

Spring 2021

控制系統  
Control Systems

Unit 7B  
Control System Design:  
Satellite and Airplane

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NTU-EE

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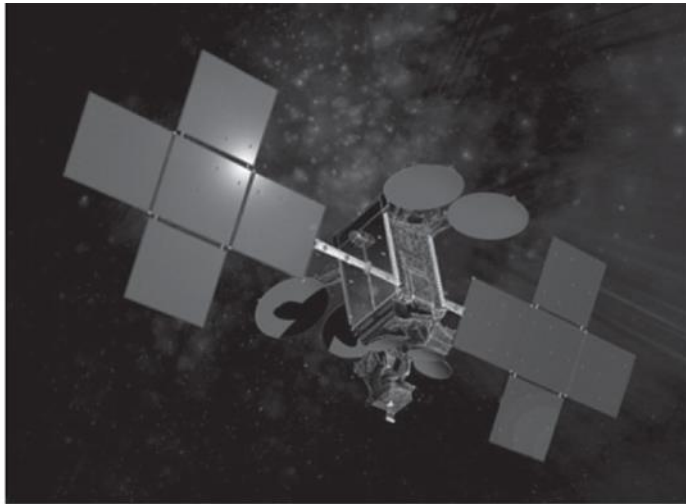
## ▪ Examples of Control Systems Design

- Satellite's Attitude Control
- Lateral & Longitudinal Control of Boeing
- Read Write Head of a Hard Disk
- RTP Systems in Wafer Manufacturing
- Chemotaxis Swims Away from Trouble
- Quadrotor Drone

## ▪ Control Tutorials Website

- Cruise Control
- Motor Speed
- Motor Position
- Suspension
- Inverted Pendulum
- Aircraft Pitch
- Ball & Beam

- (STEP 1) Understand the process and its performance specifications
  - the vehicle has an astronomical survey mission requiring accurate pointing of a scientific sensor package.



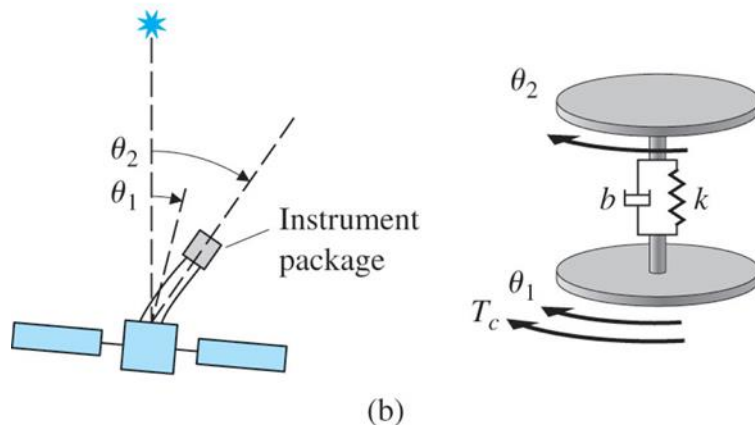
(a)

$\theta_1$  : the angle of the main satellite with respect to the star

$\theta_2$  : satellite attitude

SPEC:

- a transient settling time of 20 sec
- an overshoot of no more than 15 %



(b)

## ■ (STEP 2) Select sensors

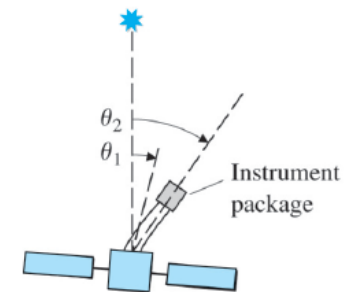
- Star tracker to obtain  $\theta_2$
- Rate gyto to have  $\dot{\theta}_2$

SPEC:

- a transient settling time of 20 sec
- an overshoot of no more than 15 %

## ■ (STEP 3) Select actuators

- Cold-gas jets as being fast and adequately accurate



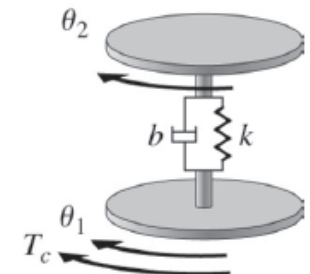
## ■ (STEP 4) Make a linear model

$$J_1 \ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = T_c$$

$$J_2 \ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$$

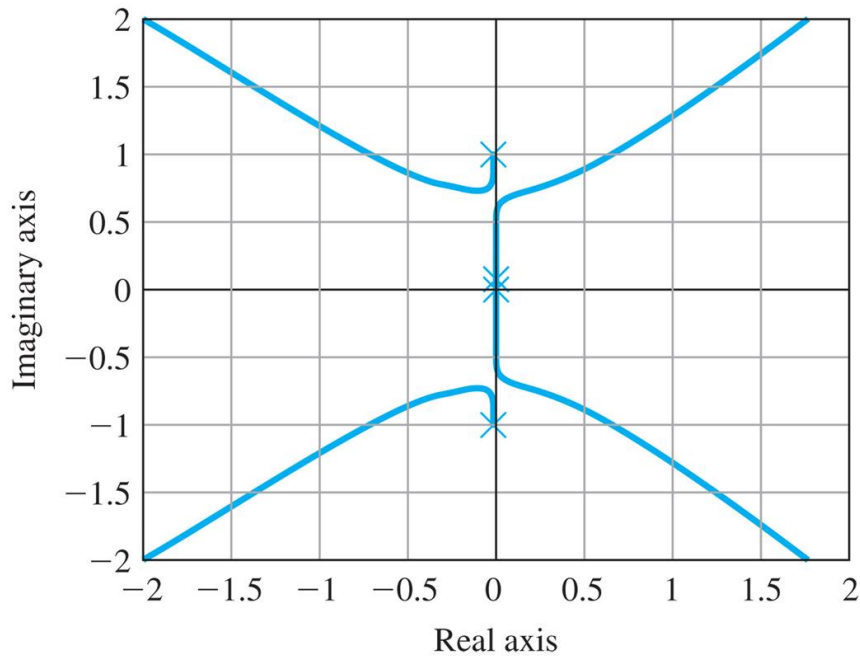
$$G(s) = \frac{\Theta_2(s)}{T_c(s)} = \frac{10bs + 10k}{s^2(s^2 + 11bs + 11k)}$$

$$(J_1 = 1, J_2 = 0.1)$$



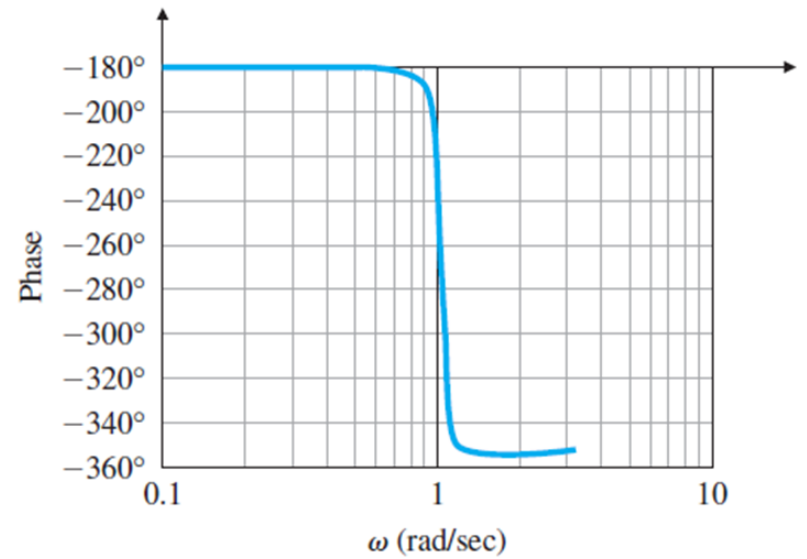
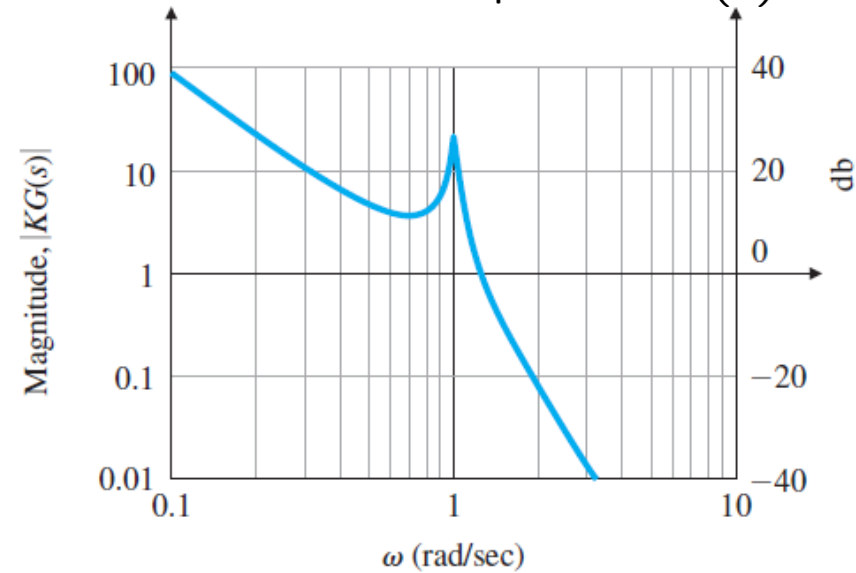
- (STEP 5) Try a lead-lag or P/D controller

proportional gain root locus



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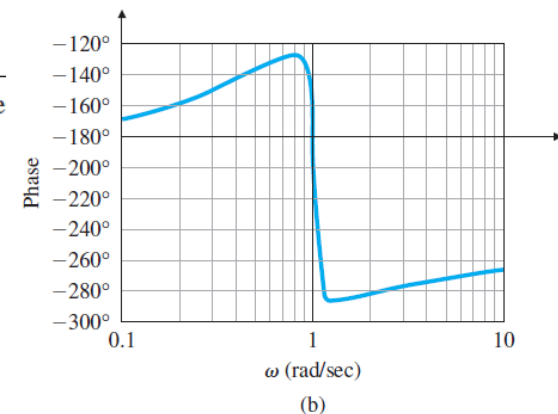
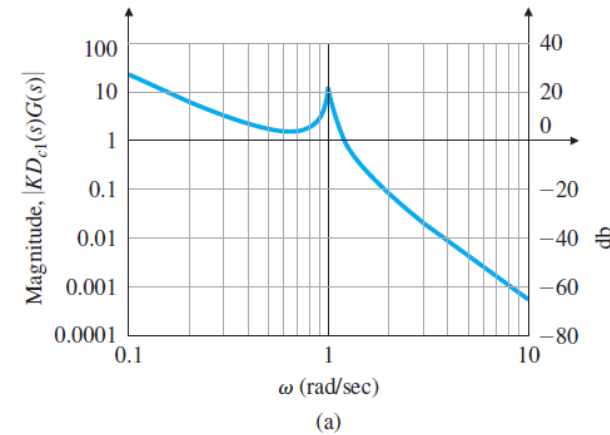
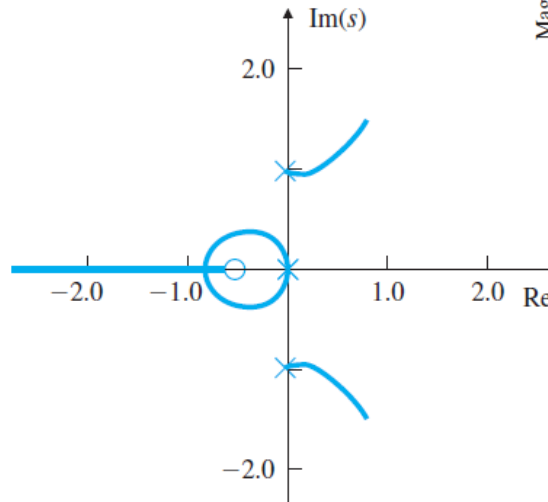
Bode plot of  $G(s)$



(b)

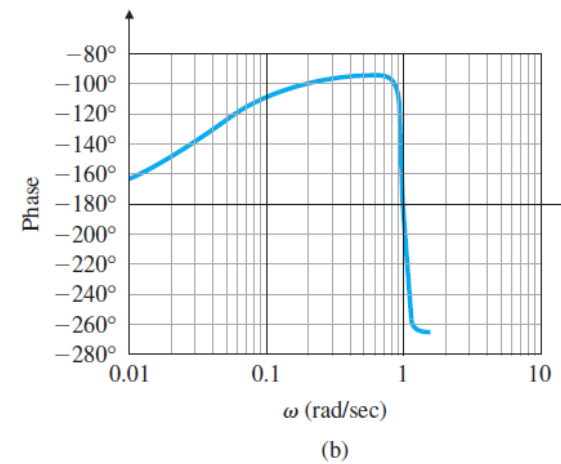
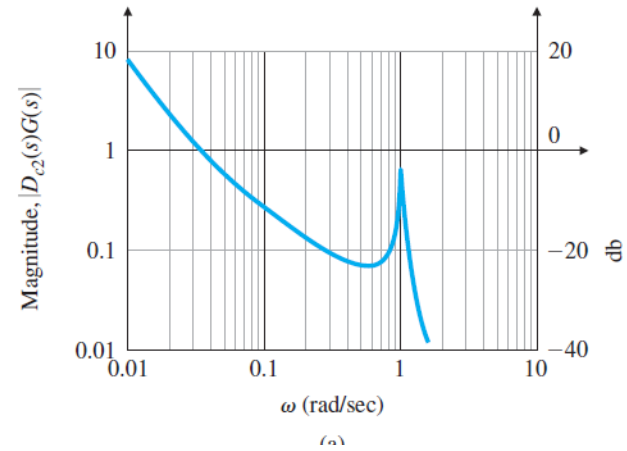
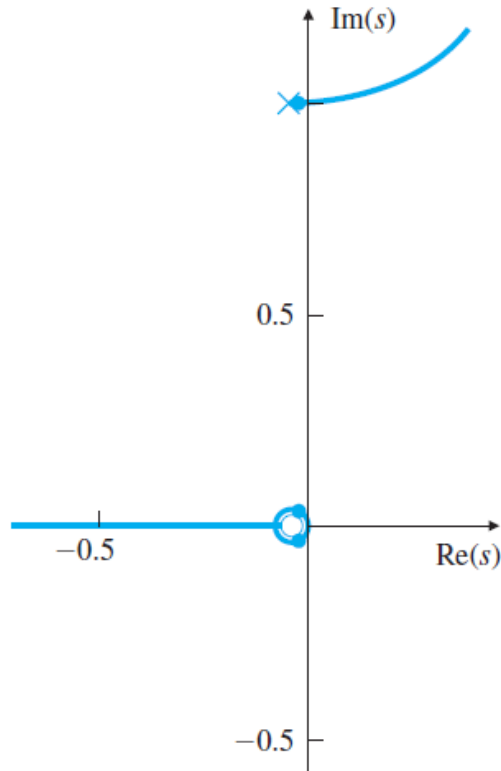
## ■ (STEP 5) Try a lead-lag or P/D controller

- First ignore the resonance and generate a design that would be acceptable for the rigid body alone
- Take the process transfer function to be  $1/s^2$
- Consider the PD control,  $D_c(s) = K(sT_D + 1)$
- The response objective is  $\omega_n = 0.5(\text{rad/sec})$ ,  $\zeta = 0.5$
- For  $D_{c1}(s) = 0.25(2s + 1)$
- Unstable



## ■ (STEP 5) Try a lead-lag or P/D controller

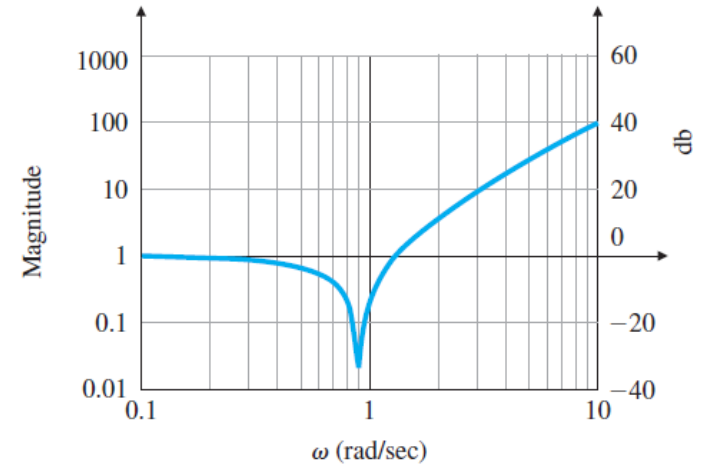
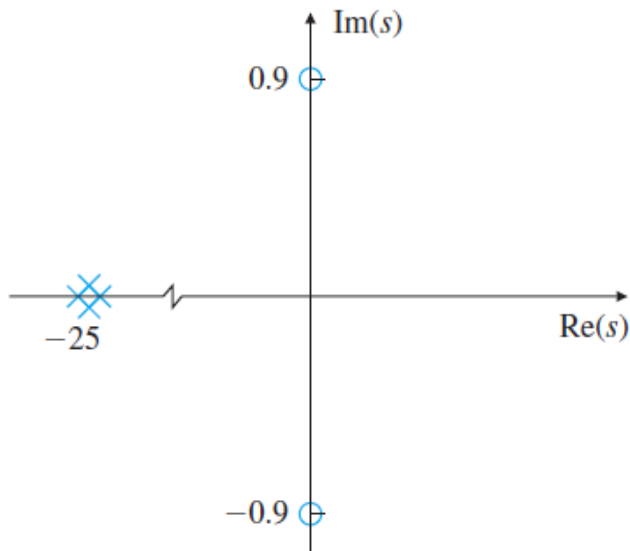
- Lower the gain for  $D_{c2}(s) = 0.001(30s + 1)$



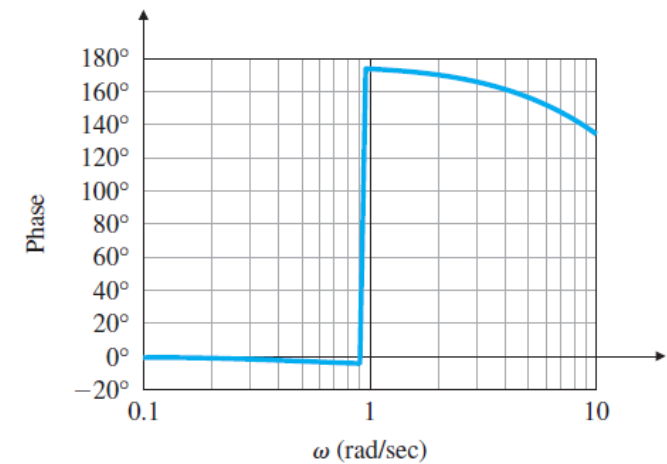
■ (STEP 5) Try a lead-lag or P/D controller

- Consider a notch filter

$$D_{c3}(s) = 0.25(2s + 1) \frac{(s/09)^2 + 1}{[(s/25) + 1]^2}$$



(a)

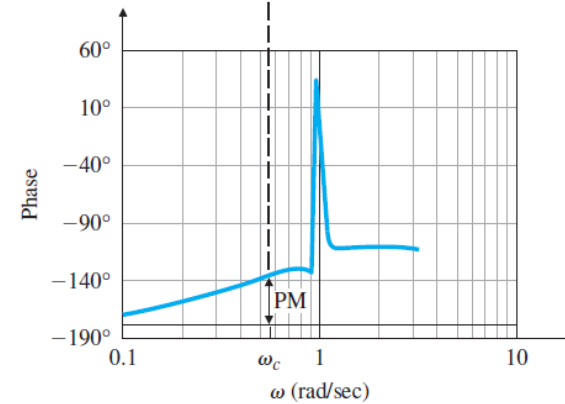
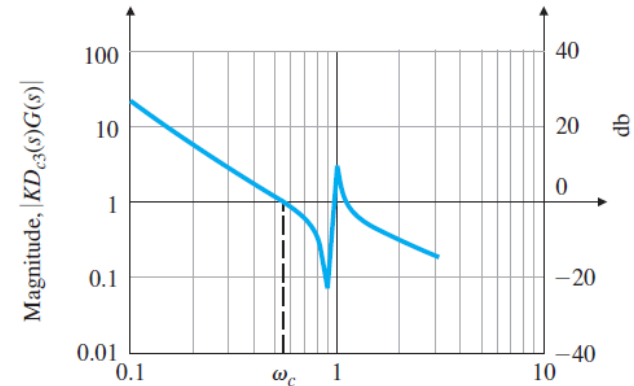
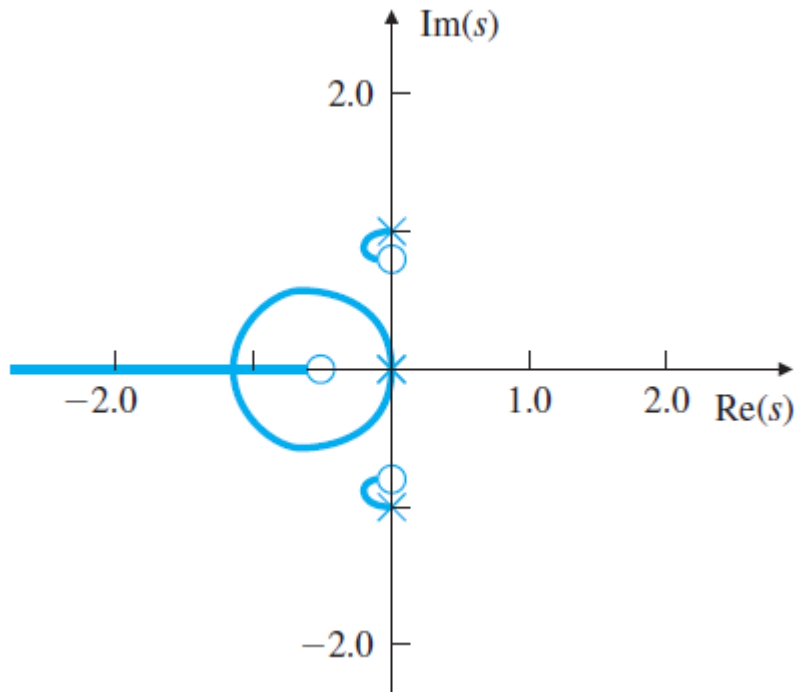


(b)



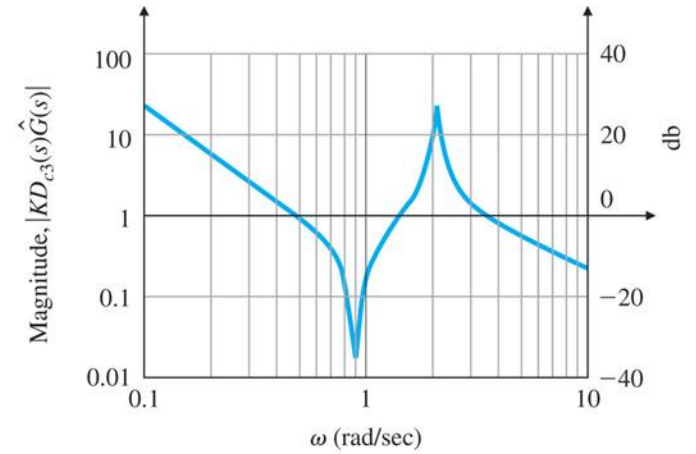
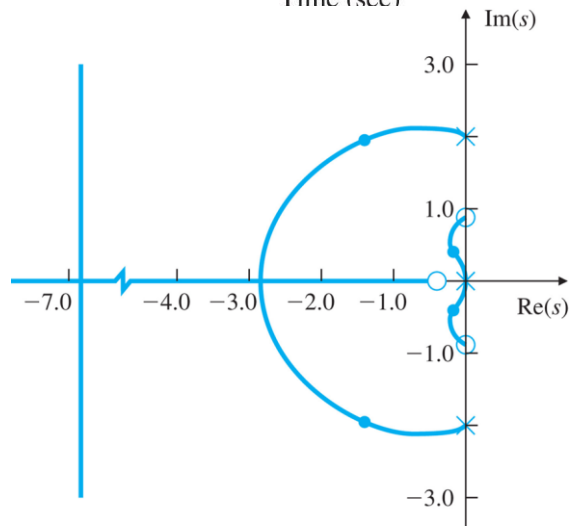
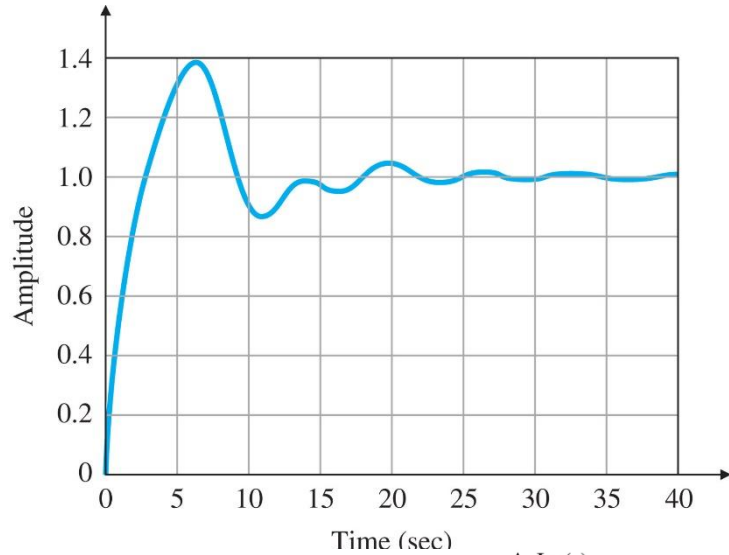
- (STEP 5) Try a lead-lag or P/D controller

- Response of  $KD_{c3}(s)G(s)$

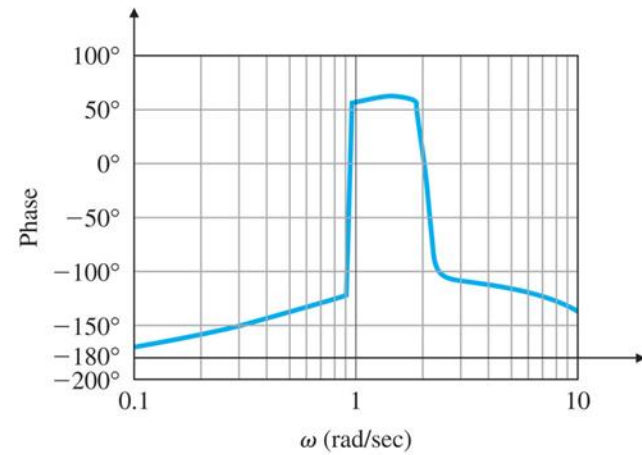


## ■ (STEP 5) Try a lead-lag or P/D controller

- Closed-loop response ( $\theta_2(0) = 0.2$ )



(a)



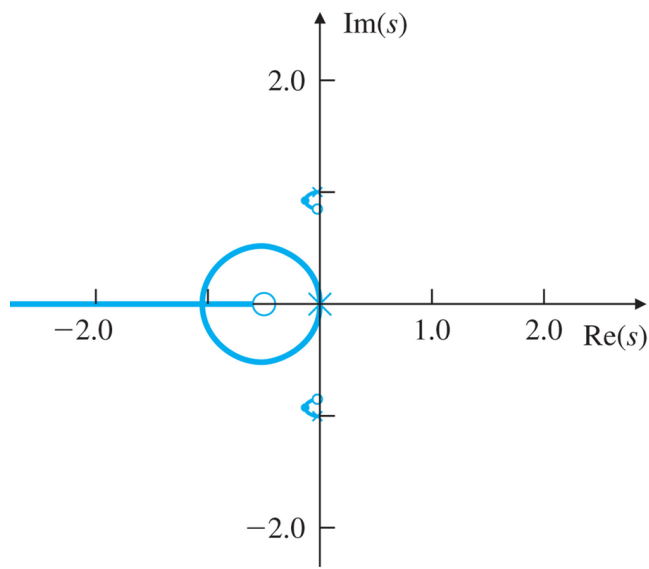
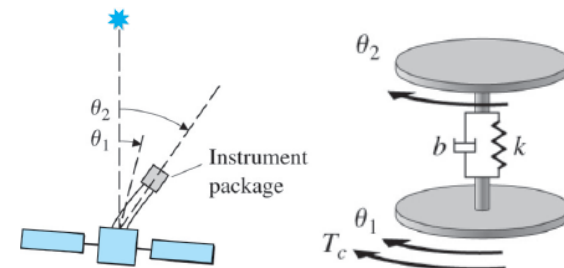
(b)

## ■ (STEP 6) Evaluate/verify the plant

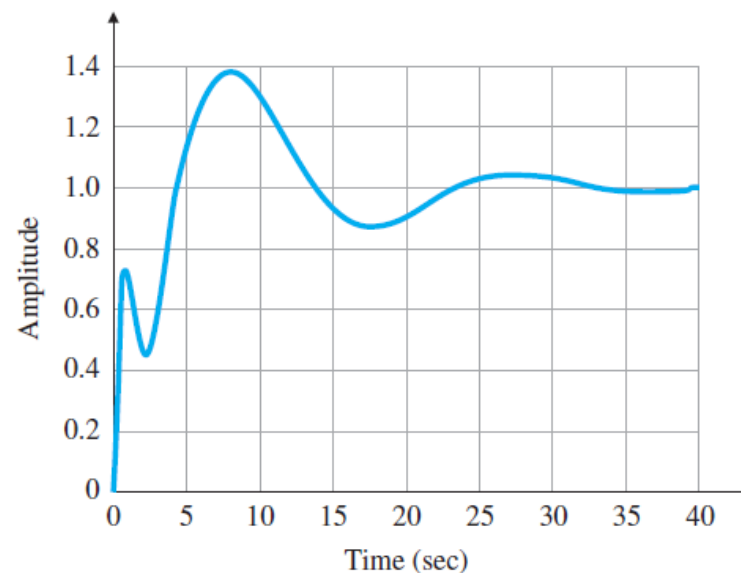
- Moving the sensor from a noncollocated position to one collocated with the actuator

$$G_0(s) = \frac{\Theta_1(s)}{T_c(s)} = \frac{(s + 0.018 \pm 0.954j)}{s^2(s + 0.02 \pm j)}$$

- For  $D_{c5}(s) = 0.25(2s+1)$ , closed-loop response of  $D_{c5}(s)G_0(s)$



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- (STEP 7)  
Try an optimal design (State space design)
- (STEP 8)  
Build a computer model, and  
compute (simulate) the performance of the design
- (STEP 9)  
Build a prototype

Equations of motion: Boeing 747

$$m(\dot{U} + qW - rV = X - mg \sin \theta + \kappa T \cos \theta$$

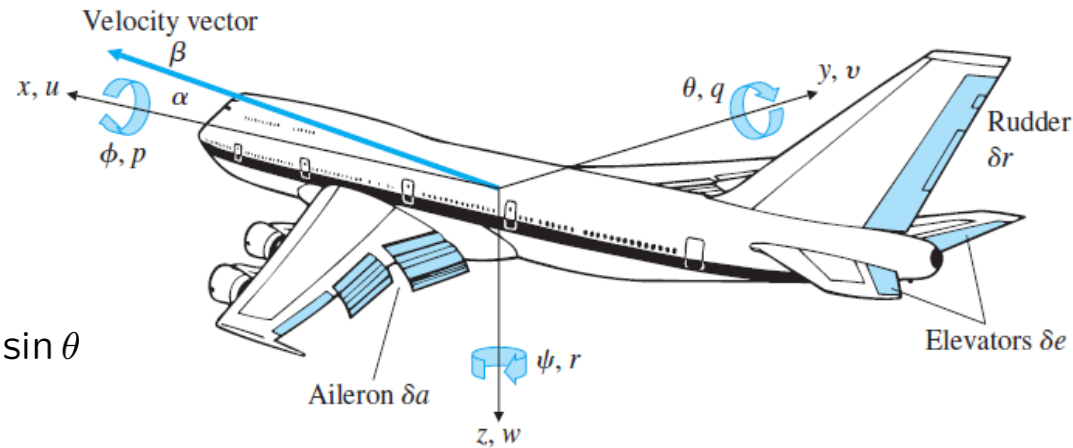
$$m(\dot{V} + rU - pW = Y + mg \cos \theta \sin \phi$$

$$m(\dot{W} + pV - qU = Z + mg \cos \theta \cos \phi - \kappa T \sin \theta$$

$$I_x \dot{p} + I_{xz} \dot{r} + (I_z - I_y)qr + I_{xz}qp = L$$

$$I_y \dot{q} + (I_x - I_z)pr + I_{xz}(r^2 - p^2) = M$$

$$I_z \dot{r} + (I_y - I_x)qp - I_{xz}qr = N$$



$x, y, z$  = position coordinates  
 $u, v, w$  = velocity coordinates  
 $p$  = roll rate  
 $q$  = pitch rate  
 $r$  = yaw rate

$\phi$  = roll angle  
 $\theta$  = pitch angle  
 $\psi$  = yaw angle  
 $\beta$  = side-slip angle  
 $\alpha$  = angle of attack

Linearization of the system:

$$\dot{U} = \dot{V} = \dot{W} = \dot{p} = \dot{q} = \dot{r} = 0$$

$$p_o = q_o = r_o = 0 \text{ (reference angular velocities)}$$

▪ **Yaw damper**

- Linearized lateral motion equation

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & -U_o & V_o & g_o \cos \theta_o \\ N_v & N_r & N_p & 0 \\ L_v & L_r & L_p & 0 \\ 0 & \tan \theta_o & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} Y_{\delta r} & Y_{\delta a} \\ N_{\delta r} & N_{\delta a} \\ L_{\delta r} & L_{\delta a} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta r \\ \delta a \end{bmatrix}$$

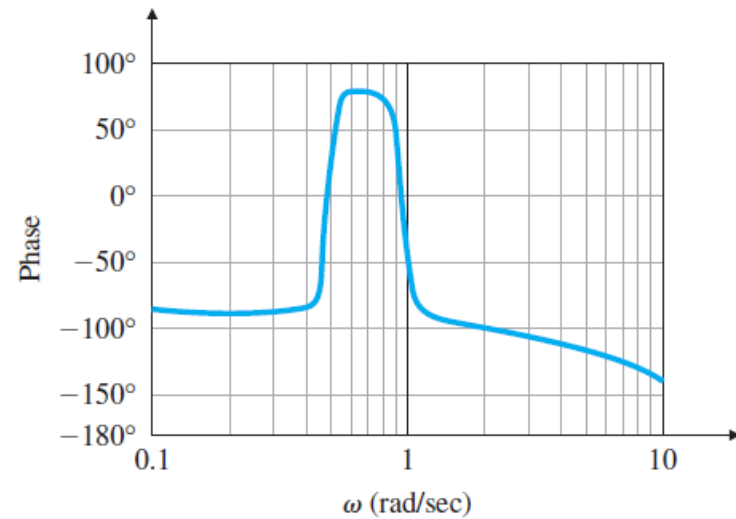
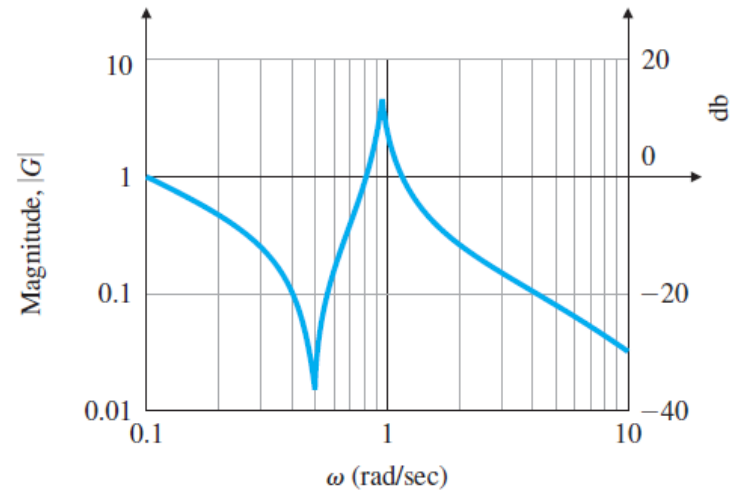
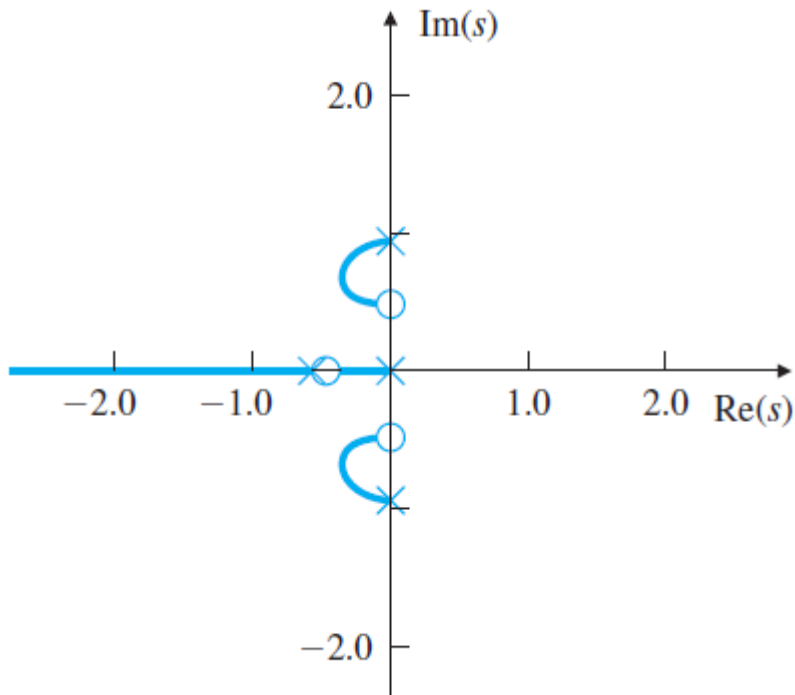
(Input:  $\delta r$  rudder, output:  $r$  yaw rate)

- (STEP 1) Understand the process and its performance specifications
  - Modify the the natural dynamics so that the plane is acceptable for the pilot to fly
  - $\omega_n \leq 0.5$  and damping ratio of  $\zeta \geq 0.5$  approximately

- (STEP 2) Select sensors
  - Measure the angular velocity (gyro)
  
- (STEP 3) Select actuators
  - Rudder
  
- (STEP 4) Make a linear model

$$G(s) = \frac{r(s)}{\delta r(s)} = \frac{-0.475(s + 0.498)(s + 0.012 \pm 0.488j)}{(s + 0.0073)(s + 0.563)(s + 0.033 \pm 0.947j)}$$

- (STEP 5) Try a lead-lag or PID design
  - For a proportional feedback

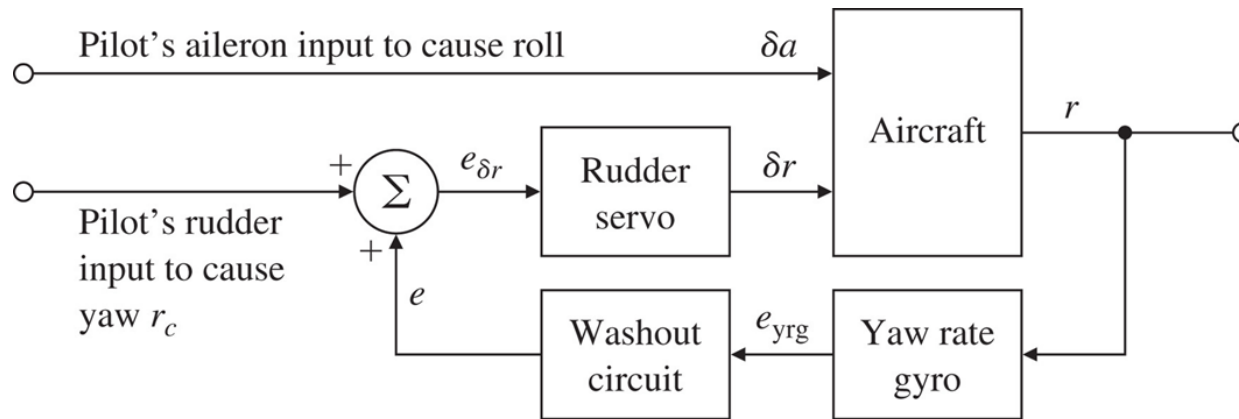


(b)

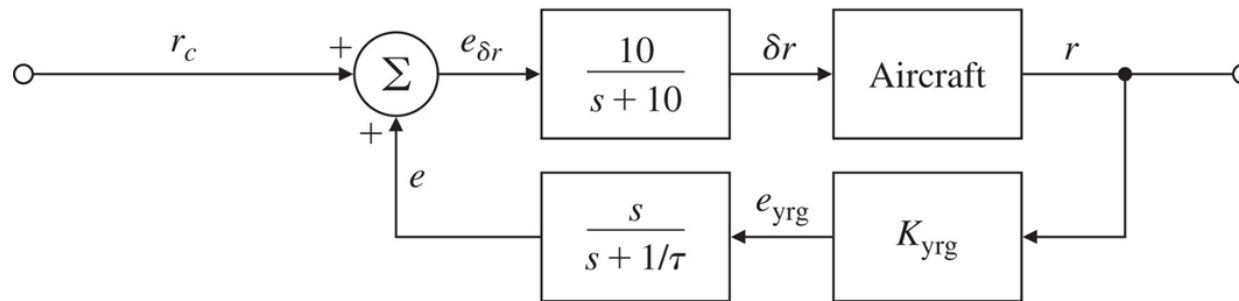


■ (STEP 5) Try a lead-lag or PID design

- Add  $H(s) = \frac{s}{1+s/\tau}$



(a)

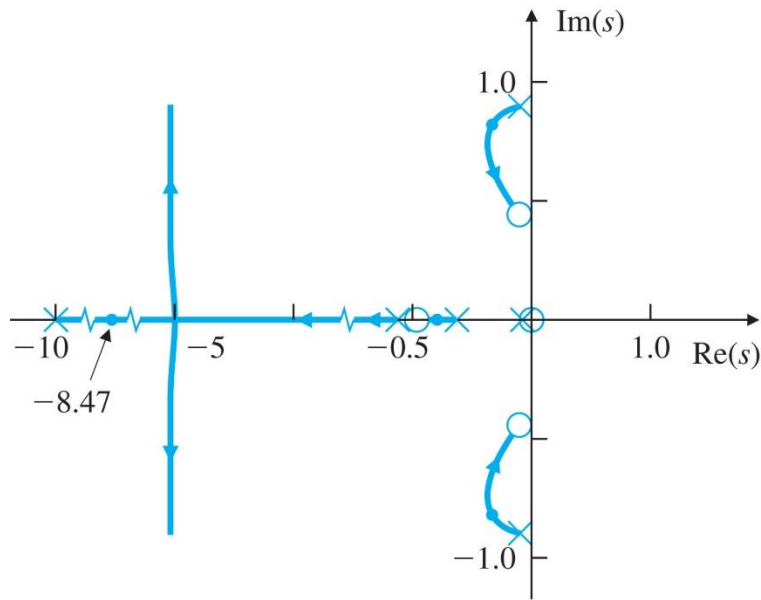


(b)

Yaw damper: (a) functional block diagram; (b) block diagram for analysis

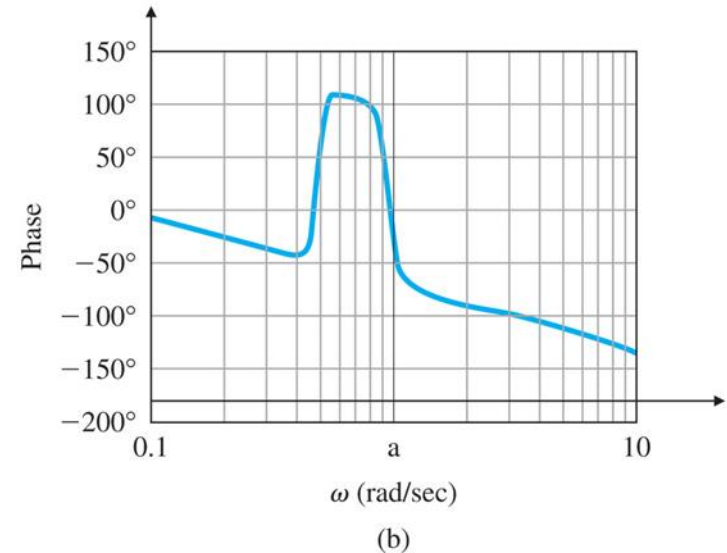
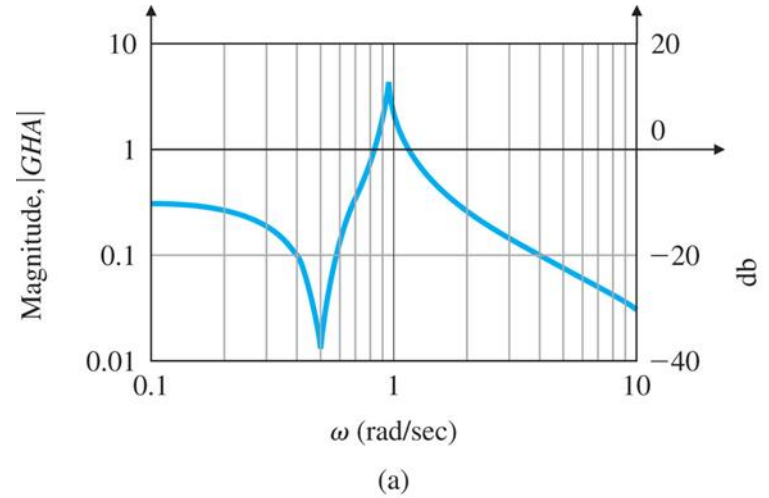
■ (STEP 5) Try a lead-lag or PID design

- With  $\tau = 3$



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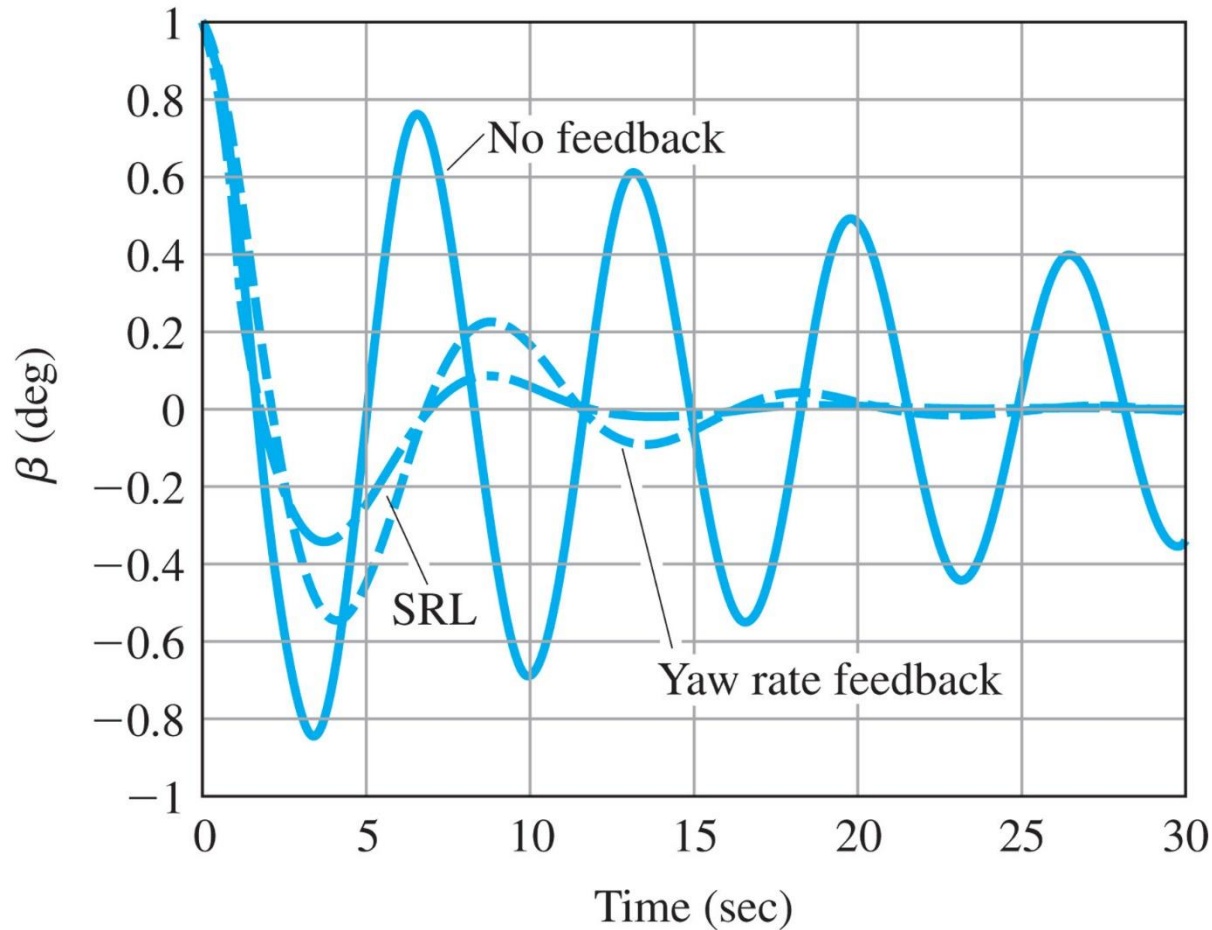
(damping ratio from 0.03 to 0.35)



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## ■ (STEP 5) Try a lead-lag or PID design

- With  $\tau = 3$



▪ **Altitude hold autopilot**

▪ **(STEP 1) Understand the process and its performance specifications**

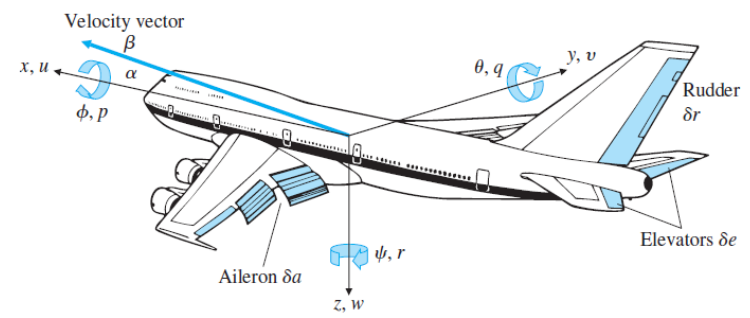
- The design should provide the kind of ride that pilot and passenger like
- Damping ratio  $\zeta \approx 0.5$
- The natural frequency should be “much” less than  $\omega_n = 1$

▪ **(STEP 2) Select sensors**

- Measure the altitude

▪ **(STEP 3) Select actuators**

- Elevator  $\delta e$

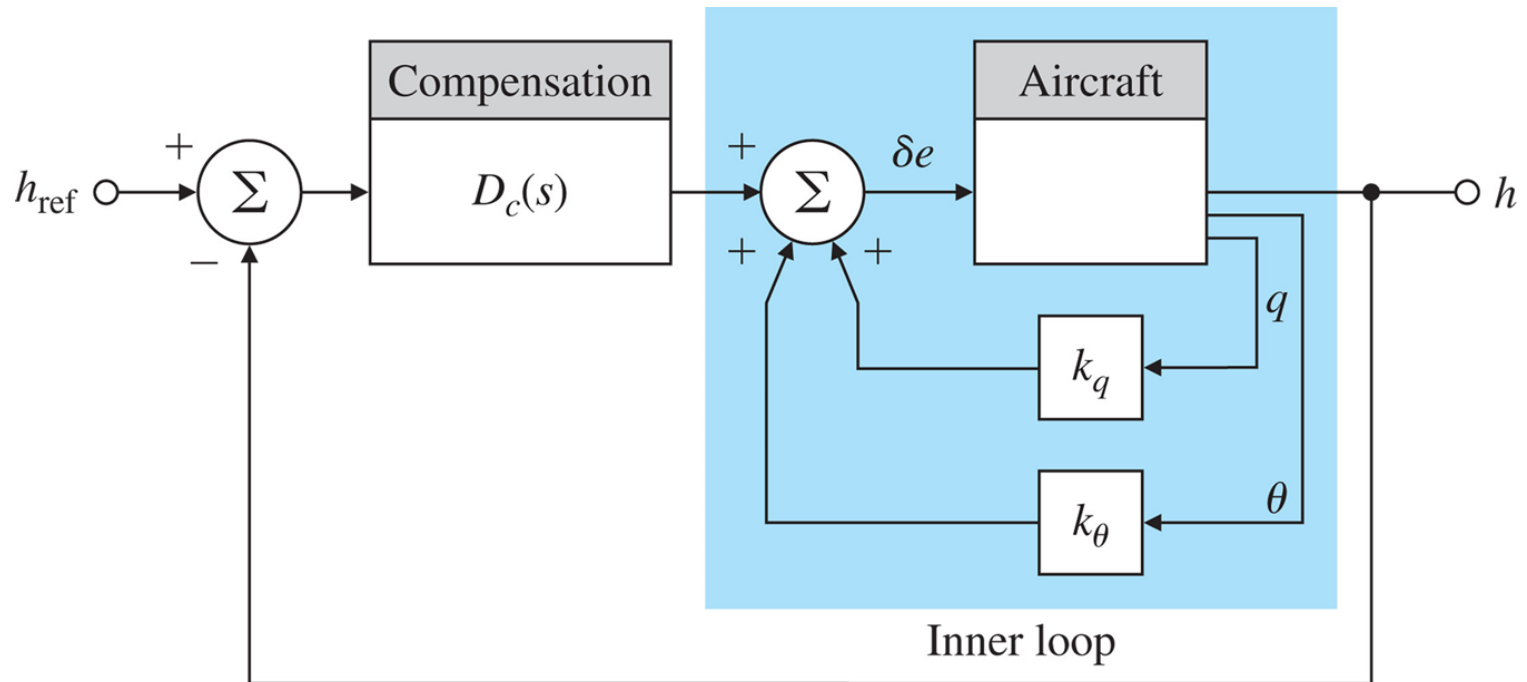


|                                  |                            |
|----------------------------------|----------------------------|
| $x, y, z =$ position coordinates | $\phi =$ roll angle        |
| $u, v, w =$ velocity coordinates | $\theta =$ pitch angle     |
| $p =$ roll rate                  | $\psi =$ yaw angle         |
| $q =$ pitch rate                 | $\beta =$ side-slip angle  |
| $r =$ yaw rate                   | $\alpha =$ angle of attack |

- (STEP 4) Make a linear model

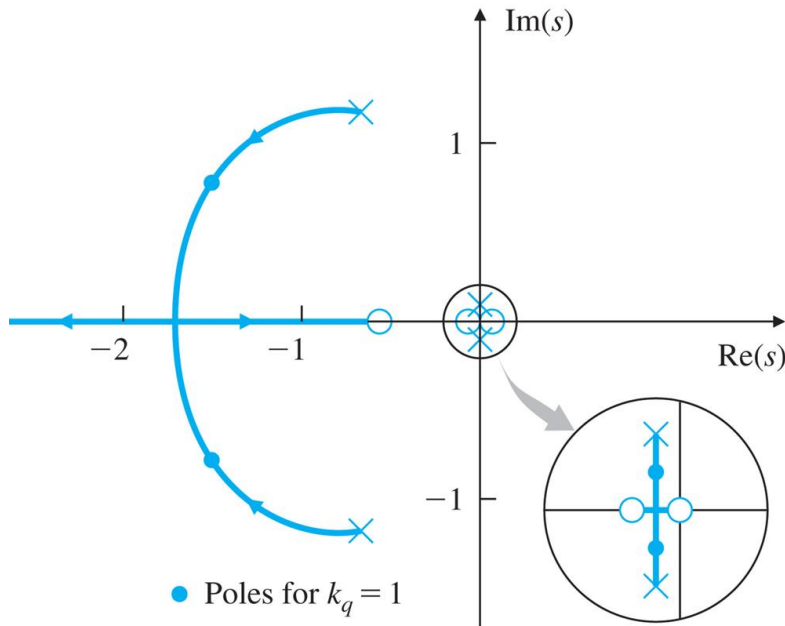
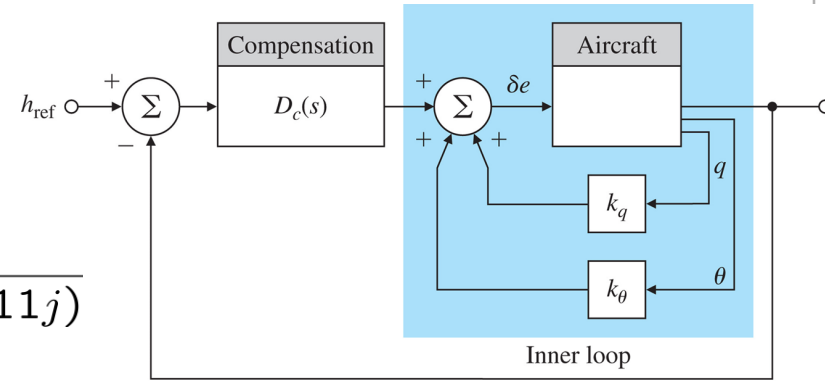
$$\frac{h(s)}{\delta e(s)} = \frac{32.7(s + 0.0045)(s + 5.645)(s - 5.61)}{s(s + 0.003 \pm 0.0098j)(s + 0.6463 \pm 1.1211j)}$$

- (STEP 5) Try a lead-lag or PID design



- (STEP 5) Try a lead-lag or PID design

$$\frac{h(s)}{\delta e(s)} = \frac{2.08s(s + 0.0105)(s + 0.596)}{(s + 0.003 \pm 0.0098j)(s + 0.646 \pm 1.1211j)}$$



if  $k_q = 1$ , poles are  
 $-0.0039 \pm 0.0067j, -1.683 \pm 0.277j$

■ (STEP 5) Try a lead-lag or PID design

- root locus with feedback of  $h$  and derivative of  $h$

