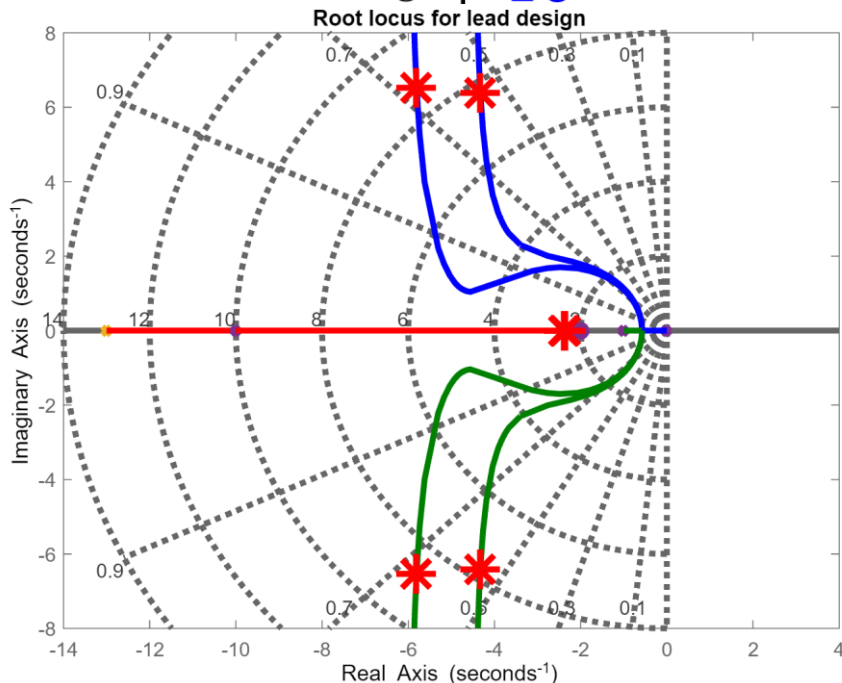
$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = 70 \frac{s+2}{s+10}$$

$$\Rightarrow \zeta \geq 0.5$$

$$\Rightarrow \omega_n \approx \frac{1.8}{0.3} \approx 6 \Rightarrow \omega_n \geq 7$$

$$D_c(s) = 91 \frac{s+2}{s+13}$$





## 2nd-Order Position Control System

$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = \frac{s+z}{s+p}, \quad z > p$$

$$D_{c2}(s) = \frac{s+0.05}{s+0.01}$$

$$D_c(0) = \frac{z}{p} = 3 \text{ to } 10$$

## Lead Compensation:

$$KD_{c1}(s) = 91 \frac{s+2}{s+13}$$

$$K_v = \lim_{s \rightarrow 0} s K D_{c1}(s) G(s)$$

$$= \lim_{s \rightarrow 0} s (91) \frac{s+2}{s+13} \frac{1}{s(s+1)}$$

$$= 14$$

IF  $K_v = 70 \text{ sec}^{-1}$

## Reduce the velocity error

by a factor of 5

$$\Rightarrow \frac{z}{p} = 5$$

$$\Rightarrow z = 0.05$$

$$\Rightarrow p = 0.01$$

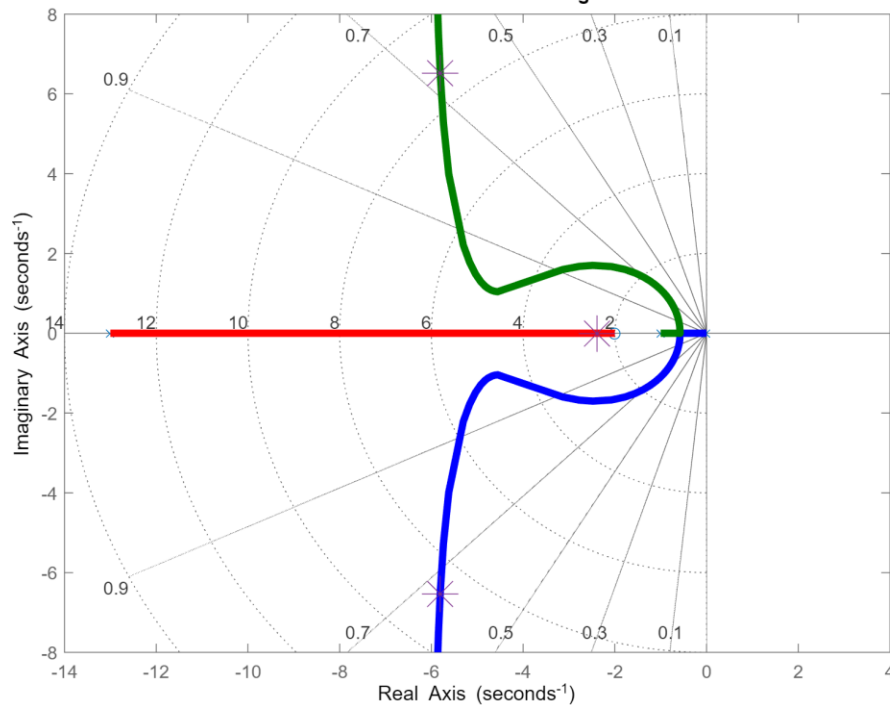
## 2nd-Order Position Control System

$$G(s) = \frac{1}{s(s+1)}$$

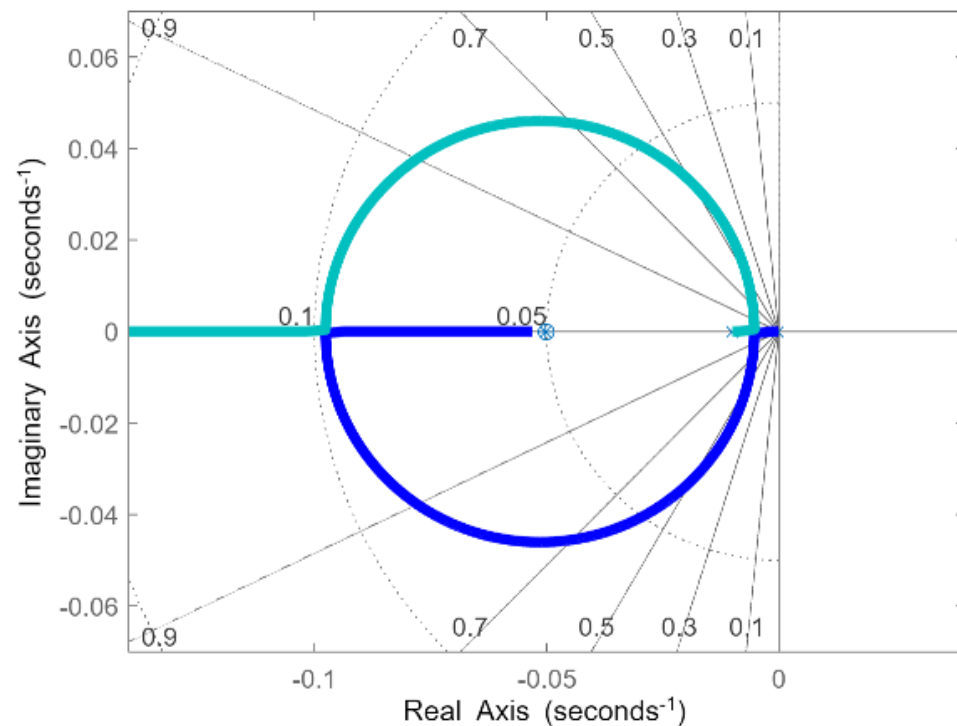
$$KD_{c1}(s) = 91 \frac{s+2}{s+13}$$

$$KD_{c2}(s) = 91 \frac{s+0.05}{s+0.01}$$

Root locus for lead design



Root locus for lead plus lag



- Suppose the design with lead and lag compensation

$$KD_c(s) = 91 \frac{s + 2}{s + 13} \frac{s + 0.05}{s + 0.01}$$

- Has a **substantial oscillation** at about 50 rad/sec

$$G(s) = \frac{2500}{s(s + 1)(s^2 + s + 2500)}$$

- Notch Compensation (Phase Stabilization)**

$$\begin{aligned} D_{notch}(s) &= \frac{s^2 + 2\zeta w_0 s + w_0^2}{(s + w_0)^2} \\ &= \frac{s^2 + 0.8s + 3600}{(s + 60)^2} \end{aligned}$$

- Suppose the design with lead and lag compensation

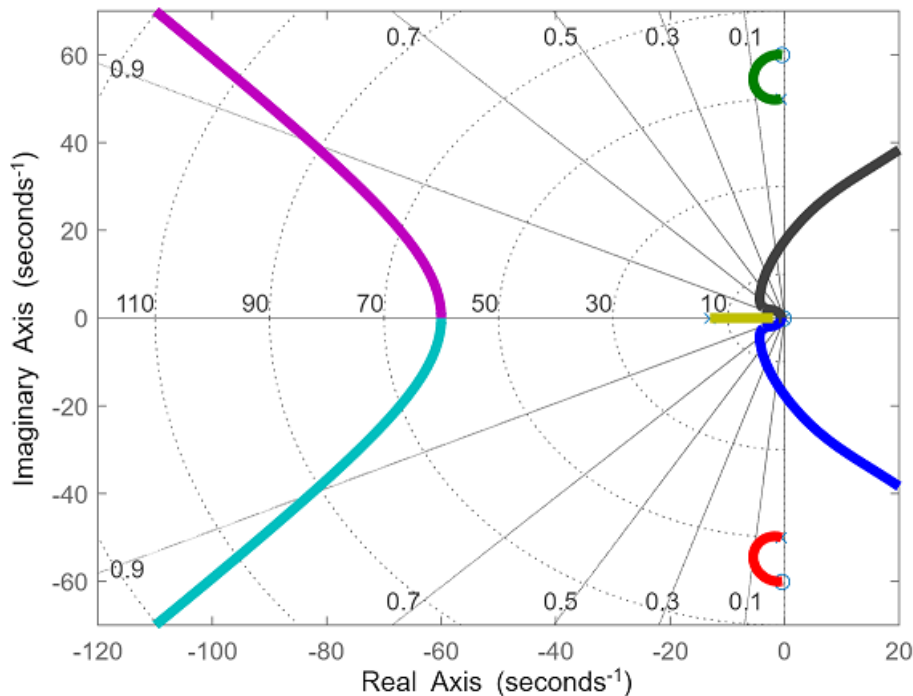
$$G(s) = \frac{2500}{s(s+1)(s^2+s+2500)}$$

$$D_{c1}(s) = \frac{s+2}{s+13}$$

$$K = 91 \quad D_{c2}(s) = \frac{s+0.05}{s+0.01}$$

$$D_{notch}(s) = \frac{s^2 + 0.8s + 3600}{(s+60)^2}$$

Root locus, lead, lag, notch



Step response for, lead, lag, notch

