Fall 2021 (110-1)

# 控制系統
# Control Systems

## Unit 2E
## Electric Circuits

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NTU-EE

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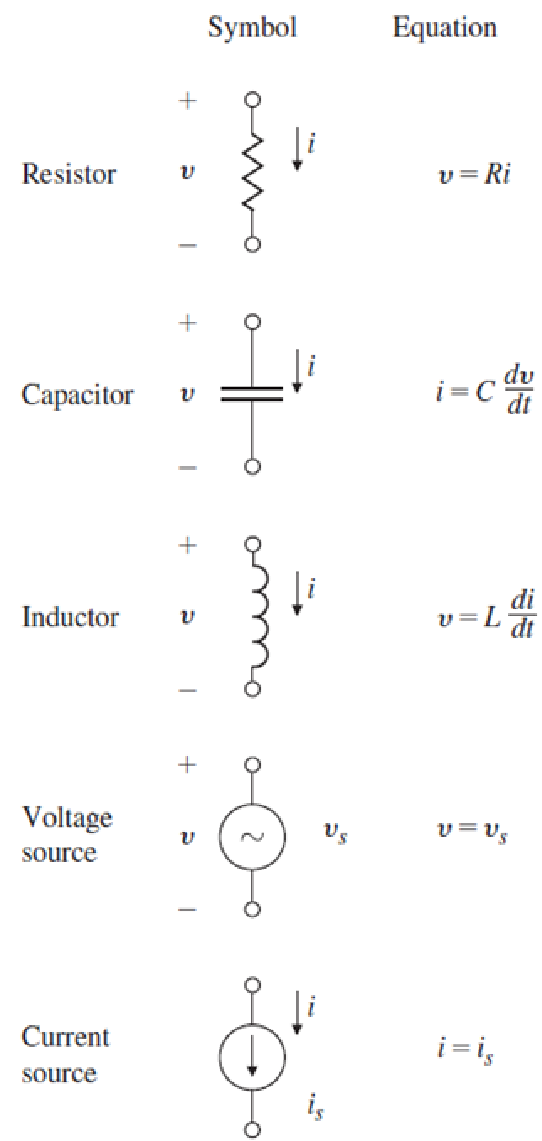
- The basic equations of electric circuits are the Kirchhoff's laws




- Kirchhoff's Current Law (KCL):

  The algebraic sum of the currents leaving a node

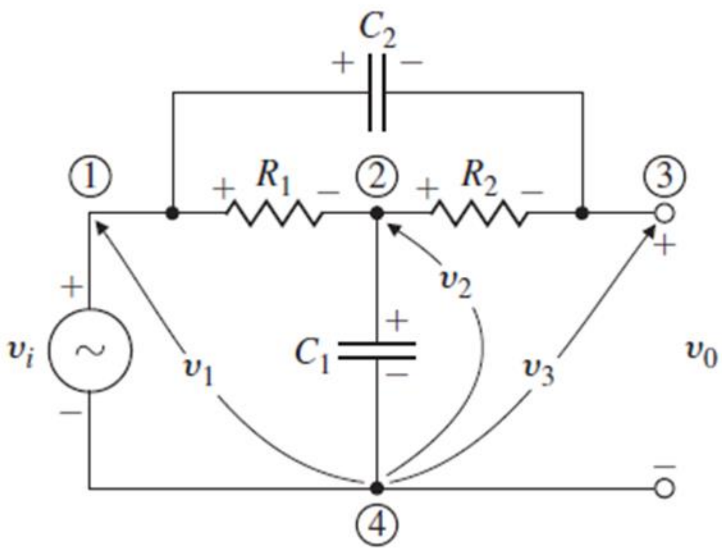
  ||

  The algebraic sum of the currents entering that node

- Kirchhoff's Voltage Law (KVL):

  The algebraic sum of all voltages taken around a closed path in a circuit is zero

- Bridged Tee Circuit

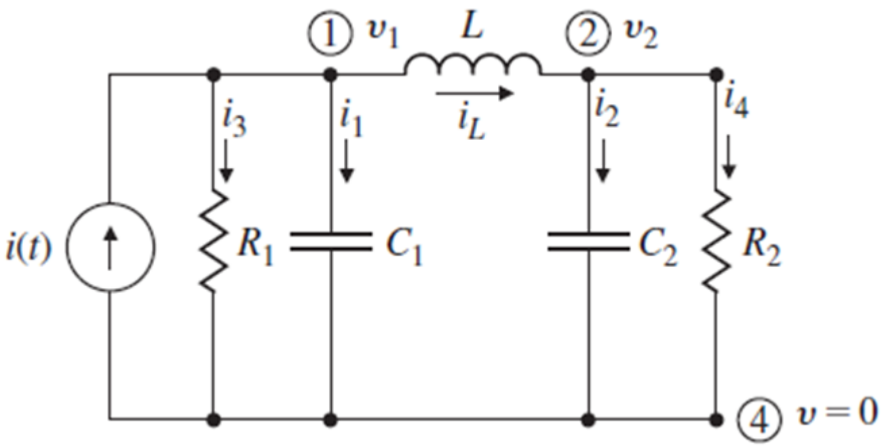



## Model (Equations of Motion)

- Select node 4 as the reference
- $v_1,\ v_2,\ v_3$ as the unknowns
- By KVL, $v_1 = v_i$

- At node 2, the KCL is $$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1 \frac{dv_2}{dt} = 0$$
- At node 3, the KCL is $$\frac{v_3 - v_2}{R_2} + C_2 \frac{d(v_3 - v_1)}{dt} = 0$$
- Transfer function from input $v_i$ to output $v_o$ can be derived

- Circuit with a current source




## Model (Equations of Motion)

- Select node 4 as the reference
- $v_1, \; v_2, \; i_L$ as the unknowns
- At node 1, the KCL is

$$i(t) = i_3 + i_1 + i_L$$

- At node 2, the KCL is

$$i_L = i_2 + i_4$$

- We also have the relations

$$i_3 = \frac{v_1}{R_1}, \quad i_1 = C_1 \frac{dv_1}{dt},$$

$$i_2 = C_2 \frac{dv_2}{dt}, \quad i_4 = \frac{v_2}{R_2},$$

$$v_1 - v_2 = L \frac{di_L}{dt},$$

$\Rightarrow$

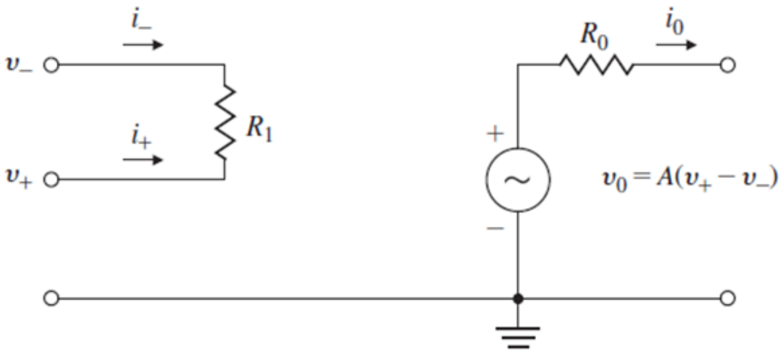
$$i(t) = \frac{v_1}{R_1} + C_1 \frac{dv_1}{dt} + i_L,$$

$$i_L = C_2 \frac{dv_2}{dt} + \frac{v_2}{R_2}$$

$$v_1 = L \frac{di_L}{dt} + v_2$$

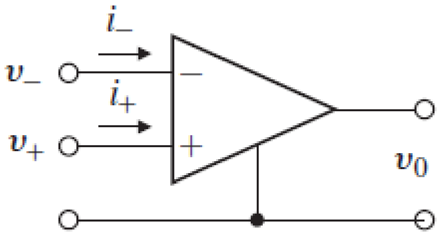
# Example 2.10 Operational Amplifier Summer

- Simplified circuit of op-amp



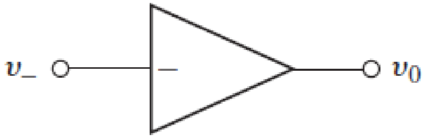
(a)

- op-amp schematic symbol



(b)

- Assume connected to ground, $v_+ = 0$

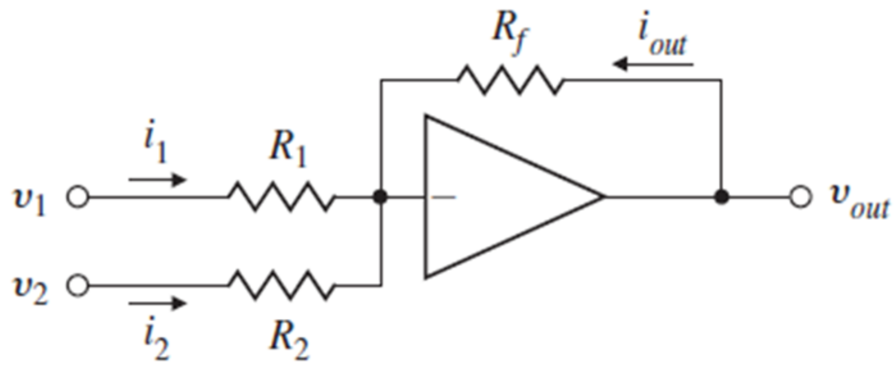


(c)

- Assume ideal op-amp, $R_1 = \infty,\ R_0 = 0,\ A = \infty$

$$i_+ = i_- = 0, \quad v_+ - v_- = 0$$

- The op-amp summer




- From $v_+ - v_- = 0$, we have $v_- = 0$

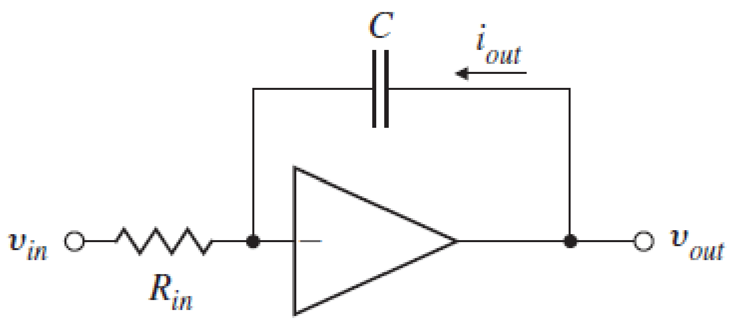
- Thus, $i_1 = \frac{v_1}{R_1}, \ i_2 = \frac{v_2}{R_2}, \ i_{out} = \frac{v_{out}}{R_f}$

- From $i_+ = i_- = 0$, we have $i_1 + i_2 + i_{out} = 0, \quad \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_{out}}{R_f} = 0$

## Model (Equations of Motion)

$$v_{out} = -\left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right]$$ (Output is the weighted sum of input voltages)

- The op-amp integrator




$$i_{in} + i_{out} = 0$$

$$\frac{v_{in}}{R_{in}} + C\frac{dv_{out}}{dt} = 0$$

## Model (Equations of Motion)

$$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}(\tau)d\tau + v_{out}(0)$$

## Transfer Function

$$V_{out}(s) = -\frac{1}{s}\frac{V_{in}(s)}{R_{in}C}$$

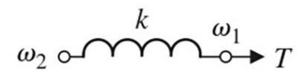
(Assume zero initial condition)

- Table 2.1 [Dorf & Bishop 2017]

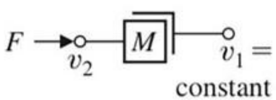
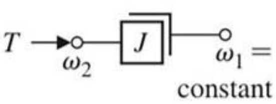
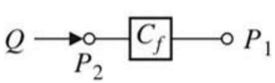
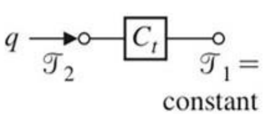
| System | Variable Through Element | Integrated Through-Variable | Variable Across Element | Integrated Across-Variable |
|---|---|---|---|---|
| Electrical | Current, $i$ | Charge, $q$ | Voltage difference, $v_{21}$ | Flux linkage, $\lambda_{21}$ |
| Mechanical translational | Force, $F$ | Translational momentum, $P$ | Velocity difference, $v_{21}$ | Displacement difference, $y_{21}$ |
| Mechanical rotational | Torque, $T$ | Angular momentum, $h$ | Angular velocity difference, $\omega_{21}$ | Angular displacement difference, $\theta_{21}$ |
| Fluid | Fluid volumetric rate of flow, $Q$ | Volume, $V$ | Pressure difference, $P_{21}$ | Pressure momentum, $\gamma_{21}$ |
| Thermal | Heat flow rate, $q$ | Heat energy, $H$ | Temperature difference, $\mathcal{T}_{21}$ | |

# Summary of Governing Differential Equations for Ideal Elements

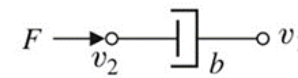
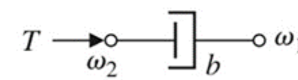
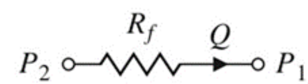
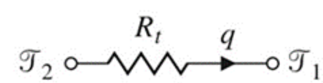
- Table 2.2-1 [Dorf & Bishop 2017]

| Type of Element | Physical Element | Governing Equation | Energy $E$ or Power $\mathscr{P}$ | Symbol |
|---|---|---|---|---|
| Inductive storage | Electrical inductance | $v_{21} = L\frac{di}{dt}$ | $E = \frac{1}{2}Li^2$ |   |
| | Translational spring | $v_{21} = \frac{1}{k}\frac{dF}{dt}$ | $E = \frac{1}{2}\frac{F^2}{k}$ |   |
| | Rotational spring | $\omega_{21} = \frac{1}{k}\frac{dT}{dt}$ | $E = \frac{1}{2}\frac{T^2}{k}$ |   |
| | Fluid inertia | $P_{21} = I\frac{dQ}{dt}$ | $E = \frac{1}{2}IQ^2$ |   |

- Table 2.2-2

| Type of Element | Physical Element | Governing Equation | Energy $E$ or Power $\mathscr{P}$ | Symbol |
|---|---|---|---|---|
| Capacitive storage | Electrical capacitance | $i = C\frac{dv_{21}}{dt}$ | $E = \frac{1}{2}C{v_{21}}^2$ |   |
| | Translational mass | $F = M\frac{dv_2}{dt}$ | $E = \frac{1}{2}M{v_2}^2$ |   |
| | Rotational mass | $T = J\frac{d\omega_2}{dt}$ | $E = \frac{1}{2}J{\omega_2}^2$ |   |
| | Fluid capacitance | $Q = C_f\frac{dP_{21}}{dt}$ | $E = \frac{1}{2}C_f{P_{21}}^2$ |   |
| | Thermal capacitance | $q = C_t\frac{d\mathscr{T}_2}{dt}$ | $E = C_t\mathscr{T}_2$ |   |

- Table 2.2.3 [Dorf & Bishop 2017]

| Type of Element | Physical Element | Governing Equation | Energy $E$ or Power $\mathcal{P}$ | Symbol |
|---|---|---|---|---|
| Energy dissipators | Electrical resistance | $i = \frac{1}{R} v_{21}$ | $\mathcal{P} = \frac{1}{R} {v_{21}}^2$ |   |
| | Translational damper | $F = b v_{21}$ | $\mathcal{P} = b {v_{21}}^2$ |   |
| | Rotational damper | $T = b \omega_{21}$ | $\mathcal{P} = b {\omega_{21}}^2$ |   |
| | Fluid resistance | $Q = \frac{1}{R_f} P_{21}$ | $\mathcal{P} = \frac{1}{R_f} {P_{21}}^2$ |   |
| | Thermal resistance | $q = \frac{1}{R_t} \mathcal{T}_{21}$ | $\mathcal{P} = \frac{1}{R_t} \mathcal{T}_{21}$ |   |