

Fall 2021 (110-1)

控制系統  
Control Systems

Unit 3A  
Laplace Transforms

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- The two-sided (or bilateral) Laplace Transform

$$F(s) \triangleq \int_{-\infty}^{\infty} f(t)e^{-st} dt$$

- The one-sided (or unilateral) Laplace Transform

$$F(s) \triangleq \int_{0^-}^{\infty} f(t)e^{-st} dt$$

- The Fourier Transform  $s = \sigma + j\omega$

$$F(j\omega) \triangleq \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

$$\mathcal{L}\{f(t)\} = \mathcal{F}\{f(t)e^{-\sigma t}\}$$

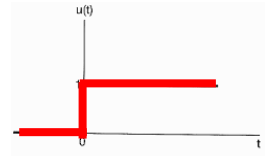
$$F(s) \Big|_{s=j\omega} = \mathcal{L}\{f(t)\} \Big|_{s=j\omega} = \mathcal{F}\{f(t)\} = F(j\omega)$$

- The Inverse Laplace Transform

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)e^{st} ds$$

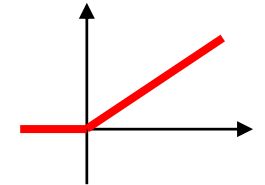
## LT of Step Functions

$$\mathcal{L}\{a \mathbf{1}(t)\} = \int_0^{\infty} a \mathbf{1}(t) e^{-st} dt = \frac{a}{s}, \quad \mathcal{R}e\{s\} > 0$$



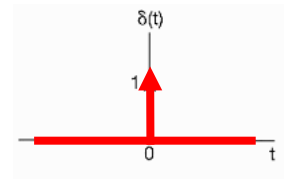
## LT of Ramp Functions

$$\mathcal{L}\{b t \mathbf{1}(t)\} = \int_0^{\infty} b t \mathbf{1}(t) e^{-st} dt = \frac{b}{s^2}, \quad \mathcal{R}e\{s\} > 0$$



## LT of Impulse Functions

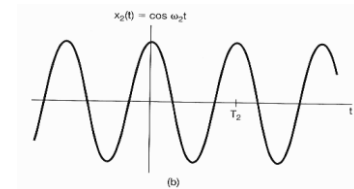
$$\mathcal{L}\{\delta(t)\} = \int_0^{\infty} \delta(t) e^{-st} dt = 1$$



## LT of Sinusoid Functions

$$\mathcal{L}\{\sin(\omega t)\} = \int_0^{\infty} \sin(\omega t) e^{-st} dt = \frac{\omega}{s^2 + \omega^2}, \quad \mathcal{R}e\{s\} > 0$$

$$\mathcal{L}\{\cos(\omega t)\} = \int_0^{\infty} \cos(\omega t) e^{-st} dt = \frac{s}{s^2 + \omega^2}, \quad \mathcal{R}e\{s\} > 0$$



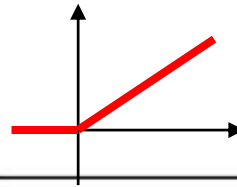
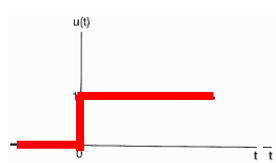
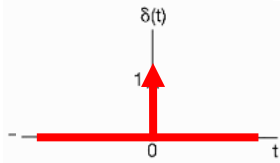
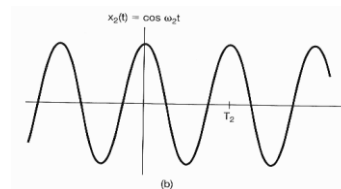
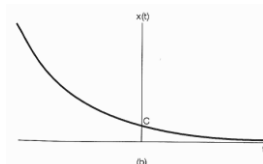
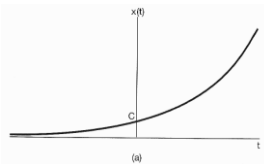


Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	$t$
4	$2!/s^3$	$t^2$
5	$3!/s^4$	$t^3$
6	$m!/s^{m+1}$	$t^m$
7	$\frac{1}{s+a}$	$e^{-at}$
8	$\frac{1}{(s+a)^2}$	$te^{-at}$
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1 - e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at - 1 + e^{-at})$

Continued

Number	$F(s)$	$f(t), t \geq 0$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt} - ae^{-at}$
17	$\frac{a}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1 - e^{-at} \left( \cos bt + \frac{a}{b} \sin bt \right)$



**TABLE 9.2** LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All $s$
2	$u(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} < -\alpha$
10	$\delta(t - T)$	$e^{-sT}$	All $s$
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
13	$[e^{-\alpha t} \cos \omega_0 t]u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
14	$[e^{-\alpha t} \sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n \delta(t)}{dt^n}$	$s^n$	All $s$
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re\{s\} > 0$

# Some Laplace Transform Pairs

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. $t$	$\frac{1}{s^2}$
3. $t^n$	$\frac{n!}{s^{n+1}}$ , $n$ a positive integer
4. $t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$
5. $t^{1/2}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
6. $t^\alpha$	$\frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}$ , $\alpha > -1$
7. $\sin kt$	$\frac{k}{s^2 + k^2}$
8. $\cos kt$	$\frac{s}{s^2 + k^2}$
9. $\sin^2 kt$	$\frac{2k^2}{s(s^2 + 4k^2)}$
10. $\cos^2 kt$	$\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
11. $e^{at}$	$\frac{1}{s - a}$
12. $\sinh kt$	$\frac{k}{s^2 - k^2}$
13. $\cosh kt$	$\frac{s}{s^2 - k^2}$
14. $\sinh^2 kt$	$\frac{2k^2}{s(s^2 - 4k^2)}$
15. $\cosh^2 kt$	$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
16. $te^{at}$	$\frac{1}{(s - a)^2}$
17. $t^n e^{at}$	$\frac{n!}{(s - a)^{n+1}}$ , $n$ a positive integer

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
18. $e^{at} \sin kt$	$\frac{k}{(s - a)^2 + k^2}$
19. $e^{at} \cos kt$	$\frac{s - a}{(s - a)^2 + k^2}$
20. $e^{at} \sinh kt$	$\frac{k}{(s - a)^2 - k^2}$
21. $e^{at} \cosh kt$	$\frac{s - a}{(s - a)^2 - k^2}$
22. $t \sin kt$	$\frac{2ks}{(s^2 + k^2)^2}$
23. $t \cos kt$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$
24. $\sin kt + kt \cos kt$	$\frac{2ks^2}{(s^2 + k^2)^2}$
25. $\sin kt - kt \cos kt$	$\frac{2k^3}{(s^2 + k^2)^2}$
26. $t \sinh kt$	$\frac{2ks}{(s^2 - k^2)^2}$
27. $t \cosh kt$	$\frac{s^2 + k^2}{(s^2 - k^2)^2}$
28. $\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s - a)(s - b)}$
29. $\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s - a)(s - b)}$
30. $1 - \cos kt$	$\frac{k^2}{s(s^2 + k^2)}$
31. $kt - \sin kt$	$\frac{k^3}{s^2(s^2 + k^2)}$
32. $\frac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$
33. $\frac{\cos bt - \cos at}{a^2 - b^2}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
34. $\sin kt \sinh kt$	$\frac{2ks}{s^4 + 4k^4}$
35. $\sin kt \cosh kt$	$\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
36. $\cos kt \sinh kt$	$\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
37. $\cos kt \cosh kt$	$\frac{s^3}{s^4 + 4k^4}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
38. $J_0(kt)$	$\frac{1}{\sqrt{s^2 + k^2}}$
39. $\frac{e^{bt} - e^{at}}{t}$	$\ln \frac{s - a}{s - b}$
40. $\frac{2(1 - \cos kt)}{t}$	$\ln \frac{s^2 + k^2}{s^2}$
41. $\frac{2(1 - \cosh kt)}{t}$	$\ln \frac{s^2 - k^2}{s^2}$
42. $\frac{\sin at}{t}$	$\arctan\left(\frac{a}{s}\right)$
43. $\frac{\sin at \cos bt}{t}$	$\frac{1}{2} \arctan \frac{a + b}{s} + \frac{1}{2} \arctan \frac{a - b}{s}$
44. $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$
45. $\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
46. $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$
47. $2\sqrt{\frac{t}{\pi}} e^{-a^2/4t} - a \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s\sqrt{s}}$
48. $e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$
49. $-e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \frac{a}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{be^{-a\sqrt{s}}}{s(\sqrt{s} + b)}$
50. $e^{at} f(t)$	$F(s - a)$
51. $\mathcal{U}(t - a)$	$\frac{e^{-as}}{s}$
52. $f(t - a)\mathcal{U}(t - a)$	$e^{-as}F(s)$
53. $g(t)\mathcal{U}(t - a)$	$e^{-as}\mathcal{L}\{g(t + a)\}$
54. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
55. $t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
56. $\int_0^t f(\tau)g(t - \tau) d\tau$	$F(s)G(s)$
57. $\delta(t)$	1
58. $\delta(t - t_0)$	$e^{-st_0}$

# Properties of Laplace Transforms

- Superposition
- Time Delay
- Time Scaling
- Shift in Frequency
- Differentiation
- Integration
- Convolution
- Time Product
- Multiplication by Time
  
- Initial-Value Theorem
- Final-Value Theorem

## Properties of Laplace Transforms

Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t - \lambda)$	Time delay ( $\lambda \geq 0$ )
3	$\frac{1}{ a } F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s + a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \dots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s} F(s)$	$\int_0^t f(\zeta) d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
8	$\lim_{s \rightarrow \infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s \rightarrow 0} sF(s)$	$\lim_{t \rightarrow \infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F_1(\zeta)F_2(s - \zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi} \int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega) d\omega$	$\int_0^\infty y(t)u(t) dt$	Parseval's Theorem
12	$-\frac{d}{ds} F(s)$	$tf(t)$	Multiplication by time

- Superposition
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**TABLE 9.1** PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	$R$
		$x_1(t)$	$X_1(s)$	$R_1$
		$x_2(t)$	$X_2(s)$	$R_2$
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0} X(s)$	$R$
9.5.3	Shifting in the $s$ -Domain	$e^{st_0} x(t)$	$X(s - s_0)$	Shifted version of $R$ (i.e., $s$ is in the ROC if $s - s_0$ is in $R$ )
9.5.4	Time scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., $s$ is in the ROC if $s/a$ is in $R$ )
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	$R$
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt} x(t)$	$sX(s)$	At least $R$
9.5.8	Differentiation in the $s$ -Domain	$-tx(t)$	$\frac{d}{ds} X(s)$	$R$
9.5.9	Integration in the Time Domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{s} X(s)$	At least $R \cap \{\Re\{s\} > 0\}$

Initial- and Final-Value Theorems	
9.5.10	<p>If <math>x(t) = 0</math> for <math>t &lt; 0</math> and <math>x(t)</math> contains no impulses or higher-order singularities at <math>t = 0</math>, then</p> $x(0^+) = \lim_{s \rightarrow \infty} sX(s)$ <p>If <math>x(t) = 0</math> for <math>t &lt; 0</math> and <math>x(t)</math> has a finite limit as <math>t \rightarrow \infty</math>, then</p> $\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$



## ■ Inverse Laplace Transforms by Partial-Fraction Expansion

$$F(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

$$= K \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)} \quad \begin{array}{l} s = z_i, \text{ zero} \\ s = p_i, \text{ pole} \end{array}$$

$$= K \frac{\prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}$$

$$= \frac{C_1}{s - p_1} + \frac{C_2}{s - p_2} + \cdots + \frac{C_n}{s - p_n}$$

$$f(t) = C_1 e^{p_1 t} \mathbf{1}(t) + C_2 e^{p_2 t} \mathbf{1}(t) + \cdots + C_n e^{p_n t} \mathbf{1}(t)$$

- Example 3.11: Partial-Fraction Expansion: Distinct Real Roots

$$Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

$$= \frac{\frac{8}{3}}{s} + \frac{-\frac{3}{2}}{s+1} + \frac{-\frac{1}{6}}{s+3}$$

$$y(t) = \frac{8}{3} \mathbf{1}(t) - \frac{3}{2} e^{-t} \mathbf{1}(t) - \frac{1}{6} e^{-3t} \mathbf{1}(t)$$

- Example 3.12: The Final Value Theorem

$$Y(s) = \frac{3(s+2)}{s(s^2+2s+10)}$$

$$y(\infty) = s Y(s) \Big|_{s=0} = \frac{3 \cdot 2}{10} = 0.6$$

- Example 3.15: Homogeneous Differential Equation

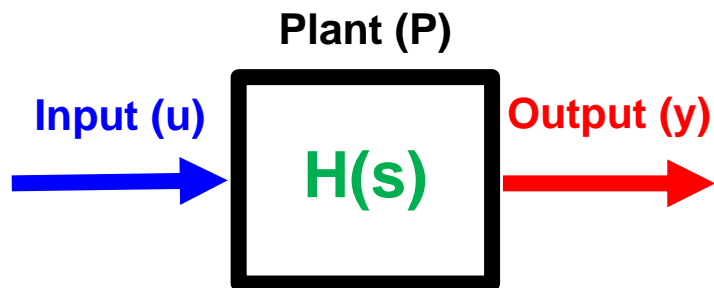
$$\ddot{y}(t) + y(t) = 0 \quad \text{where } y(0) = a, \dot{y}(0) = b$$

$$s^2 Y(s) - a s - b + Y(s) = 0$$

$$(s^2 + 1) Y(s) = a s + b$$

$$Y(s) = \frac{a s}{s^2 + 1} + \frac{b}{s^2 + 1}$$

$$y(t) = [ a (\cos t) + b (\sin t) ] 1(t)$$



▪ Example 3.16: Forced Differential Equation

$$\ddot{y}(t) + 5 \dot{y}(t) + 4 y(t) = 3 \quad \text{where } y(0) = a, \dot{y}(0) = b$$

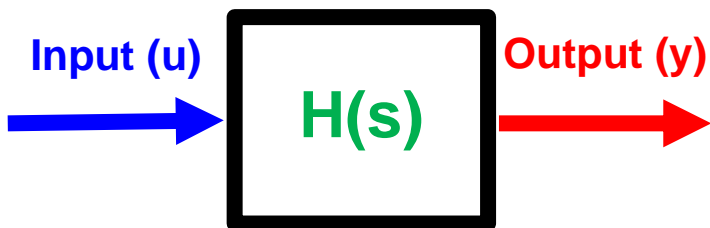
$$[s^2 Y(s) - a s - b] + 5 [s Y(s) - a] + 4 Y(s) = \frac{3}{s}$$

$$Y(s) = \frac{s (s a + b + 5 a) + 3}{s (s + 1) (s + 4)}$$

$$= \frac{3}{s} - \frac{3-b-4a}{s+1} + \frac{3-4a-4b}{s+4}$$

$$y(t) = \left( \frac{3}{4} + \frac{-3+b+4a}{3} e^{-t} + \frac{3-4a-4b}{12} e^{-4t} \right) \mathbf{1}(t)$$

Plant (P)



**Example 3.17: Forced Equation Solution with zero I.C.**

$$\ddot{y}(t) + 5 \dot{y}(t) + 4 y(t) = u(t) \quad \text{where } y(0) = 0, \dot{y}(0) = 0$$

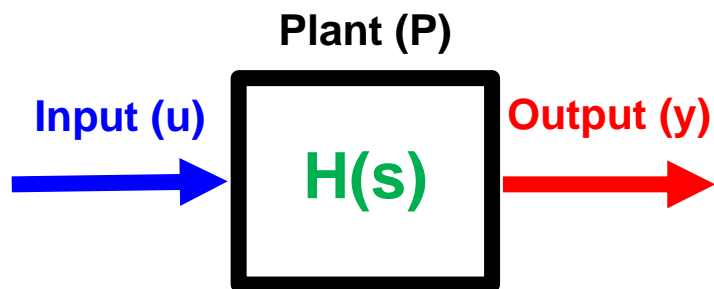
$$u(t) = 2 e^{-2t} \mathbf{1}(t)$$

$$s^2 Y(s) + 5 s Y(s) + 4 Y(s) = \frac{2}{s + 2}$$

$$Y(s) = \frac{2}{(s + 2)(s + 1)(s + 4)}$$

$$= -\frac{1}{s + 2} + \frac{\frac{2}{3}}{s + 1} + \frac{\frac{1}{3}}{s + 4}$$

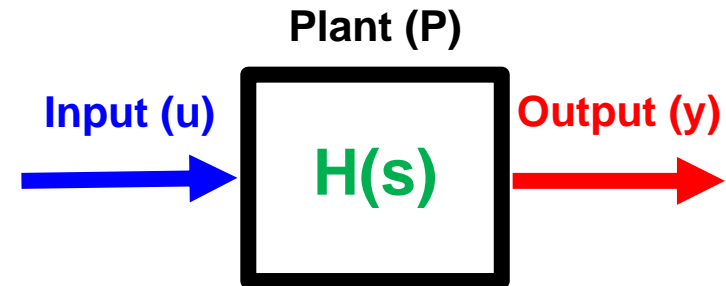
$$y(t) = \left( -1 e^{-2t} + \frac{2}{3} e^{-t} + \frac{1}{3} e^{-4t} \right) \mathbf{1}(t)$$



## Rational Transfer Function

$$H(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{s^n + a_1 s^{n-1} + \dots + a_n} = \frac{N(s)}{D(s)}$$

$$= K \frac{\prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}$$



## Zeros

$$s = z_i, \text{ zero}$$

$$\left. H(s) \right|_{s=z_i} = 0$$

## Poles

$$s = p_i, \text{ pole}$$

$$\left. H(s) \right|_{s=p_i} = \infty$$

The **Poles** are the **Modes** of the System

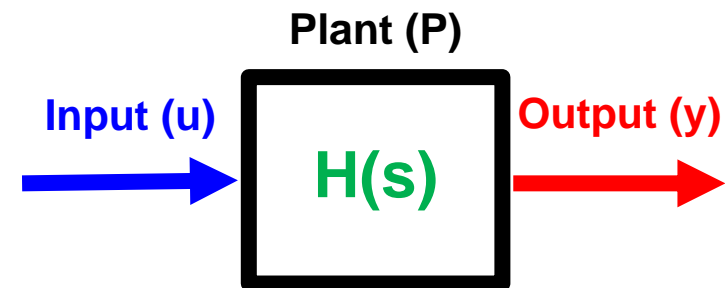
The **Poles** of the system determine the **Stability** properties and the **natural** or **unforced** behavior.

- Example 3.18: Cruise Control Transfer Function

$$H(s) = \frac{0s^2 + 0s + 0.001}{s^2 + 0.05s + 0} = \frac{0.001}{s(s + 0.05)}$$

```
num = [ 0 0 0.001 ]  
den = [ 1 0.05 0 ]  
[ z, p, k ] = tf2zp( num, den )
```

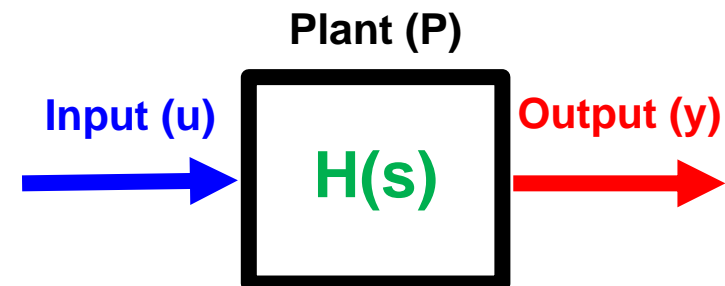
```
%z = [ ]  
%p = [ 0 -0.05 ]'  
%K = 0.001
```



## Example 3.19: DC Motor Transfer Function

$$H(s) = \frac{100}{s^3 + 10.1s^2 + 101s} = \frac{100}{s(s^2 + 10.1s + 101)}$$

```
numb = [ 0 0 100 ];  
denb = [ 1 10.1 101 0 ];  
[ z, p, k ] = tf2zp( numb, denb )  
  
%z = [ ]  
%p = [ 0 -5.0500+8.6889j -5.0500-8.6889j ]'  
%K = 100
```

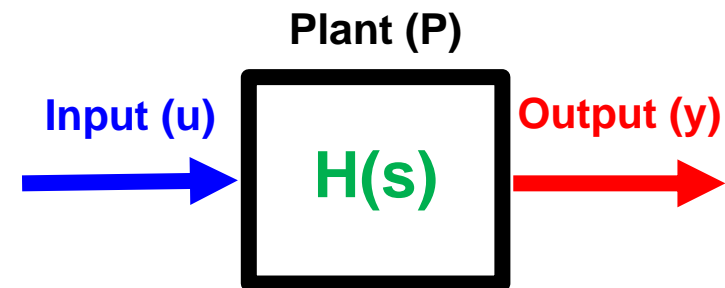




## Example 3.19: DC Motor Transfer Function

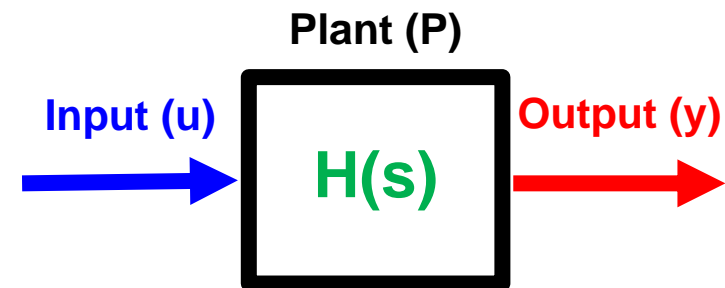
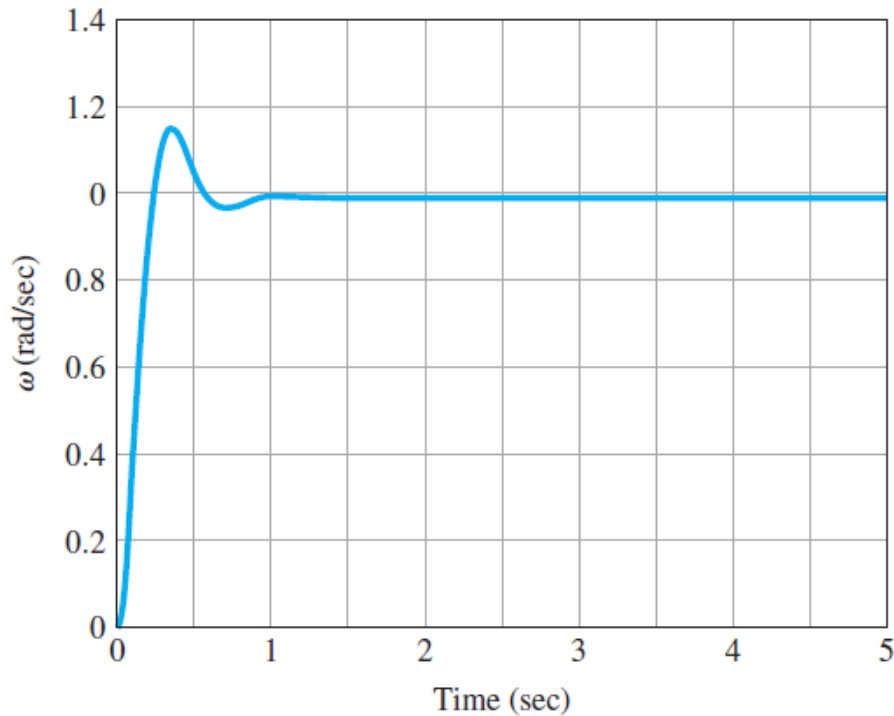
$$G(s) = \frac{100 s}{s^3 + 10.1 s^2 + 101 s} = \frac{100}{(s^2 + 10.1 s + 101)}$$

```
numb = [ 0 0 100 0 ];  
denb = [ 1 10.1 101 0 ];  
[ z, p, k ] = tf2zp( numb, denb )  
  
%z = [ ]  
%p = [ -5.0500+8.6889j -5.0500-8.6889j ]'  
%K = 100  
  
s = tf( 's' );  
sysb = 100*s/(s^3 + 10.1*s^2 + 101*s );  
t = 0:0.01:5;  
  
y = step( sysb, t );  
plot( t, y )
```



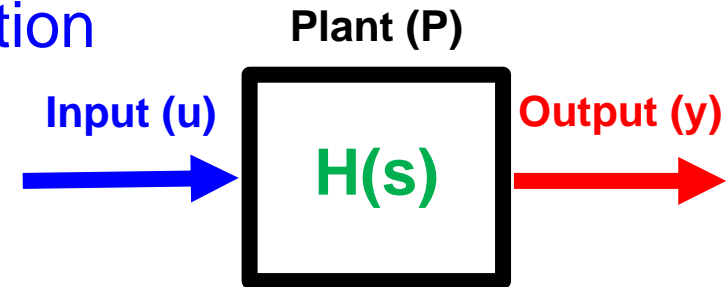
## Example 3.19: DC Motor Transfer Function

$$G(s) = \frac{100 s}{s^3 + 10.1 s^2 + 101 s} = \frac{100}{(s^2 + 10.1 s + 101)}$$



## Example 3.21: Satellite Transfer Function

$$H(s) = \frac{0,0002}{s^2}$$



```
numG = [ 0 0 0.0002 ];  
denG = [ 1 0 0 ];
```

```
s = tf( 's' )  
sysG = 0.0002/(s^2)
```

```
t = 0:0.01:10;
```

```
% u1
```

```
u1 = [ zeros( 1, 500 ) 25*ones( 1, 10 ) zeros( 1, 491 ) ];
```

```
[ y1 ] = lsim( sysG, u1, t );
```

```
ff = 180/pi;
```

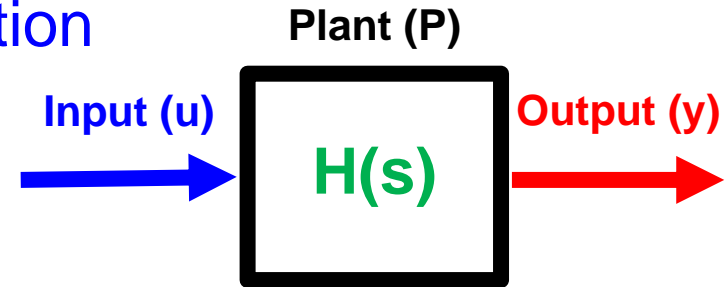
```
y1 = ff*y1;
```

```
plot( t, u1 );
```

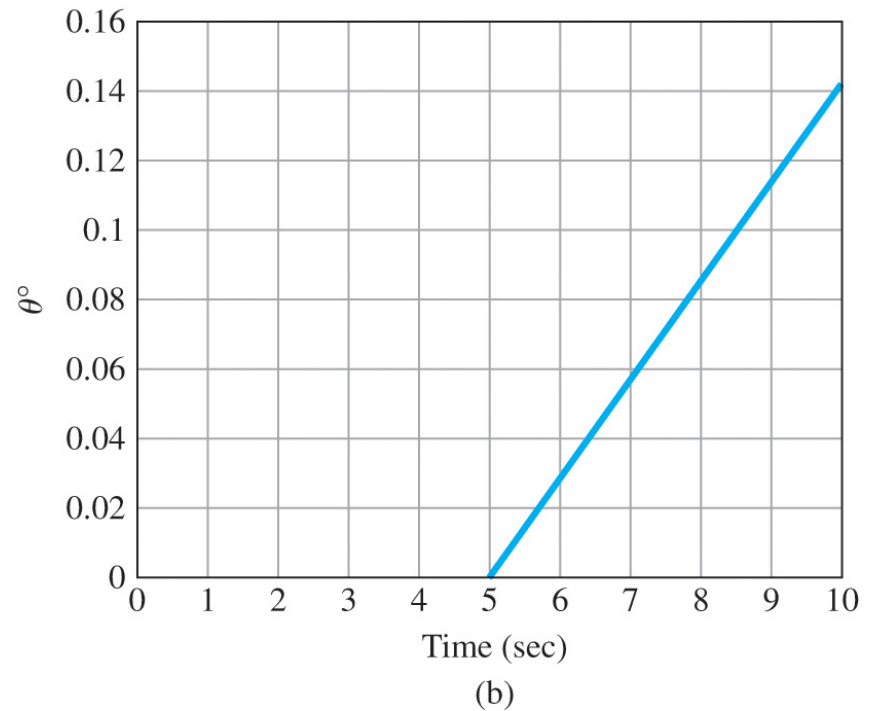
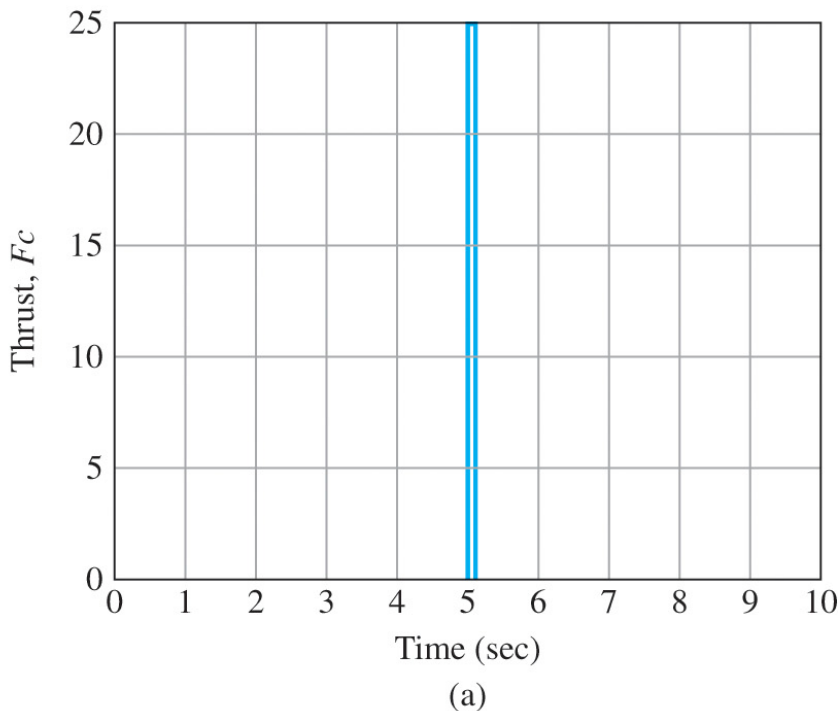
```
plot( t, y1 );
```

## Example 3.21: Satellite Transfer Function

$$H(s) = \frac{0,0002}{s^2}$$

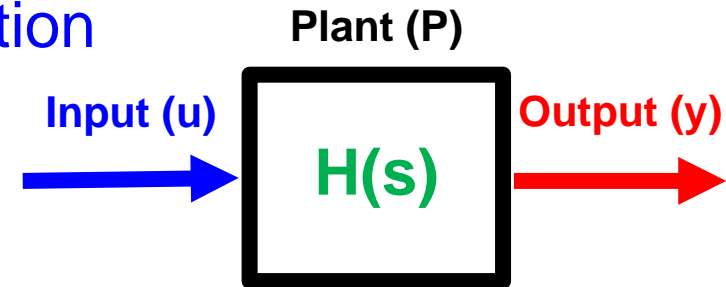


- Transient response for satellite:  
(a) thrust input (b) satellite attitude



- Example 3.21: Satellite Transfer Function

$$H(s) = \frac{0,0002}{s^2}$$



```
numG = [ 0 0 0.0002 ];  
denG = [ 1 0 0 ];
```

```
s = tf( 's' )  
sysG = 0.0002/(s^2)
```

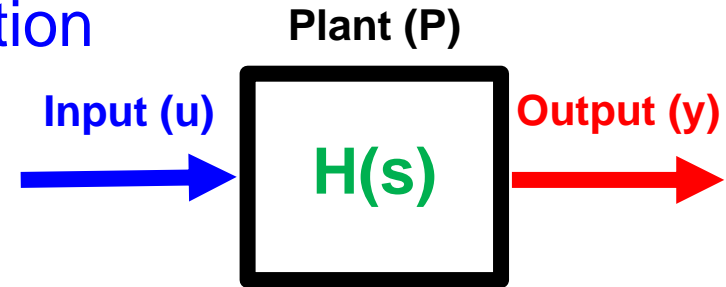
```
t = 0:0.01:10;
```

```
% u2  
u2 = [ zeros(1,500) 25*ones(1,10) zeros(1,100) (-25)*ones(1,10) zeros(1, 381) ];  
[ y2 ] = lsim( sysG, u2, t );  
ff = 180/pi;  
y2 = ff*y2;
```

```
plot( t, u2 );  
plot( t, y2 );
```

## Example 3.21: Satellite Transfer Function

$$H(s) = \frac{0,0002}{s^2}$$



- Transient response for satellite (double pulse):  
(a) thrust input (b) satellite attitude

