Fall 2021 (110-1)

控制系統
Control Systems

Unit 30
Dynamic Response

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NTU-EE

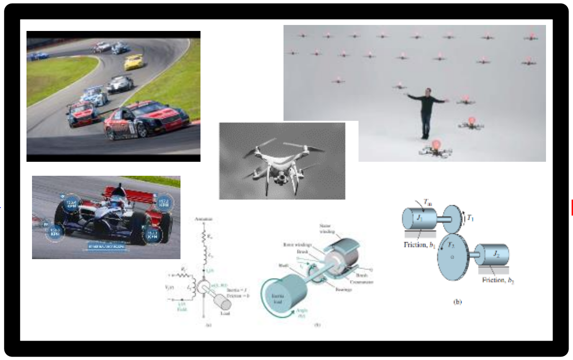
Sep 2021 – Jan 2022

- U3A Review of Laplace Transforms
  - Laplace Transforms: From Differential Eqns to Algebraic Eqns
  - Transfer Functions, Frequency Response, Poles and Zeros
  - Step, Ramp, Impulse Functions
  - Laplace Transforms and Inverse LT, The Final Value Theorem
  - Using Laplace Transforms to Solve Differential Equations
- U3B System Modeling Diagrams: Model Visualization
- U3C Effect of Pole Locations
- U3D Time-Domain Specifications
  - Rise Time
  - Overshoot and Peak Time
  - Settling Time
- U3E Effects of Zeros and Additional Poles
- U3F Stability
  - Bounded Input–Bounded Output Stability
  - Stability of LTI Systems
  - Routh's Stability Criterion

Plant (P)

Signals & Systems

Ref (r) → ○ → **Input (u)** → Plant → **Output (y)** →



$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} - 3\,y(t) = 5\,u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2\,s - 3}$$

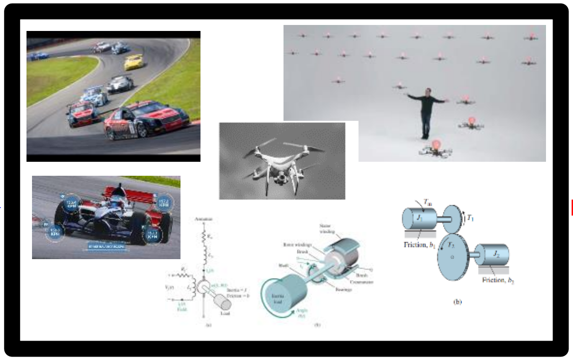
Control Systems

1. **Model**
2. **Response**
3. **Analysis**
4. **Feedback**
5. **Control**

Plant, Input, Output, Action, Goal

Signals & Systems

Plant (P)



Ref (r)

Input (u)

Output (y)

$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} - 3\,y(t) = 5\,u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2\,s - 3}$$

Controller

$$\frac{d^2y(t)}{dt^2} + 4\,\frac{dy(t)}{dt} + 3\,y(t) = 3\,r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4\,s + 3}$$

Control Systems

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Table of Laplace Transforms

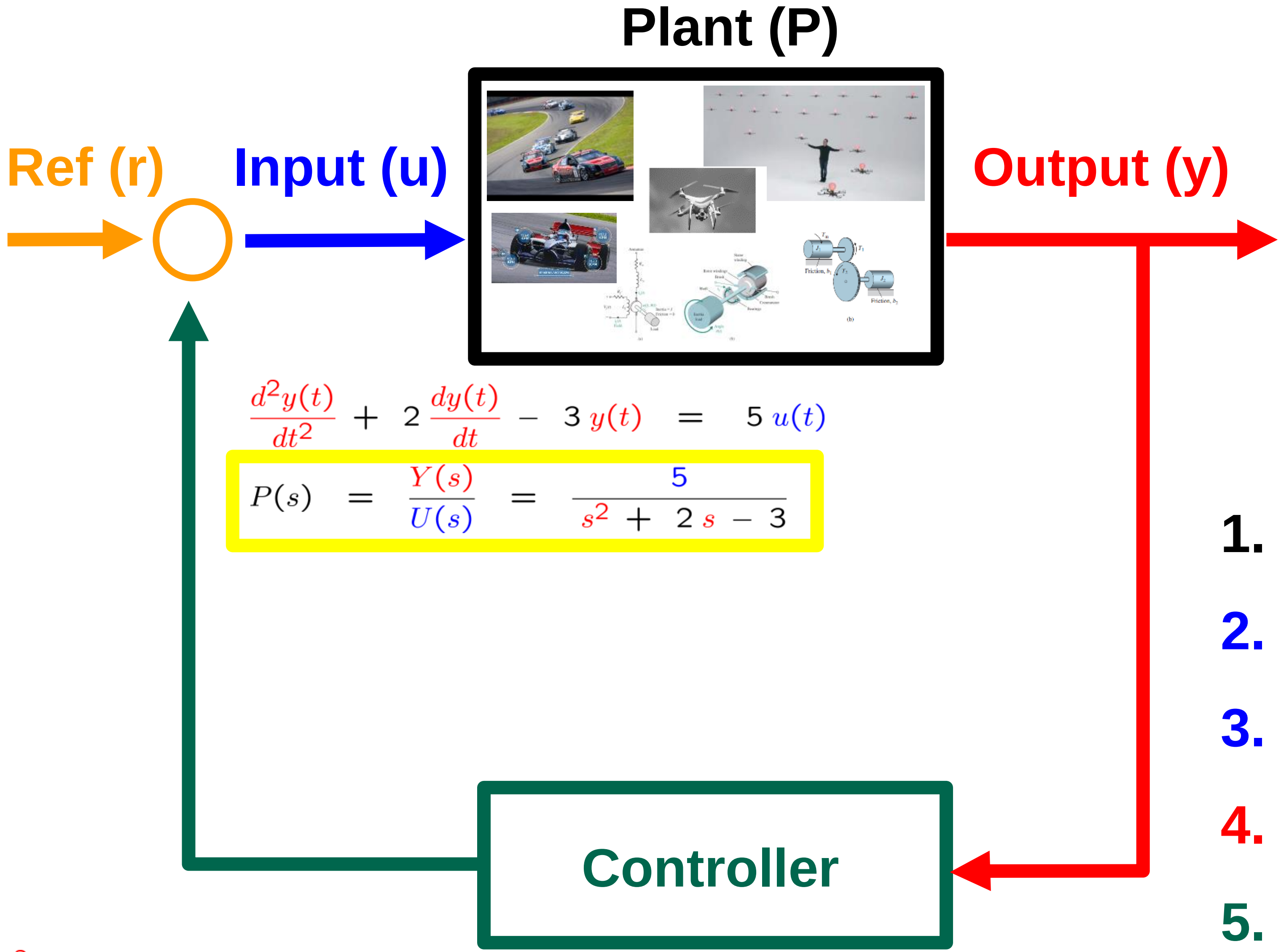
Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$\frac{1}{s+a}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$

Number	$F(s)$	$f(t), t \geq 0$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Properties of Laplace Transforms

Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s)+\beta F_2(s)$	$\alpha f_1(t)+\beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t-\lambda)$	Time delay ($\lambda \geq 0$)
3	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^mF(s)-s^{m-1}f(0)-s^{m-2}\dot{f}(0)-\cdots-f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s)$	$\int_0^t f(\zeta)\,d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t)*f_2(t)$	Convolution
8	$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)\,d\omega$	$\int_0^\infty y(t)u(t)\,dt$	Parseval's Theorem
12	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

Plant, Input, Output, Action, Goal



$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} - 3\,y(t) = 5\,u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2\,s - 3}$$

$$\frac{d^2y(t)}{dt^2} + 4\,\frac{dy(t)}{dt} + 3\,y(t) = 3\,r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4\,s + 3}$$

Signals & Systems

Control Systems

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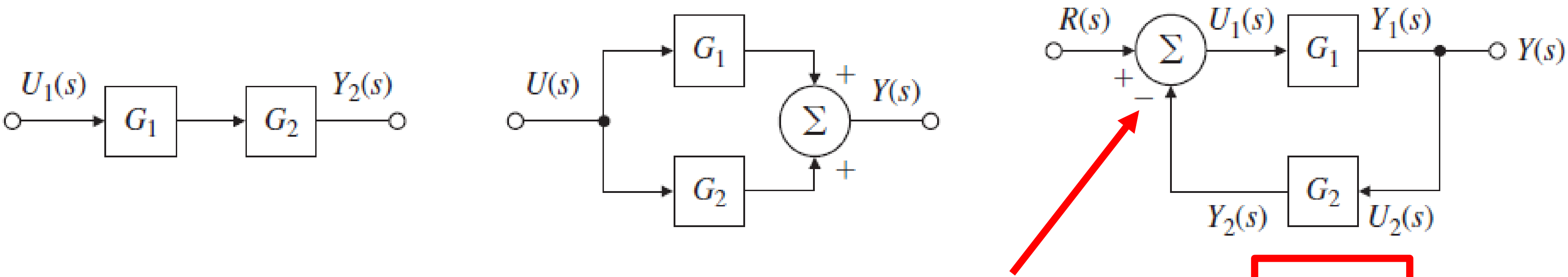
- Elementary block diagrams: (a) series; (b) parallel; (c) feedback



$$\frac{Y_2(s)}{U_1(s)} = G_2 G_1$$

(a)

$$\frac{Y(s)}{U(s)} = G_2 + G_1$$

(b)

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_2 G_1}$$

(c)

- The gain of a single-loop negative feedback system

  is given by

  the forward gain divided by the sum of 1 plus the loop gain.

- Negative Feedback

$$\frac{Y}{R} = \frac{G_1}{1 + G_2 G_1}$$

- Positive Feedback

$$\frac{Y}{R} = \frac{G_1}{1 - G_2 G_1}$$

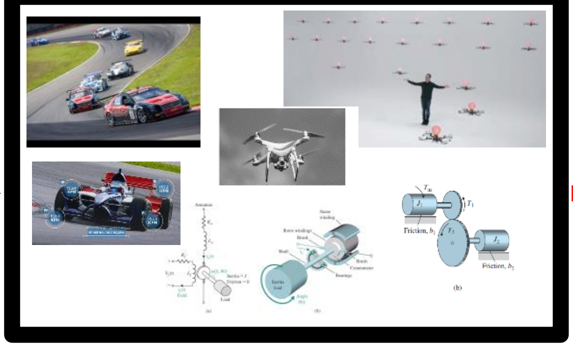
- Unity Feedback System

$$G_2 = 1$$

Plant (P)

Signals & Systems

Ref (r) → ○ **Input (u)** →



→ **Output (y)**

$$\frac{d^2y(t)}{dt^2} + 2\,\frac{dy(t)}{dt} - 3\,y(t) = 5\,u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2\,s - 3}$$

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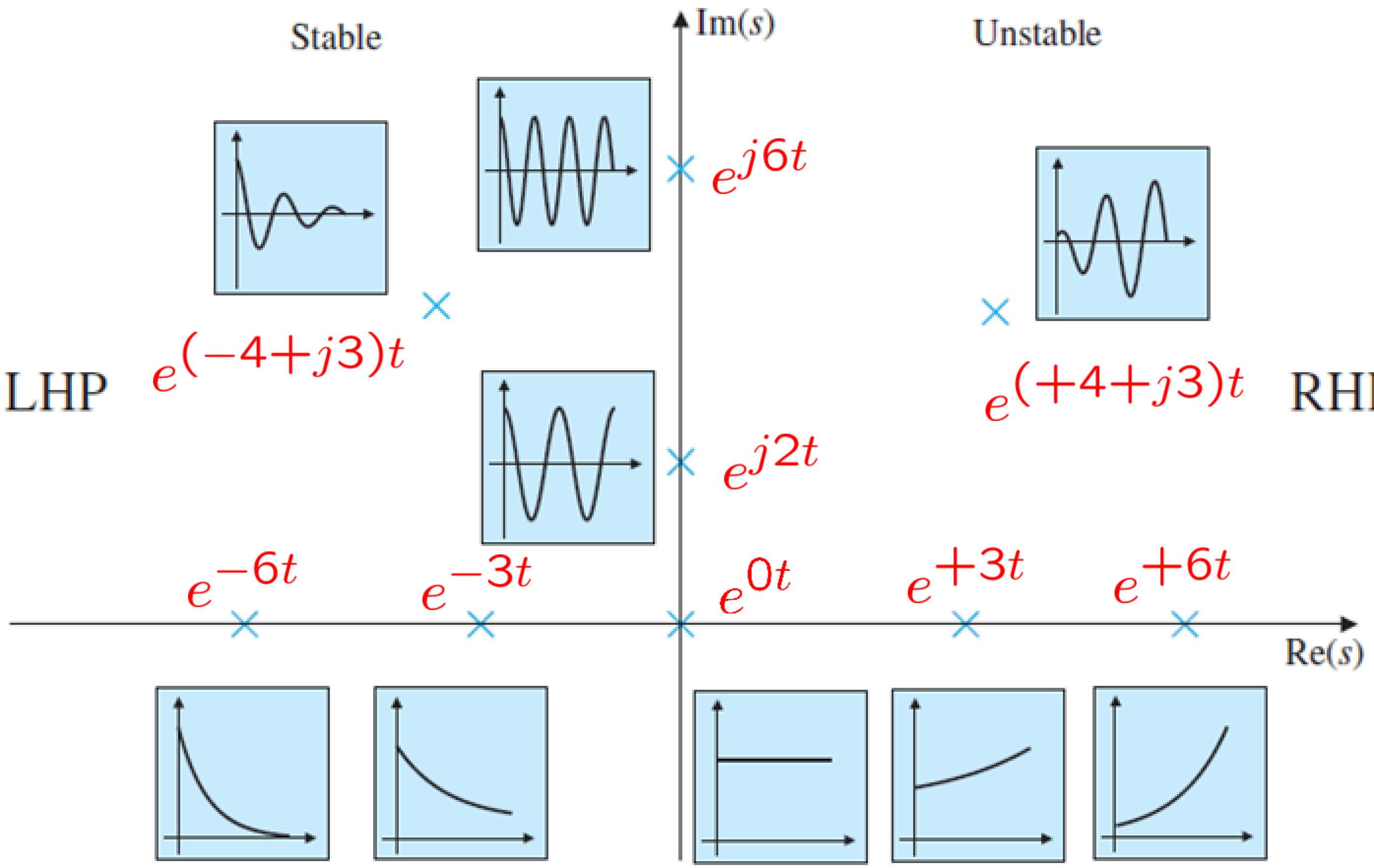
- Time functions associated with points in the s-plane
  (LHP, left half-plane; RHP, right half-plane)



Plant, Input, Output, Action, Goal

Signals & Systems

Plant (P)

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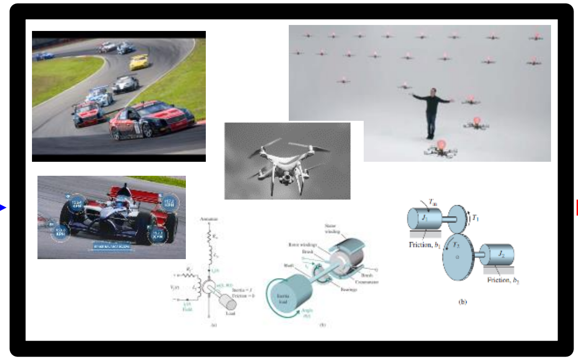

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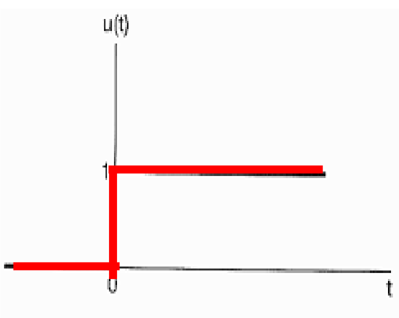
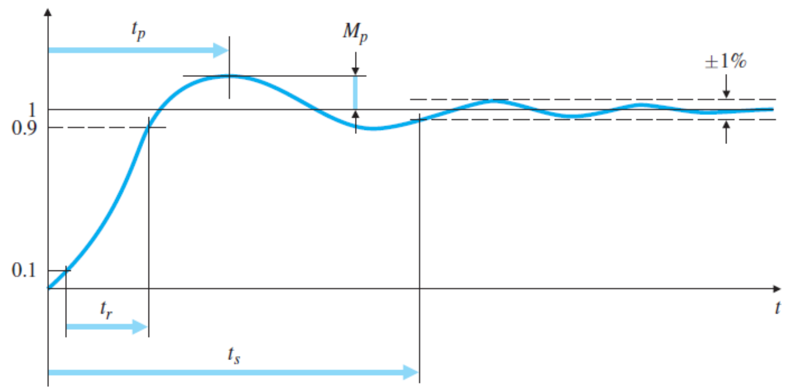
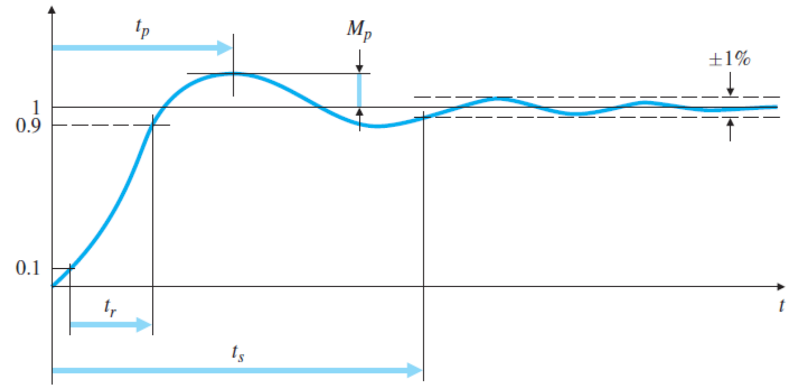




U3D Time-Domain Specifications

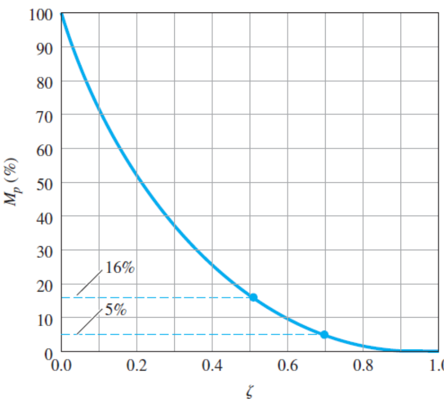
- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p

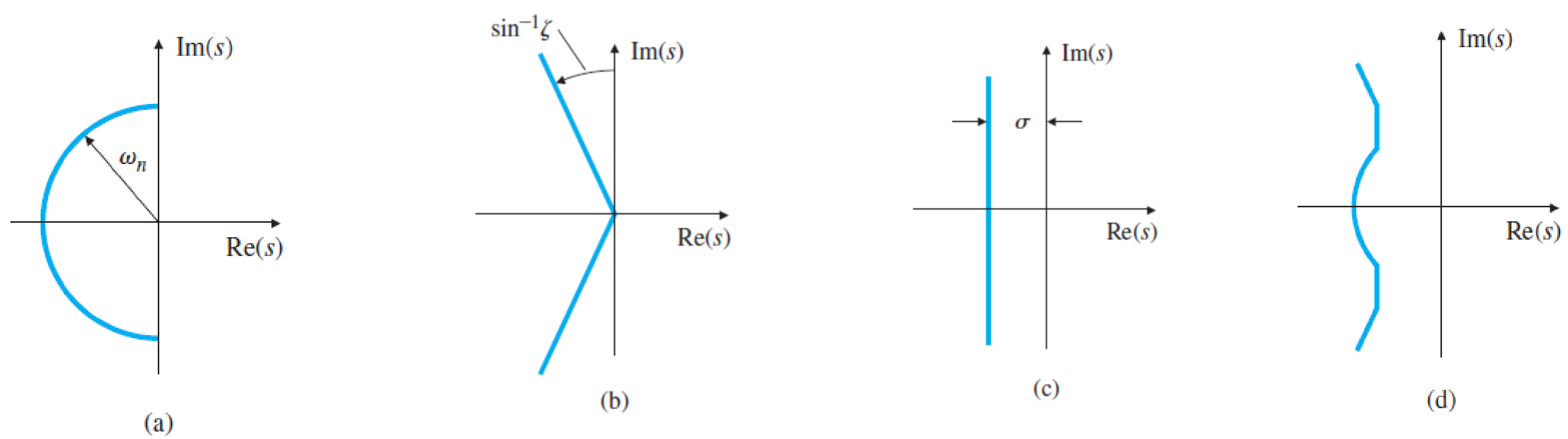


$$\Rightarrow t_r \cong \frac{1.8}{w_n} \qquad \Rightarrow w_n \ge \frac{1.8}{t_r}$$

$$\Rightarrow t_s = \frac{4.6}{\zeta w_n} = \frac{4.6}{\sigma} \qquad \Rightarrow \zeta \ge \zeta(M_p)$$

$$\Rightarrow M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad 0 \le \zeta < 1 \qquad \Rightarrow \sigma \ge \frac{4.6}{t_s}$$

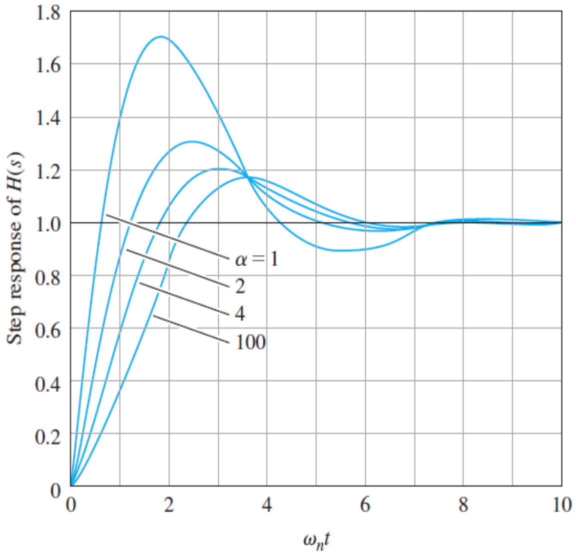


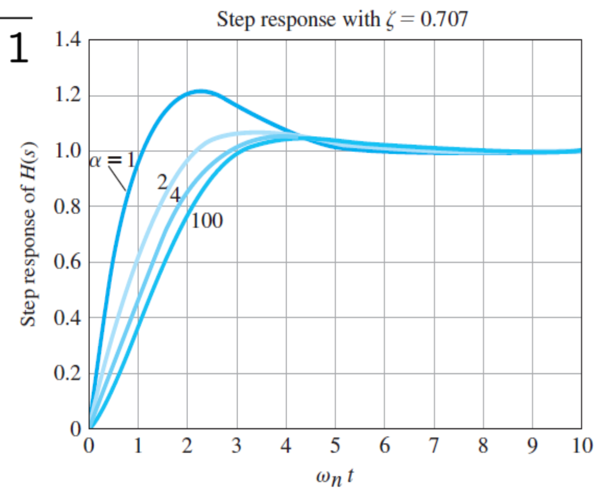


U3E Effects of Zeros and Additional Poles

▪ Same Poles, Different Zeros

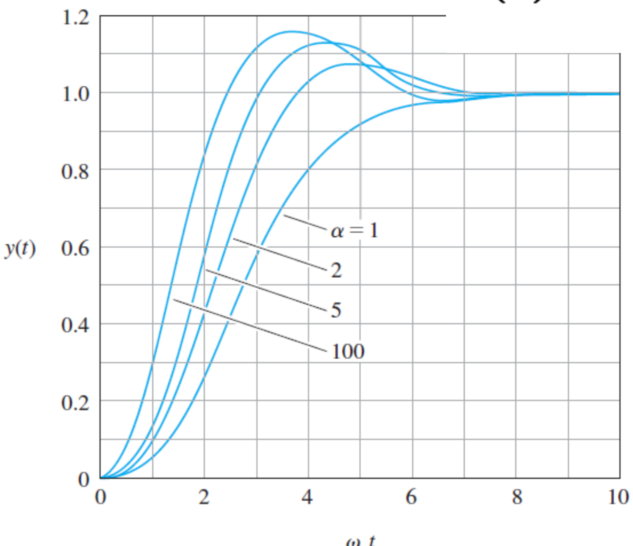
$$H(s) = \frac{\frac{s}{\alpha\zeta w_n} + 1}{(\frac{s}{w_n})^2 + 2\zeta(\frac{s}{w_n}) + 1}$$

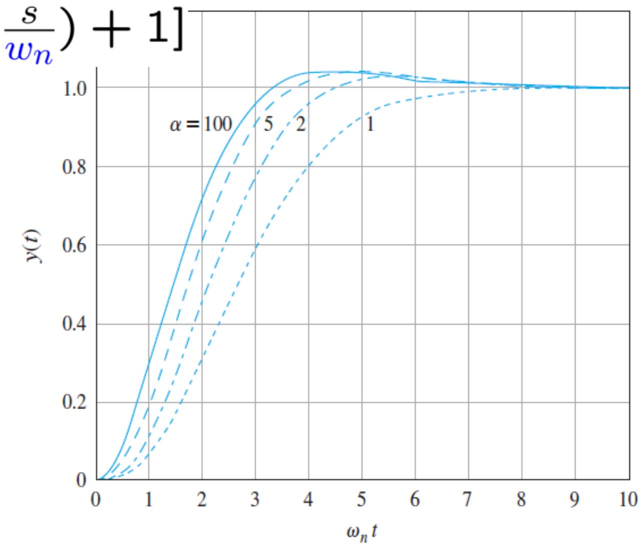




▪ Effects of Pole-Zero Patterns on Dynamic Response

$$H(s) = \frac{1}{(\frac{s}{\alpha\zeta w_n} + 1)[(\frac{s}{w_n})^2 + 2\zeta(\frac{s}{w_n}) + 1]}$$





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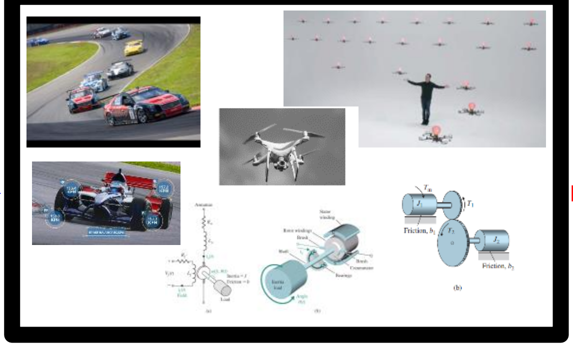

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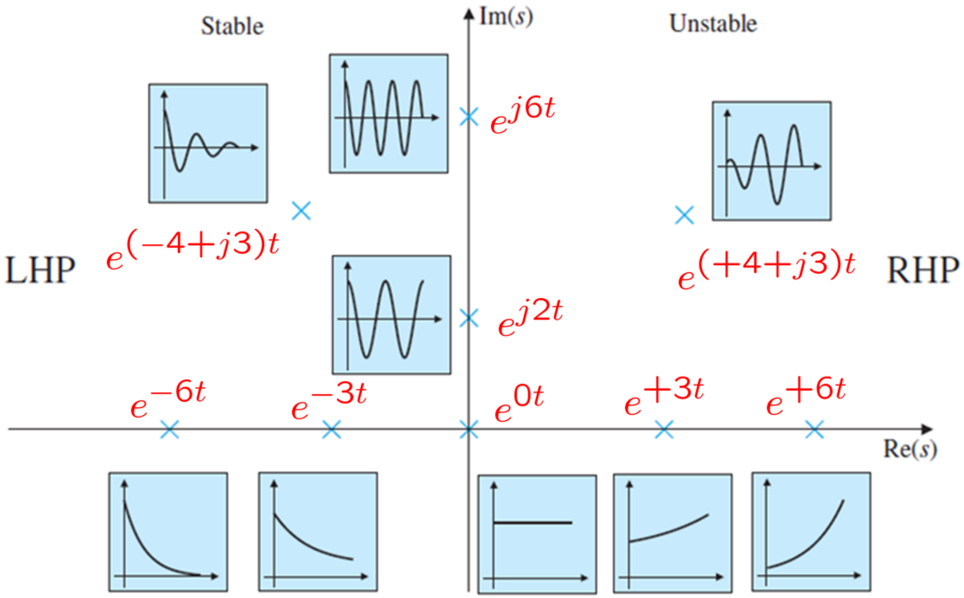
U3F Stability



stable

neutrally stable

unstable