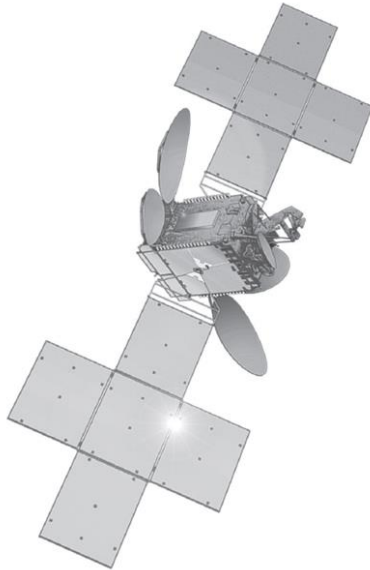# 控制系統
Control Systems

Unit 2B
Mechanical Systems – Rotational Motion

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NTU-EE

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● Communication satellite

Source: Courtesy Thaicom PLC and Space Systems/Loral

● The purpose is
to control the attitude of the satellite,
such as

✓ Antennas point toward earth
✓ Solar panels orient toward the sun

▪ Model (Equations of Motion: Rotational motion)

$$M \ = \ I \ \alpha$$

- M ($N \cdot m^2$): the sum of all external moments about the center of mass,

- I ($Kg \cdot m^2$): the body's mass moment of inertia about its center of mass,

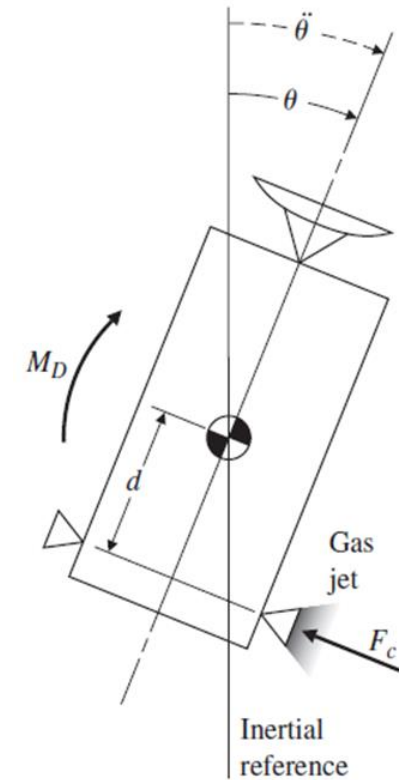- α (rad/sec^2): the angular acceleration of the body

▪ Model (Equations of Motion)

● Three axes, consider one axis at a time

$$F_c \cdot d + M_D = I \cdot \ddot{\theta}$$

- $F_c \cdot d$: Moments of control force

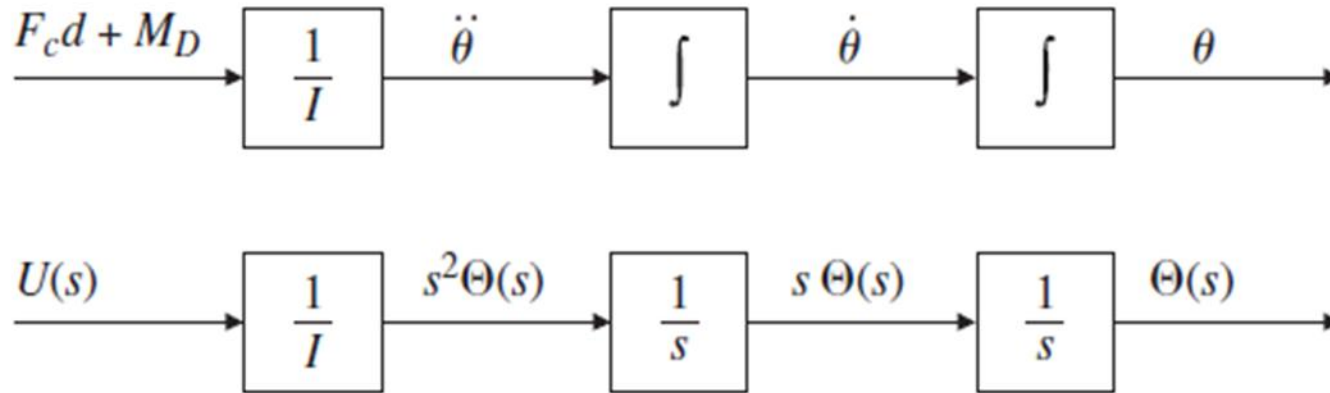- M_D : Moments of small disturbance

▪ Transfer Function

● Let $F_c \cdot d + M_D = u$

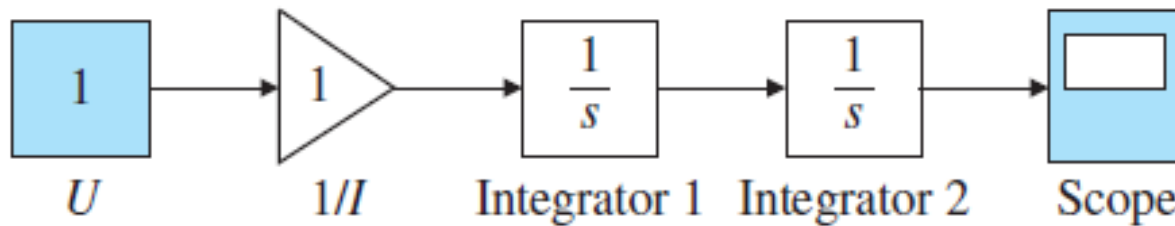$$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2} \quad \text{(Double-Integrator plant)}$$

- Block diagram $\quad \dfrac{\Theta(s)}{U(s)} = \dfrac{1}{I} \cdot \dfrac{1}{s^2}$ (Double-Integrator plant)
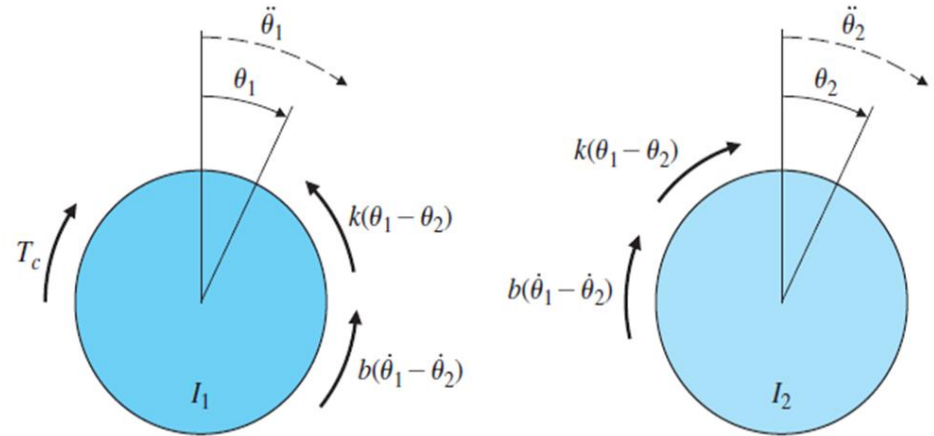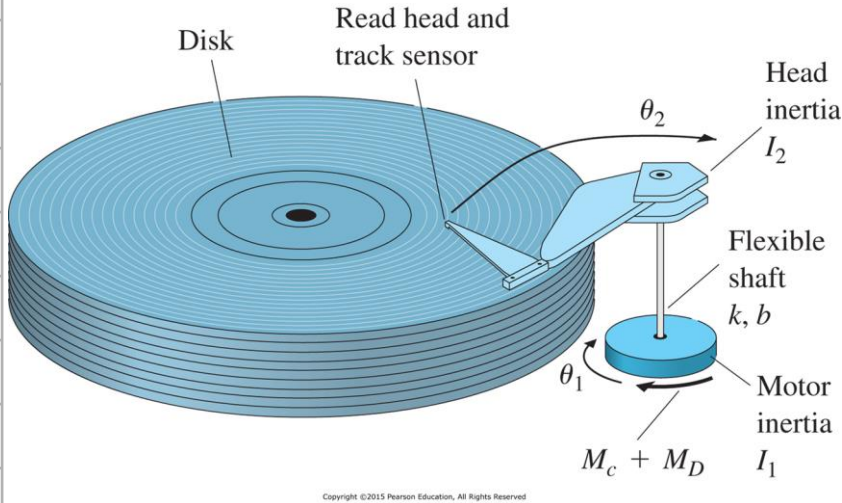

- Simulink

- **Disk Read/Write Head**

- **The moment of each body: free body diagram**

- **Model (Equations of Motion: Rotational motion)**

$$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_c + M_D$$

$$I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$$

- M_c : Moments of applied control
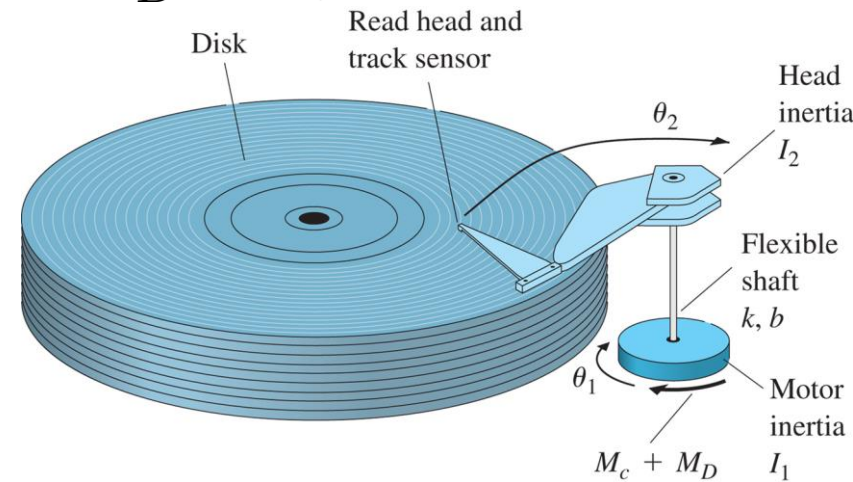- M_D : Moments of small disturbance

Model (Equations of Motion)

- Simplify the model, consider the case $M_D = 0, b = 0$

$$I_1 \ddot{\theta}_1 + k(\theta_1 - \theta_2) = M_c$$

$$I_2 \ddot{\theta}_2 + k(\theta_2 - \theta_1) = 0$$

Disk

Read head and
track sensor

Head
inertia
I_2

θ_2

Flexible
shaft
k, b

θ_1

Motor
inertia

$M_c + M_D$ I_1

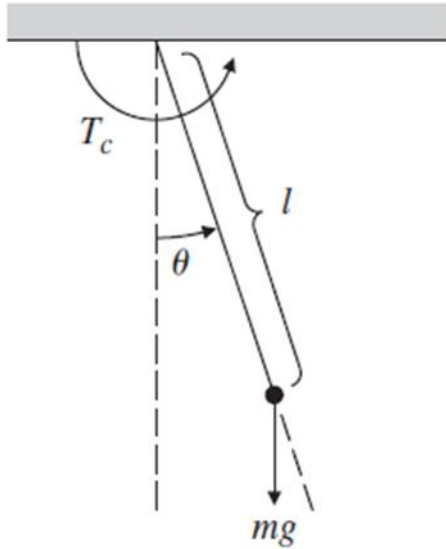Copyright ©2015 Pearson Education, All Rights Reserved

Transfer Function

$$\frac{\Theta_2(s)}{M_c(s)} = \frac{k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$$

$$\frac{\Theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$$

- **"Noncollocated case"**: there is **flexibility** between the sensor and the actuator

- **"Collocated case"**: the sensor and the actuator are **rigidly attached** to one another

Example 2.5 Pendulum

● Pendulum

■ Model (Equations of Motion)

$$T_c - mgl \sin \theta = I \ddot{\theta}$$

• The moments of inertia about the pivot point is

$$I = ml^2$$

$$\ddot{\theta} + \frac{g}{l} \sin \theta = \frac{T_c}{ml^2}$$

• The model is nonlinear due to $\sin \theta$
• When the motion is small, i.e., θ small, $\sin \theta \approx \theta$

$$\ddot{\theta} + \frac{g}{l} \theta = \frac{T_c}{ml^2}$$ (Linearization model)

Example 2.5 Pendulum
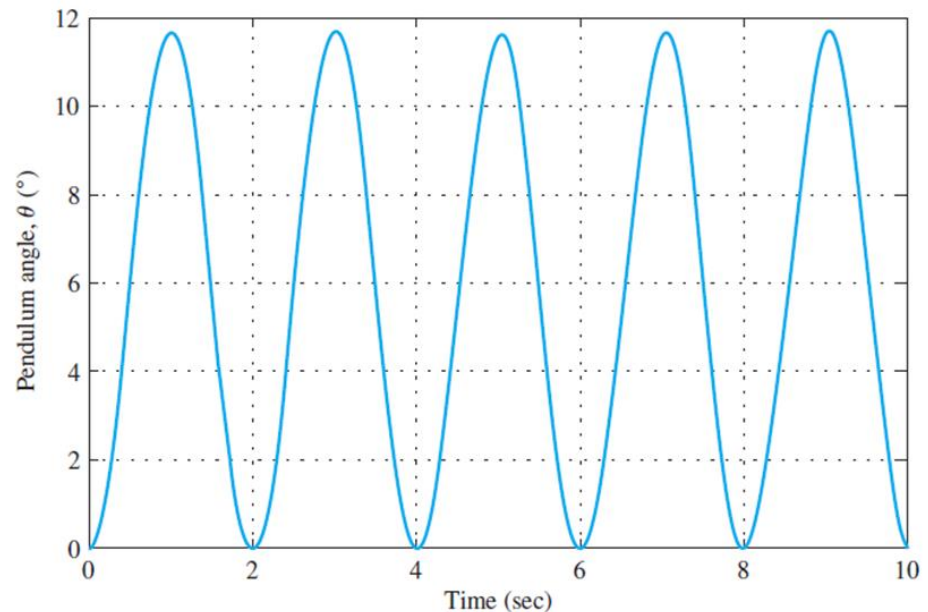
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- **Transfer Function**

$$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}}$$

- **Matlab code**
  - t = 0:0.02:10;
  - m = 1; L = 1; g = 9.81;
  - s = tf('s');
  - sys = (1/(m*L^2))/(s^2+g/L) ;
  - y1 = step(sys, t);
  - y2 = impulse(sys, t);
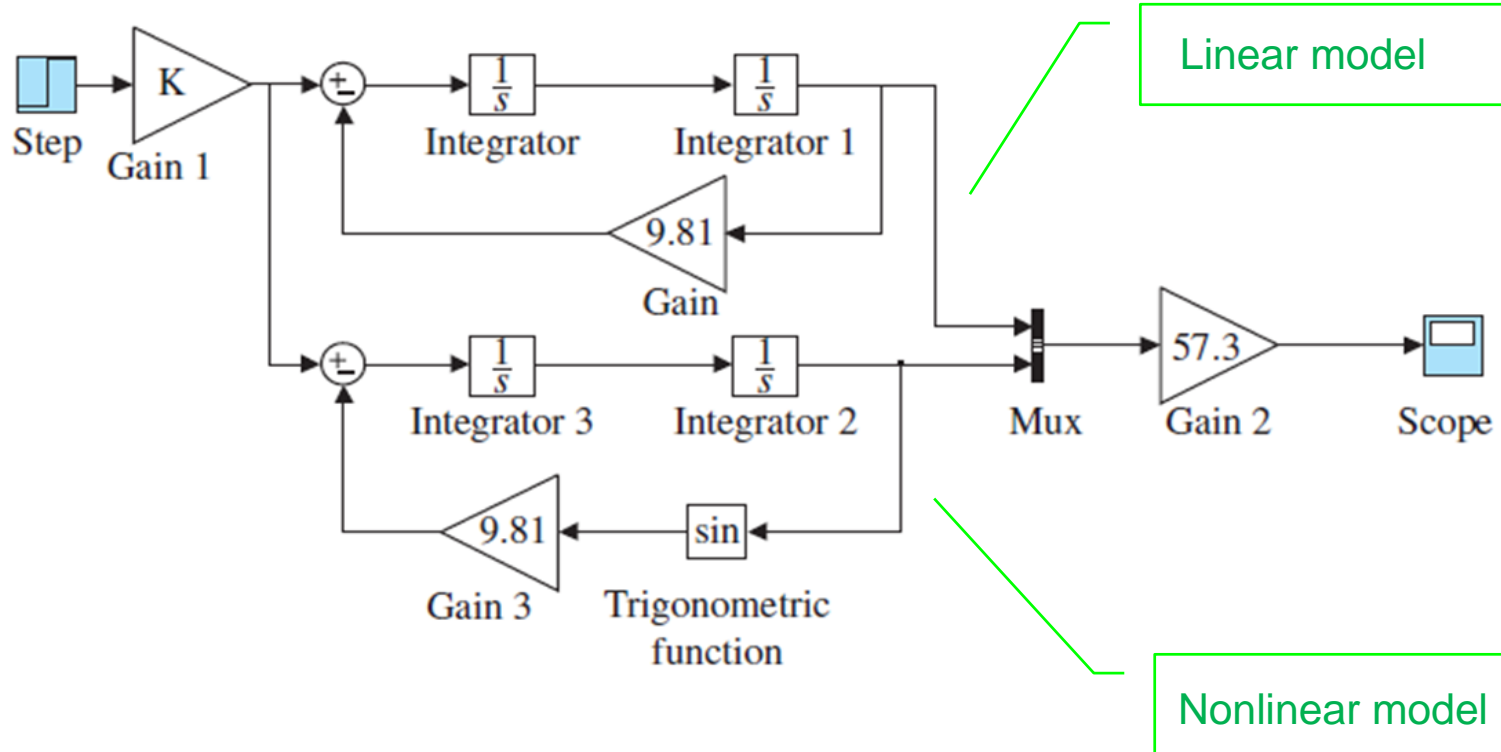
  - Rad2Deg = 57.3;
  - Plot(t, Rad2Deg*y1)   %converts output from radians to degrees
  - Plot(t, Rad2Deg*y2)   %converts output from radians to degrees

- ## Matlab Simulink   (m=1; L=1; g=9.81)

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

Linear model

Nonlinear model

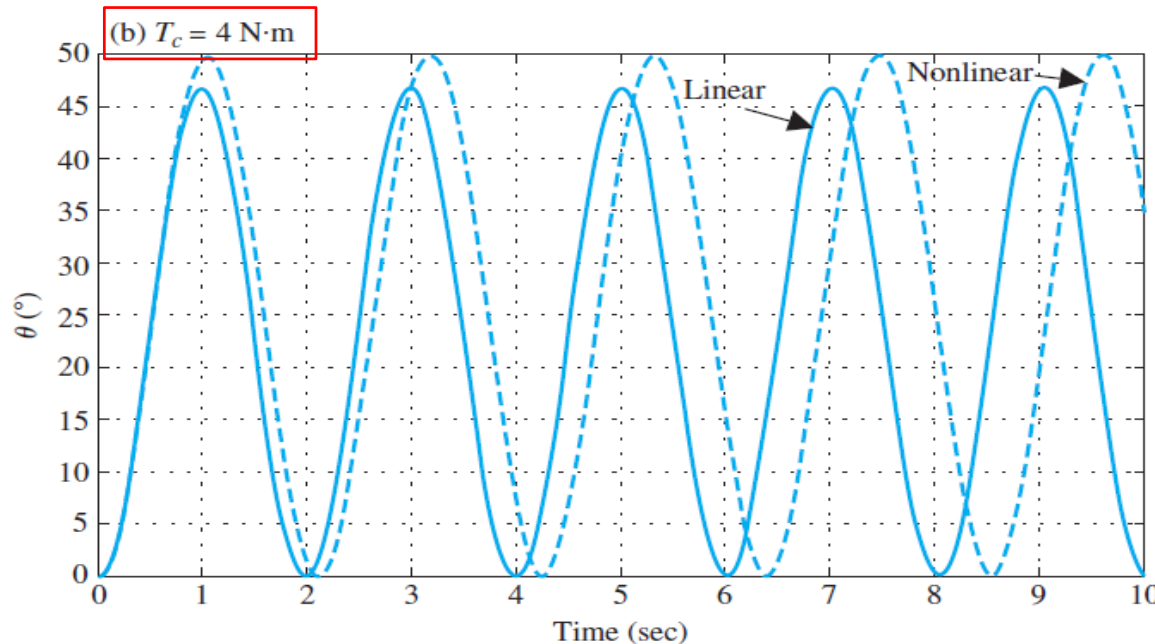$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

▪ Comparisons of linear & nonlinear responses

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

(a) $T_c = 1$ N·m

Nonlinear response

Linear response
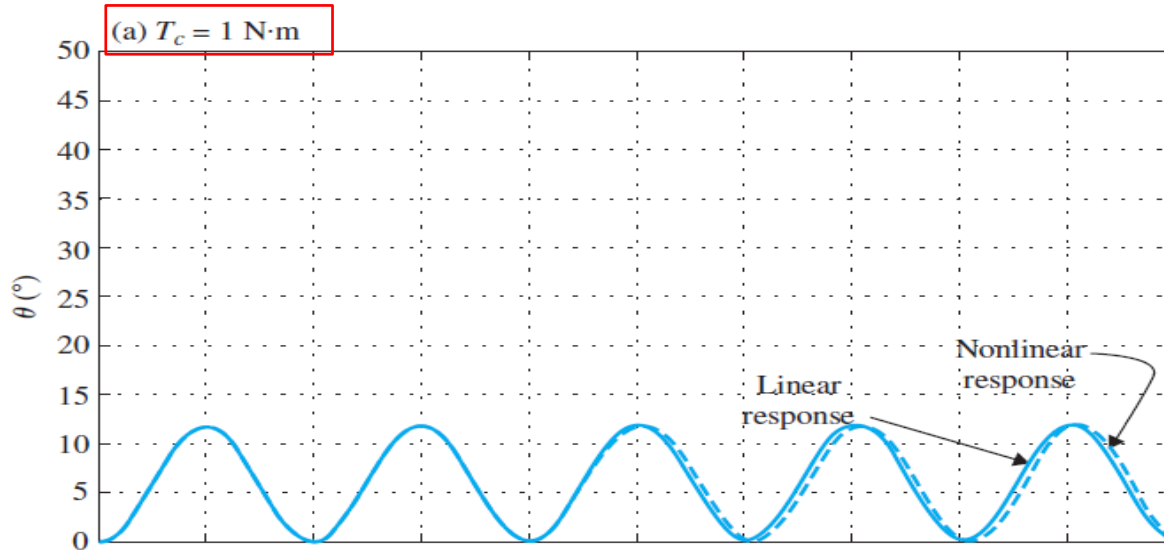
● When T_c=1,
the output θ
remains small,
thus the approximation
is still good ($\sin\theta \approx \theta$)

(b) $T_c = 4$ N·m

Linear

Nonlinear

Time (sec)

● When T_c=4,
the output θ
becomes large,
thus the approximation
is not good
($\sin\theta \approx \theta$ does not hold)