Fall 2021 (110-1)

# 控制系統
# Control Systems

## Unit 3E
## Effects of Zeros and Additional Poles

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Sep 2021 – Jan 2022

- Same Poles, Different Zeros

$$H_1(s) = \frac{2}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{2}{s+2}$$

$$H_2(s) = \frac{2(s+1.1)}{1.1(s+1)(s+2)} = \frac{2}{1.1}\left(\frac{0.1}{s+1} + \frac{0.9}{s+2}\right) = \frac{0.18}{s+1} + \frac{1.64}{s+2}$$

- One zero at z = -1.1 cancels the effect of the pole at p = -1

- Same Poles, Different Zeros

$$H(s) = \frac{\frac{s}{\alpha\zeta w_n} + 1}{(\frac{s}{w_n})^2 + 2\zeta(\frac{s}{w_n}) + 1}$$

- Zero: $s = -\alpha\zeta w_n = -\alpha\sigma$
- If $\alpha >> 1$,

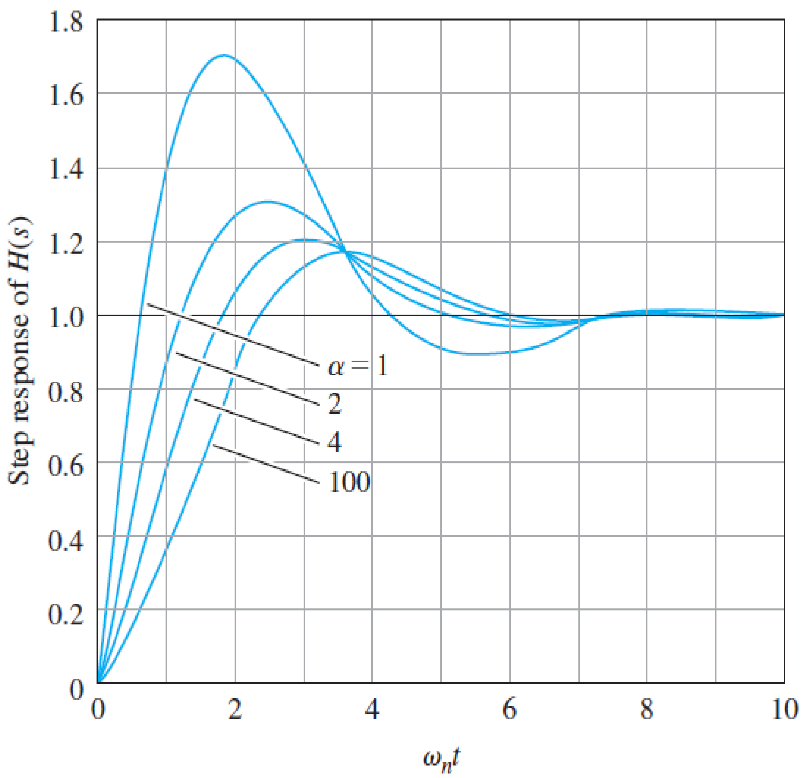
  the zero will be far removed from the poles and

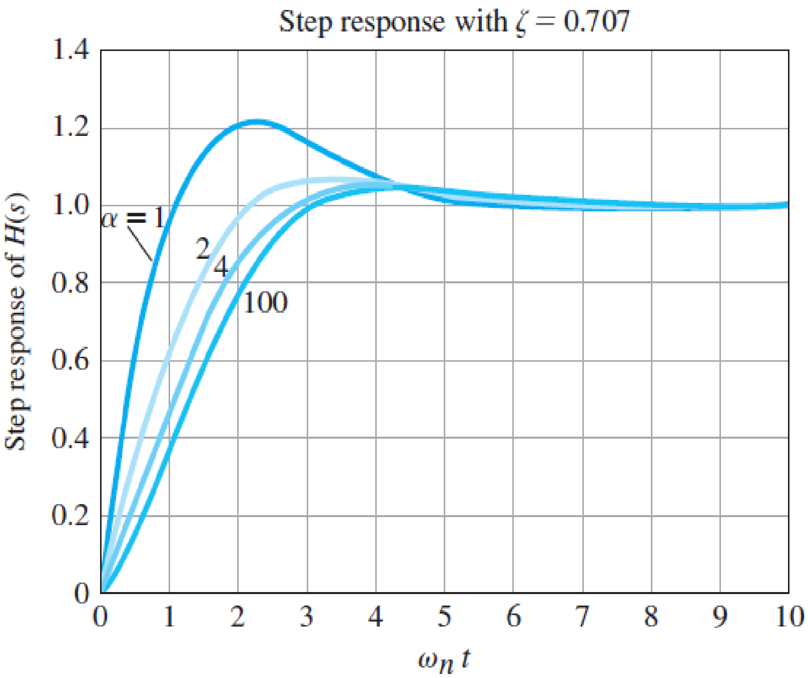
  the zero will have little effect on the response.
- If $\alpha == 1$,

  the zero will have a substantial influence on the response.

## Same Poles, Different Zeros

- Plots of the step response of a second-order system with a zero ($\zeta$ = 0.5)
- Plots of the step response of a second-order system with a zero ($\zeta$ = 0.707)

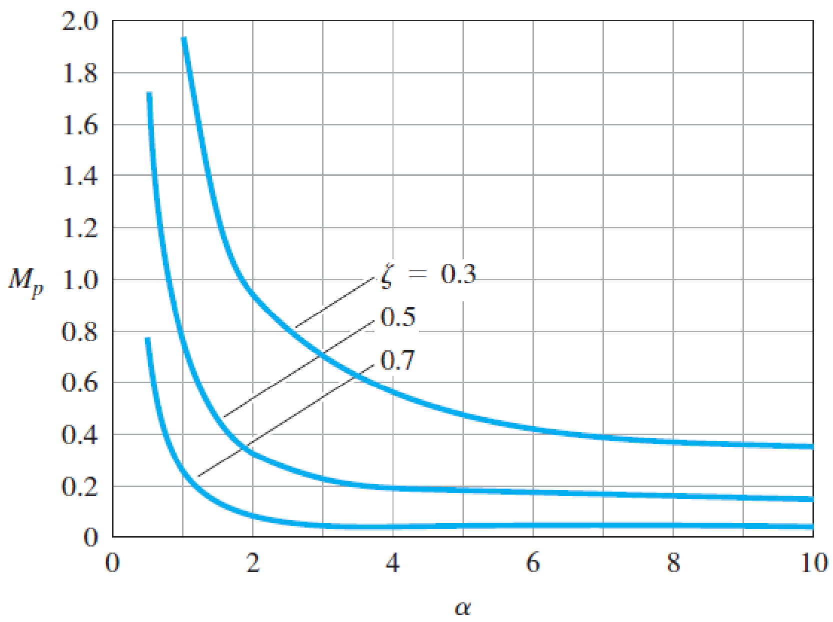



- Increase Overshoot $M_p$ and reduce Rise Time $t_r$
- Little influence on Settling Time $t_s$

## ▪ Same Poles, Different Zeros

- Plot of Overshoot $M_p$ as a function of normalized zero location α.
  At α = 1, the real part of the zero equals the real part of the poles



- Same Poles, Different Zeros

$$H(s) = \frac{\frac{s}{\alpha\zeta w_n} + 1}{(\frac{s}{w_n})^2 + 2\zeta(\frac{s}{w_n}) + 1}$$

- By normalizing frequency

$$\Rightarrow \; H(s) = \frac{\frac{s}{\alpha\zeta} + 1}{s^2 + 2\zeta s + 1}$$

$$\tau \triangleq w_n t$$

$$= \frac{1}{s^2 + 2\zeta s + 1} + \left(\frac{1}{\alpha\zeta}\right)\left(\frac{s}{s^2 + 2\zeta s + 1}\right)$$

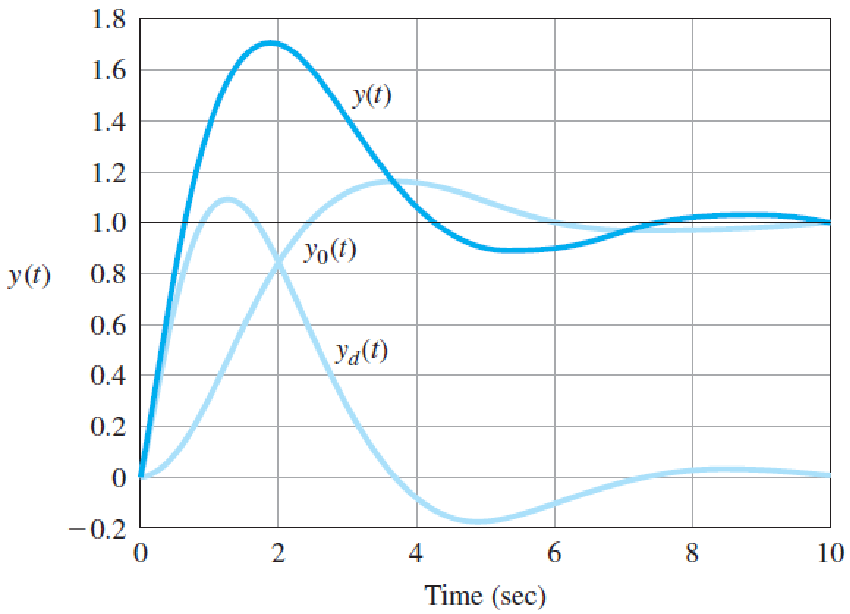
$$\triangleq H_0(s) + H_d(s)$$

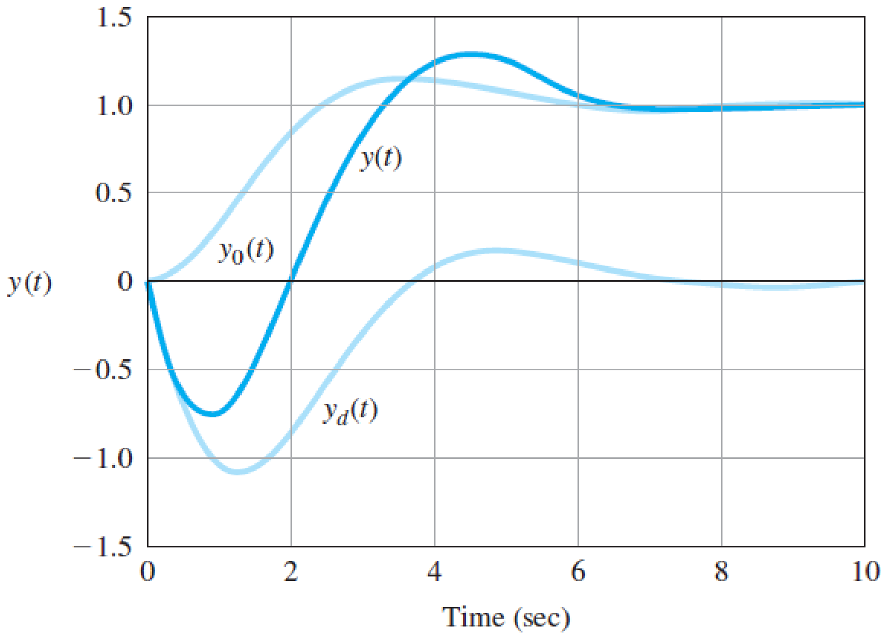
$$\Rightarrow \; y(t) = y_0(t) + y_d(t) \qquad = y_0(t) + \frac{1}{\alpha\zeta}\dot{y}_0(t)$$

## Same Poles, Different Zeros

- Second-order step responses $y(t)$ of the transfer functions $H(s)$, $H_0(s)$, and $H_d(s)$

- Step responses $y(t)$ of a second-order system with a zero in the RHP: a nonminimum-phase system




## Zero of $H_d(s)$ increase Overshoot $M_p$

- Example 3.28: Effect of Proximity of Zero to Pole Locations on the Transient Response (from Unit Step Input)

$$H(s) = \frac{24}{z} \frac{s+z}{(s+4)(s+6)} \qquad z = \{1,2,3,4,5,6\}$$

$$Y(s) = H(s)\frac{1}{s} = \frac{24}{z} \frac{s+z}{s(s+4)(s+6)}$$

$$= \frac{24}{z} \frac{s}{s(s+4)(s+6)} + \frac{24}{s(s+4)(s+6)}$$

$$y(t) = y_1(t) + y_2(t)$$

$$y_1(t) = \frac{12}{z} e^{-4t} - \frac{12}{z} e^{-6t}$$

$$y_2(t) = z \int_0^t y_1(\tau) d\tau = -3\, e^{-4t} + 2\, e^{-6t} + 1$$

$$y(t) = 1 + \left(\frac{12}{z} - 3\right) e^{-4t} + \left(2 - \frac{12}{z}\right) e^{-6t}$$

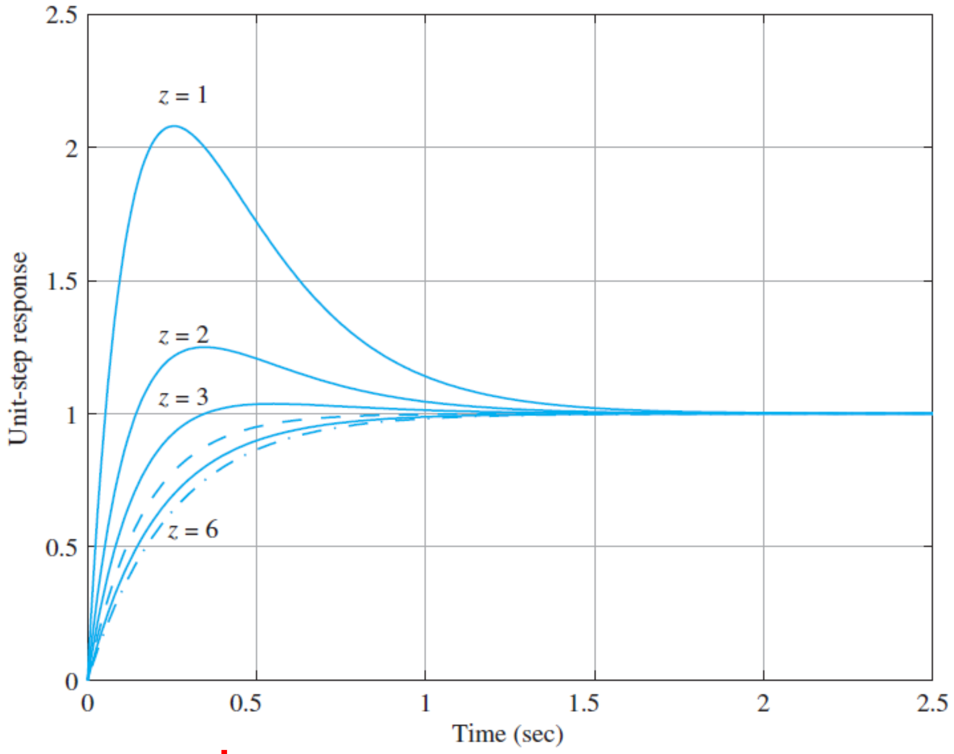
- Example 3.28: Effect of Proximity of Zero to Pole Locations on the Transient Response (from Unit Step Input)

- Effect of zero on transient response

```
z=1;
sys1 = 4*6*(1/z*s+1)/((s+4)*(s+6));
[y1] = step(sys1,t);
plot(t,y1,'LineWidth',2);
hold on;

…

z=6;
sys6 = 4*6*(1/z*s+1)/((s+4)*(s+6));
[y6] = step(sys6,t);
plot(t,y6,'-.','LineWidth',2);
```




- Influence of zero on response overshoot
- z = 4 or z = 6: absent due to zero-pole cancelations
- z = 5: no overshoot

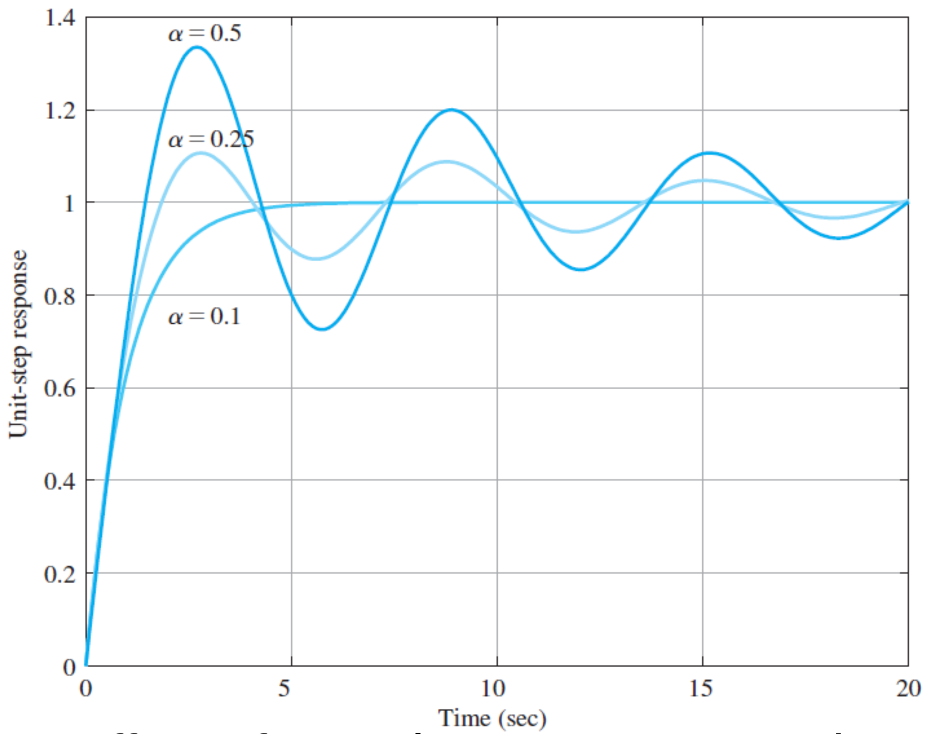
## Example 3.29: Effect of Proximity of Complex Zeros to Lightly Damped Poles (from Unit Step Input)

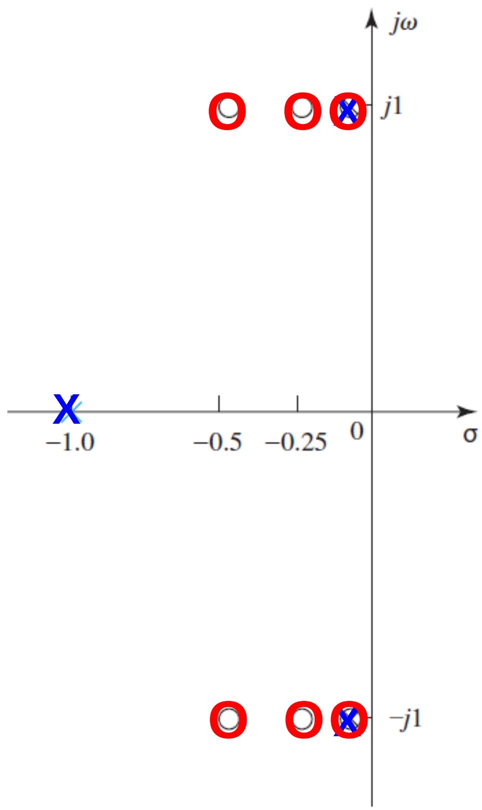
$$H(s) = \frac{(s+\alpha)^2+\beta^2}{(s+1)\left[(s+0.1)^2+1\right]}$$

$$s = -\alpha + j\beta$$

$$(\alpha, \beta) = (0.1, 1.0), (0.25, 1.0), (0.5, 1.0)$$

- Locations of complex zeros




- Effect of complex zeros on transient response

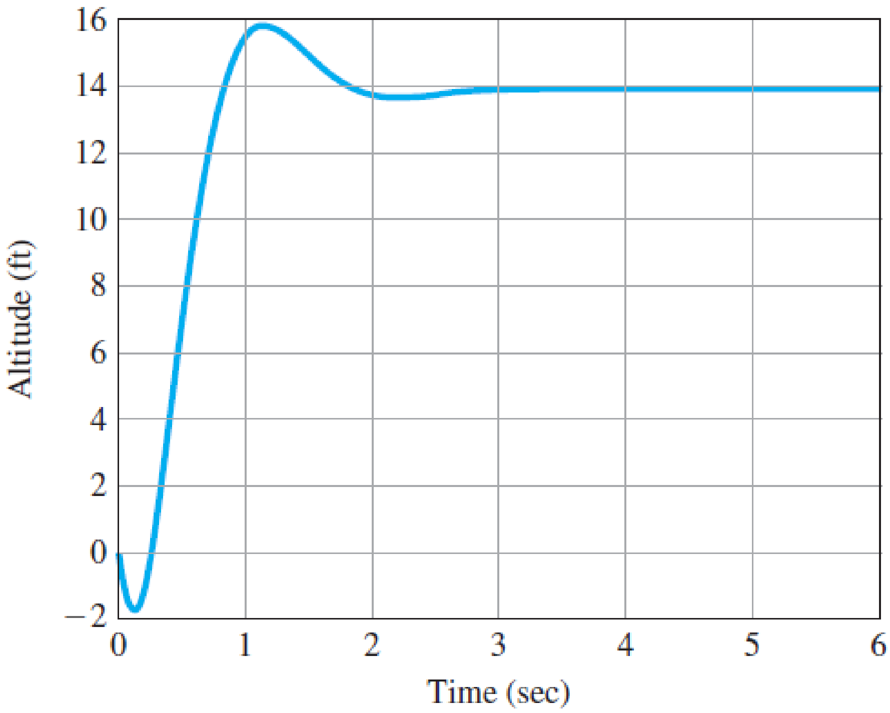
## Example 3.30: Aircraft Response Using Matlab

$$\frac{h(s)}{\delta_e(s)} = \frac{30\,(s-6)}{s(s^2+4s+13)}$$

```
s = tf( 's' )

u = -1;
sysG = u*30*(s-6)/( s^3 + 4*s^2 + 13*s );
t = 0:0.1:6;
y = impulse( sysG, t );

plot( t, y )
grid
hold on;
```



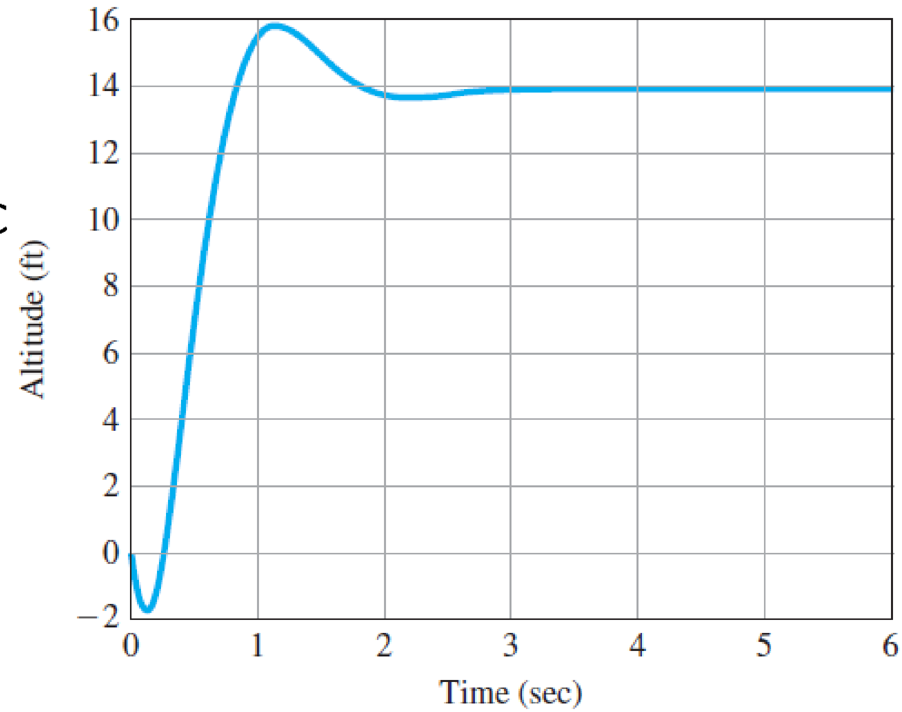

## Final Value:

$$s\,\frac{30\,(s-6)(-1)}{s(s^2+4s+13)}\bigg|_{s=0} = \frac{30\,(-6)(-1)}{13} = 13.8$$

# Example 3.30: Aircraft Response Using Matlab

## Rise Time $t_r$

$$t_r \cong \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{13}} = 0.5 \text{ sec}$$

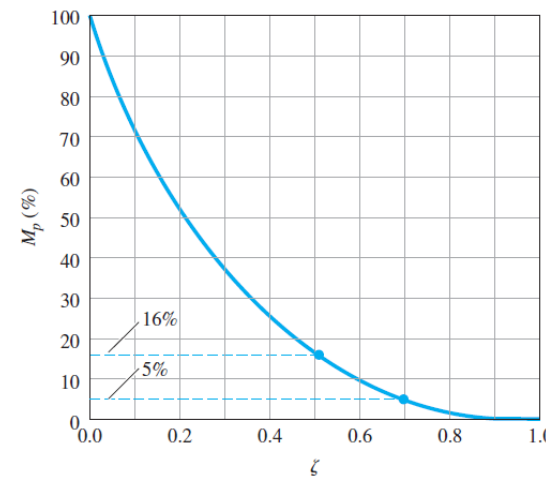



$$2\zeta\omega_n = 4$$

$$\zeta = \frac{2}{\sqrt{13}} = 0.55$$

$$\Rightarrow M_p = 14\% = 0.14$$

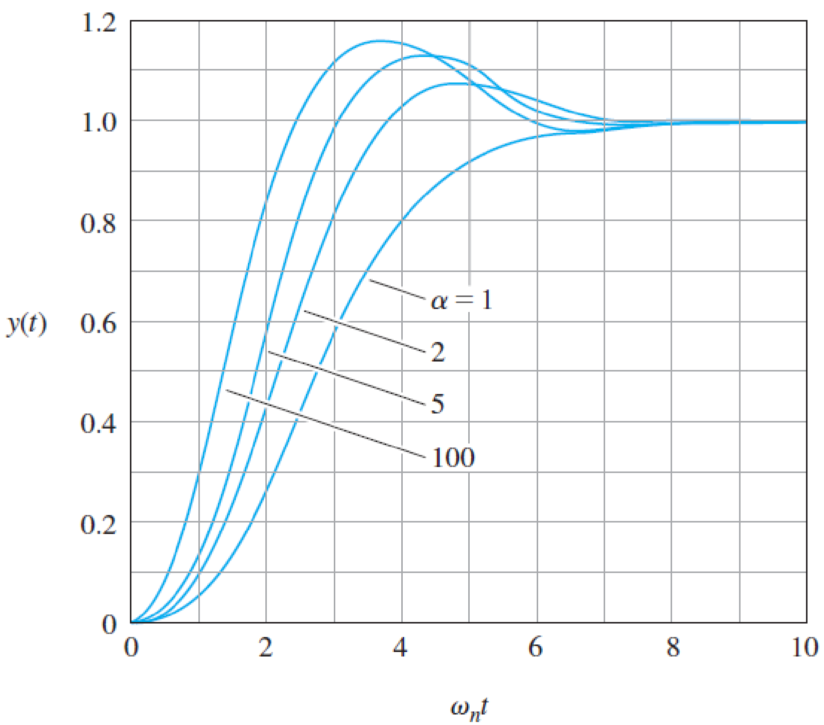
$$\Rightarrow t_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma} = \frac{4.6}{2} = 2.3 \text{ sec}$$

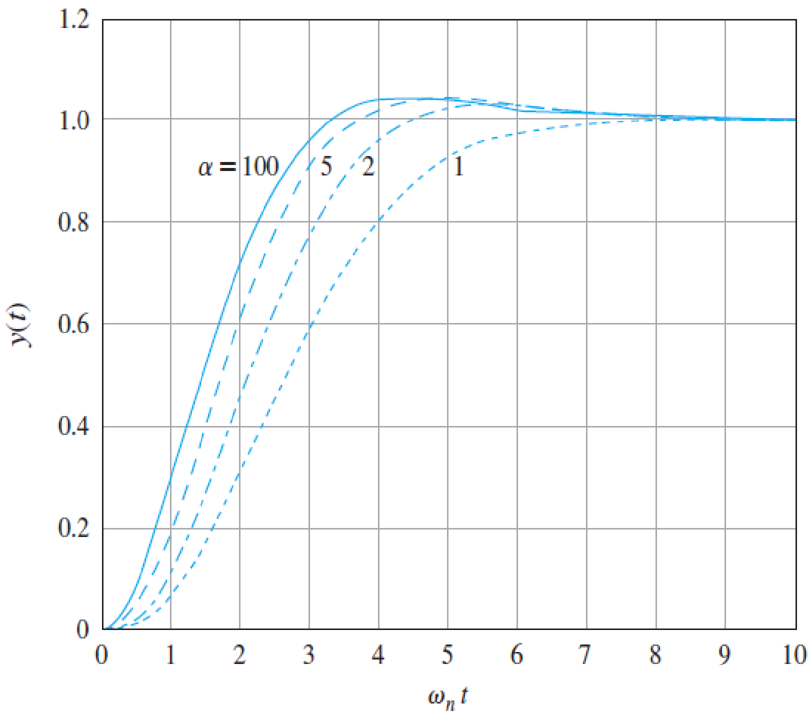


## Effects of Pole-Zero Patterns on Dynamic Response

$$H(s) = \frac{1}{(\frac{s}{\alpha\zeta w_n}+1)[(\frac{s}{w_n})^2+2\zeta(\frac{s}{w_n})+1]}$$

- Step responses for several third-order systems with $\zeta = 0.5$
- Step responses for several third-order systems with $\zeta = 0.707$



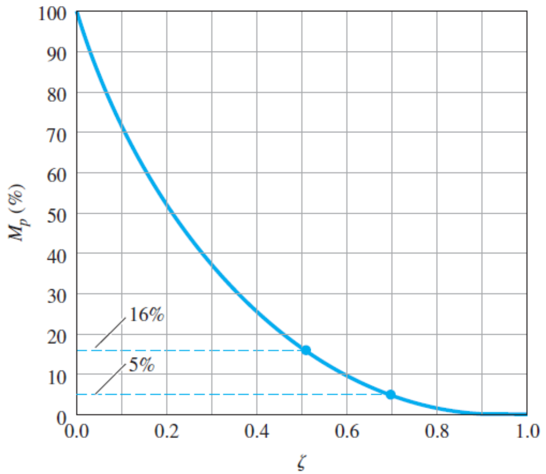



- Effects of Pole-Zero Patterns on Dynamic Response

- Rise time $t_r$ $\Rightarrow t_r \cong \dfrac{1.8}{w_n}$

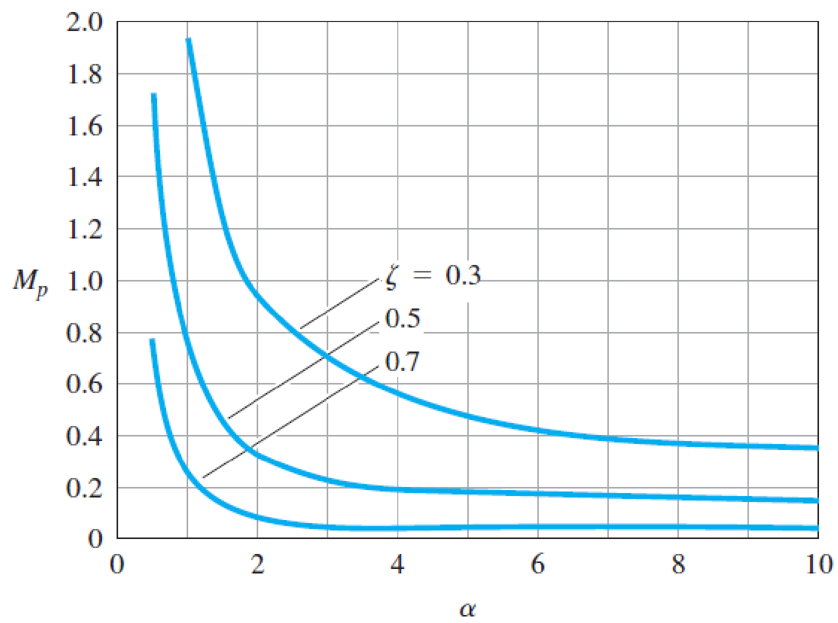
- Overshoot $M_p$ $\Rightarrow M_p = \begin{cases} 5\%, & \zeta = 0.7 \\ 16\%, & \zeta = 0.5 \\ 35\%, & \zeta = 0.3 \end{cases}$

- Settling time $t_s$ $\Rightarrow t_s = \dfrac{4.6}{\zeta w_n} = \dfrac{4.6}{\sigma}$



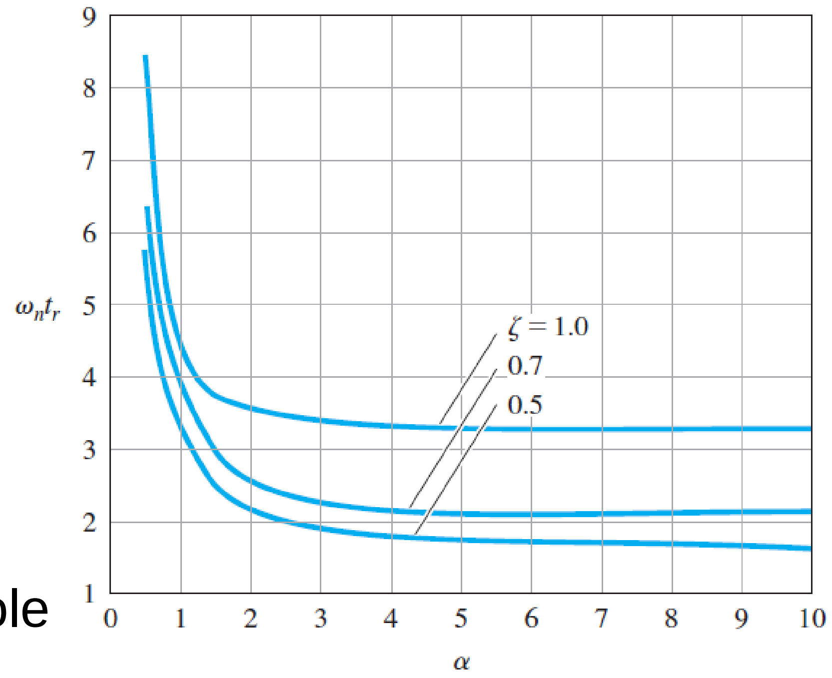
- Effects of Pole-Zero Patterns on Dynamic Response

- A zero in LHP will increase the overshoot if the zero is within a factor of 4 of the real part of the complex poles.
- A zero in RHP will depress the overshoot.



- Effects of Pole-Zero Patterns on Dynamic Response

- An additional pole in the LHP will increase the rise time significantly if the extra pole is within a factor of 4 of the real part of the complex poles.




- Normalized rise time for several locations of an additional pole