

Fall 2021 (110-1)

控制系統
Control Systems

Unit 1D

Motivating Example of Feedback and Control

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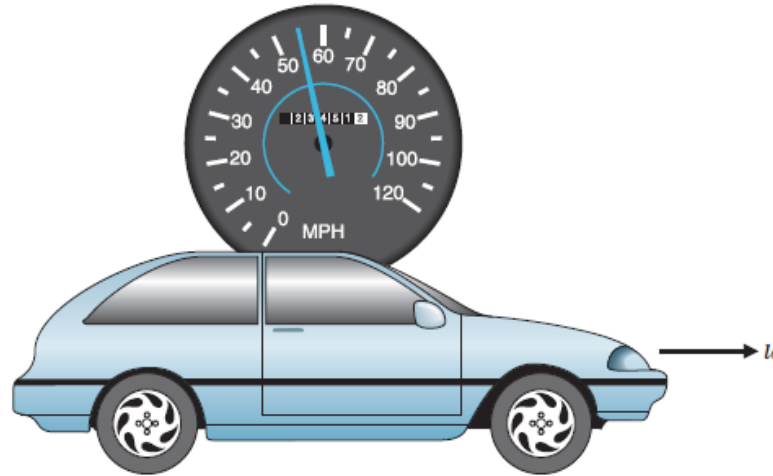
NTU-EE

Sep 2021 – Jan 2022

Control

Feedback

	Without Control	With Control
Open-Loop		
Closed-Loop		



- On level road, at 65 mph,
throttle angle = +1 (deg) ---> speed = +10 mph
- Driving up or down hills,
grade = +1% ---> speed = -5 mph
- Mathematical Model & Block Diagram (Signal & System):

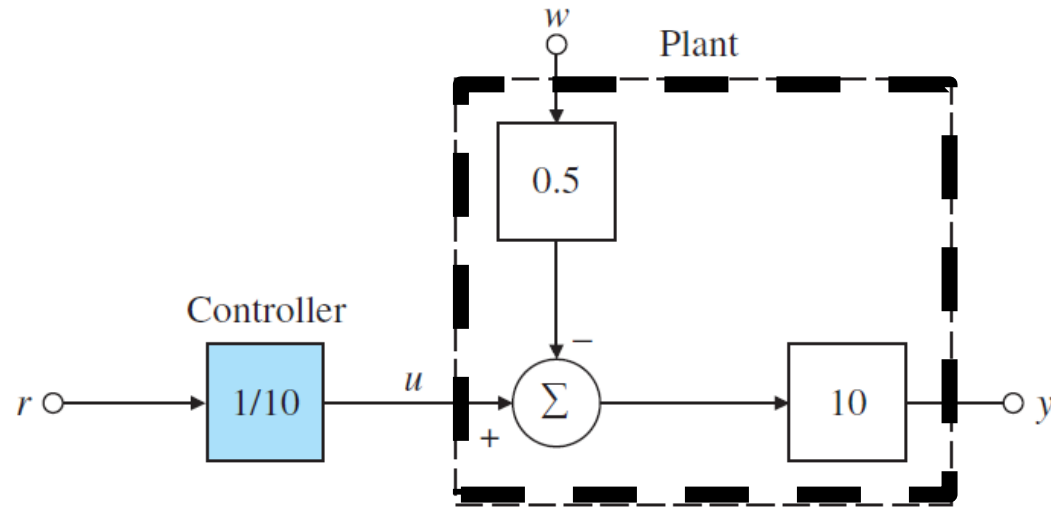
$$y = 10 u$$

$$u = \frac{1}{10} r$$

$$y = r$$



Open-Loop



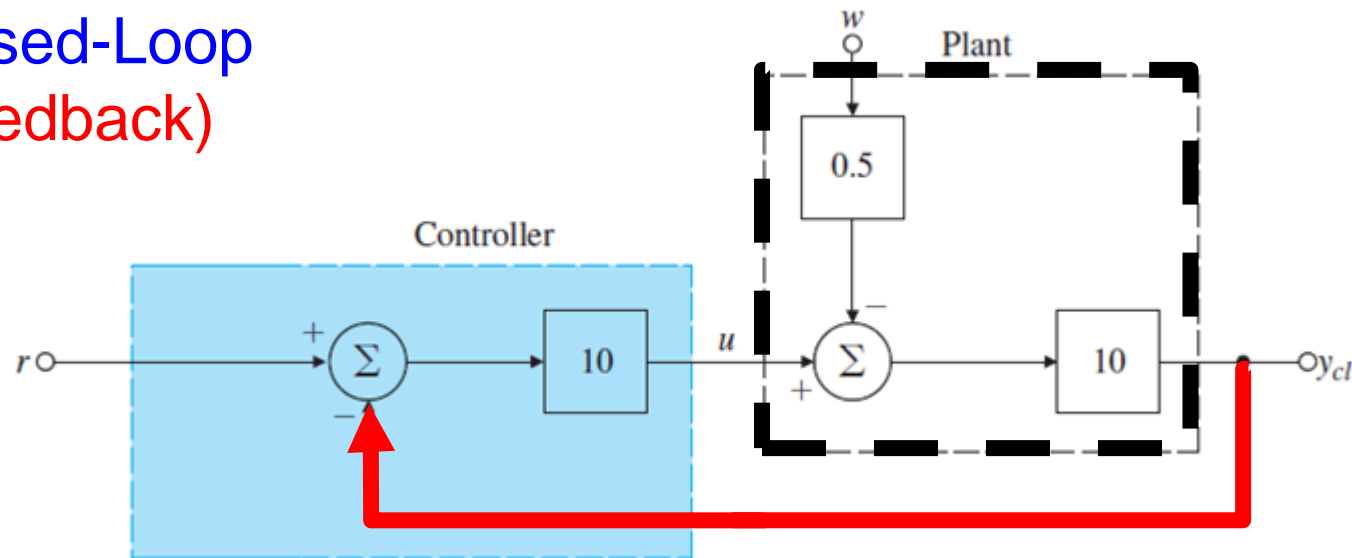
$$\begin{aligned}
 y_{ol} &= 10 (u - 0.5 w) \\
 &= 10 \left(\frac{1}{10} r - 0.5 w \right) \\
 &= r - 5 w
 \end{aligned}$$

$$\begin{aligned}
 e_{ol} &= r - y_{ol} \\
 &= 5 w
 \end{aligned}$$

$$\% \text{ error} = \frac{5 w}{r} \times 100$$

r = 65	w	error	% error
	0	0	0
	1	5	7.69%
	2	10	15.38%

■ Closed-Loop (Feedback)



$$u = 10 (r - y_{cl})$$

$$y_{cl} = 10 (u - 0.5 w)$$

$$= 100 r - 100 y_{cl} - 5 w$$

$$e_{cl} = r - y_{cl}$$

$$101 y_{cl} = 100 r - 5 w$$

$$= \frac{1}{101} r + \frac{5}{101} w$$

$$y_{cl} = \frac{100}{101} r - \frac{5}{101} w$$

$$\% \text{ error} = \frac{100}{101} + \frac{500 w}{101 r}$$

Motivating Example: Cruise Control of An Automobile

$w = 0, \quad y_{cl} = \frac{100}{101} r \quad y_{cl} = \frac{100}{101} r - \frac{5}{101} w$
 $e_{cl} = \frac{1}{101} r \quad \% \text{ error} \approx 1\% \quad e_{cl} = \frac{1}{101} r + \frac{5}{101} w$

$w = 1, \quad y_{cl} = \frac{100}{101} 65 - \frac{5}{101} 1 \quad \% \text{ error} = \frac{100}{101} + \frac{500 w}{101 r}$
 $e_{cl} = \frac{1}{101} 65 + \frac{5}{101} 1$

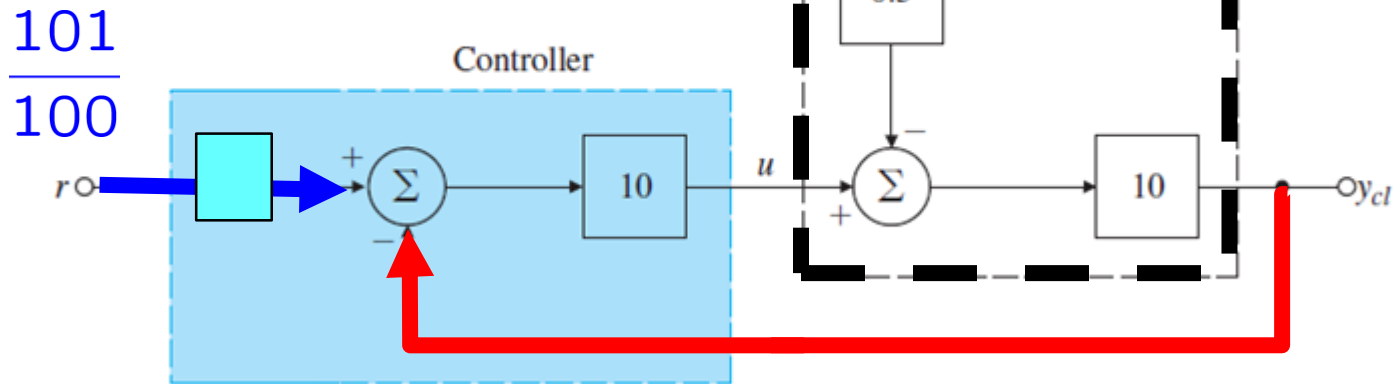
$\% \text{ error} = \frac{100}{101} + \frac{500 \times 1}{101 \times 65} = 0.99 + 0.0762 = 1.0662$

$w = 2, \quad y_{cl} = \frac{100}{101} 65 - \frac{5}{101} 2$
 $e_{cl} = \frac{1}{101} 65 + \frac{5}{101} 2$

$\% \text{ error} = \frac{100}{101} + \frac{500 \times 2}{101 \times 65} = 0.99 + 0.1523 = 1.1423$

r = 65	w	error Open	% error Open
	0	0	0
	1	5	7.69%
	2	10	15.38%

- Closed-Loop**
(Feedback + Feedforward)



$$u = 10 \left(\frac{101}{100} r - y_{cl} \right)$$

$$y_{cl} = 10 (u - 0.5 w)$$

$$= 101 r - 100 y_{cl} - 5 w$$

$$e_{cl} = r - y_{cl}$$

$$101 y_{cl} = 101 r - 5 w$$

$$= \frac{5}{101} w$$

$$y_{cl} = r - \frac{5}{101} w$$

$$\% \text{ error} = \frac{500 w}{101 r}$$

Motivating Example: Cruise Control of An Automobile

$$\begin{aligned} \blacksquare w = 0, \quad y_{cl} &= r & y_{cl} &= r - \frac{5}{101} w \\ e_{cl} &= 0 & \% \text{ error} &= 0\% \\ e_{cl} &= \frac{5}{101} w \end{aligned}$$

$$\begin{aligned} \blacksquare w = 1, \quad y_{cl} &= 65 - \frac{5}{101} 1 & \% \text{ error} &= \frac{500 w}{101 r} \\ e_{cl} &= \frac{5}{101} 1 \end{aligned}$$

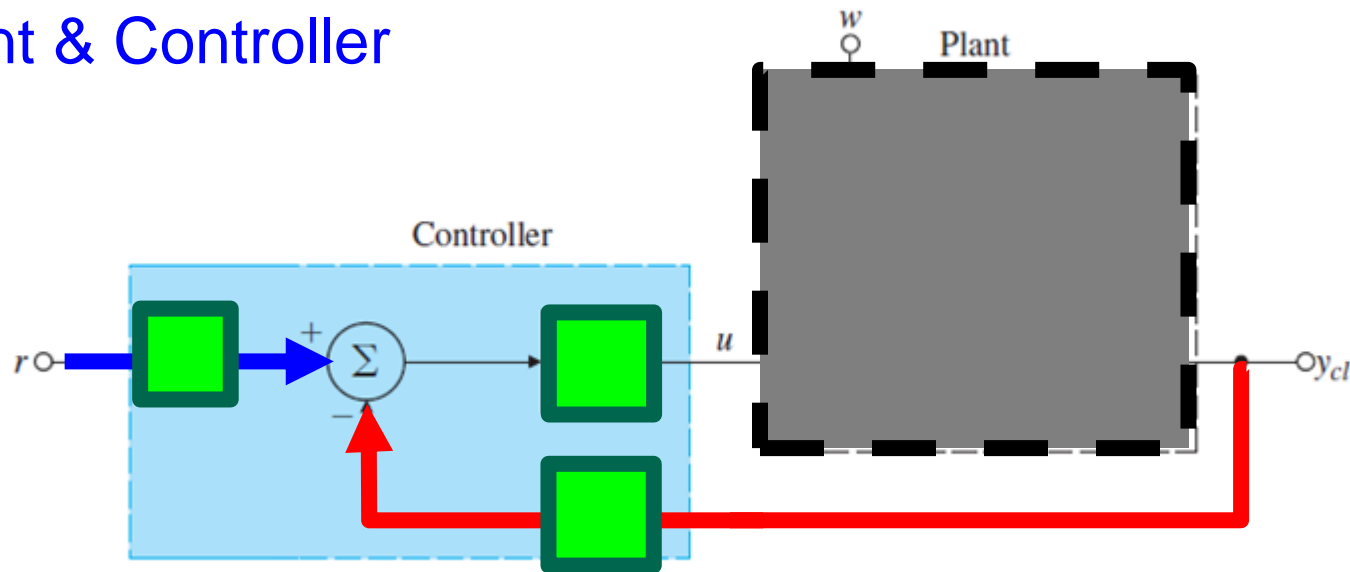
$$\% \text{ error} = \frac{500 \times 1}{101 \times 65} = 0.0762$$

$$\begin{aligned} \blacksquare w = 2, \quad y_{cl} &= 65 - \frac{5}{101} 2 \\ e_{cl} &= \frac{5}{101} 2 \end{aligned}$$

$$\begin{aligned} \% \text{ error} &= \frac{500 \times 2}{101 \times 65} \\ &= 0.1523 \end{aligned}$$

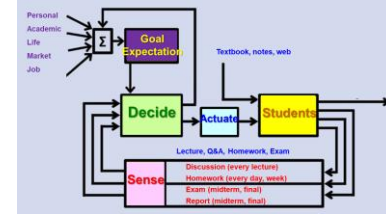
r = 65	w	error Open	% error Open
	0	0	0
	1	5	7.69%
	2	10	15.38%

Plant & Controller



$r = 65$ w	error Open	error Open %	% error Feedback + Feedforward $C=10$	% error Closed (Feedback) $C=1$	% error Closed (Feedback) $C=10$	% error Closed (Feedback) $C=100$
0	0	0	0 %	9.090 %	0.990 %	0.099 %
1	5	7.69 %	0.0762 %	9.790 %	1.066 %	0.107 %
2	10	15.38 %	0.1523 %	10.489 %	1.142 %	0.115 %

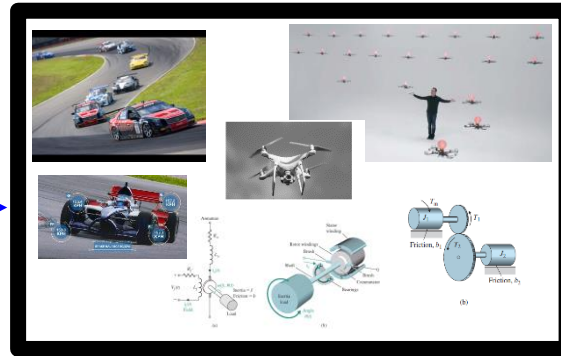
Plant (P)



Ref (r)

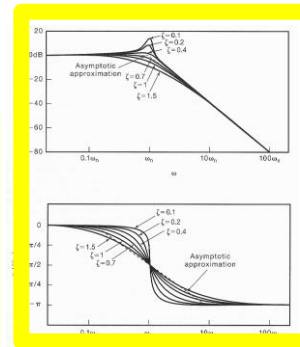
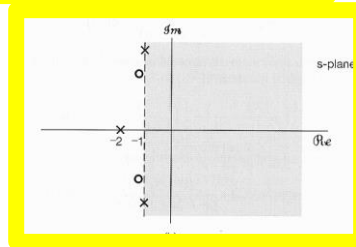
Input (u)

Output (y)



$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3y(t) = 5u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s - 3}$$



Controller

1. Model
2. Response
3. Analysis
4. Feedback
5. Control

$$\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = 3r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4s + 3}$$