

Spring 2021

控制系統
Control Systems

Unit 6K
PID Compensation

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NTU-EE

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■ PID Compensation

- Need PM improvement at ω_c
- Need low-frequency gain improvement

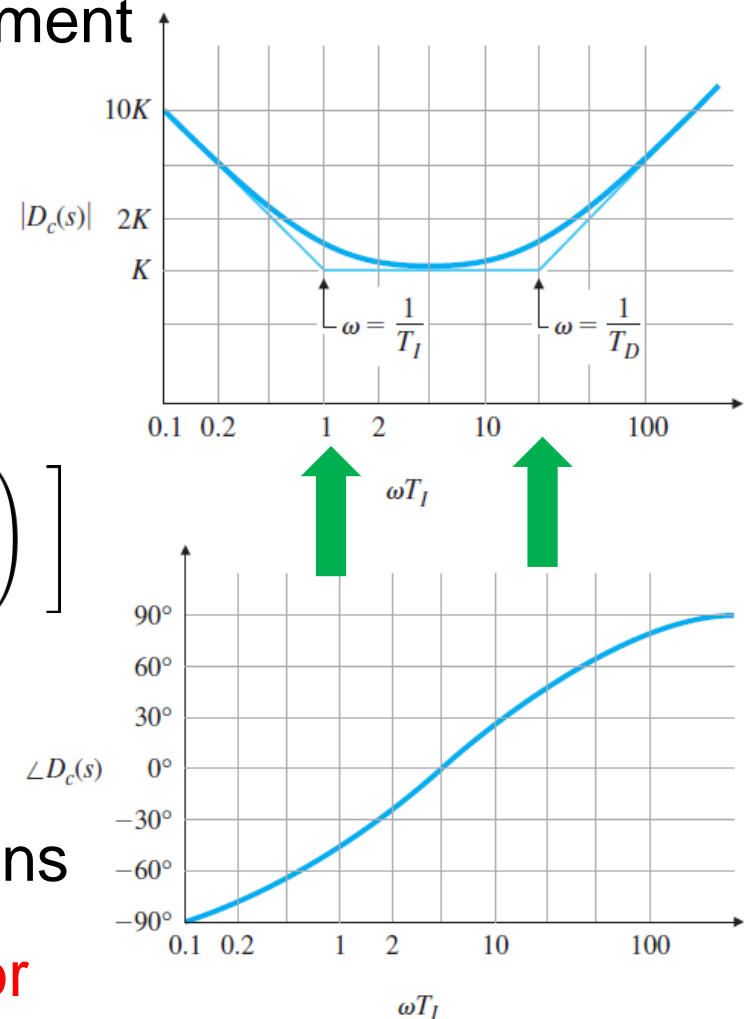
■ A common transfer function:

$$D_c(s) = k_P + \frac{k_I}{s} + k_D s$$

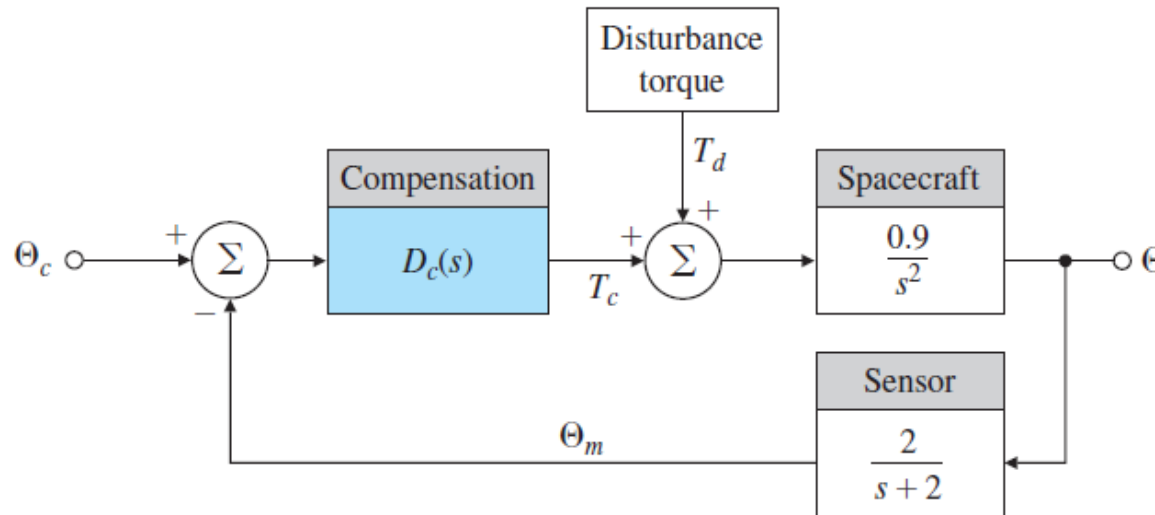
$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right]$$

■ Roughly equivalent to

combining lead and lag compensations referred to as a lead-lag compensator



Example 6.20: PID Compensation for Spacecraft Attitude Control



- Design PID controller for:
 - Zero steady-state error
 - $PM = 65^\circ$
 - As high a bandwidth as possible
 - Torque disturbance $\omega = 0.001 \text{ rad/sec}$

Example

- Example 6.20: PID Compensation
for Spacecraft Attitude Control

- For Final Steady Value

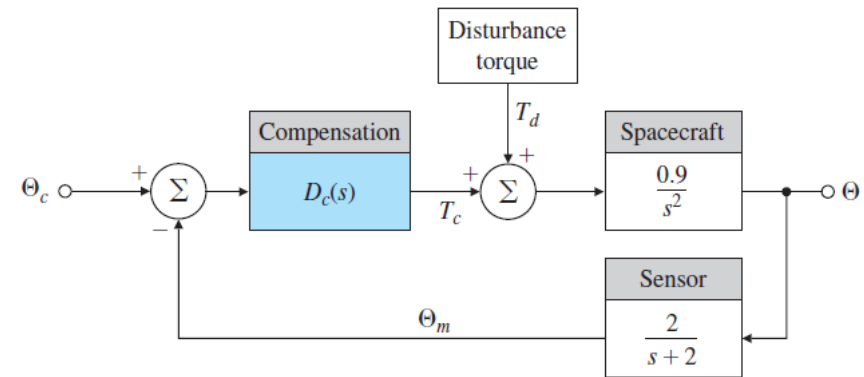
$$\rightarrow T_d + T_c = 0$$

- Therefore, if $T_d \neq 0$

$$\rightarrow T_c = -T_d$$

- The only way this can be true with no error ($e = 0$) is:

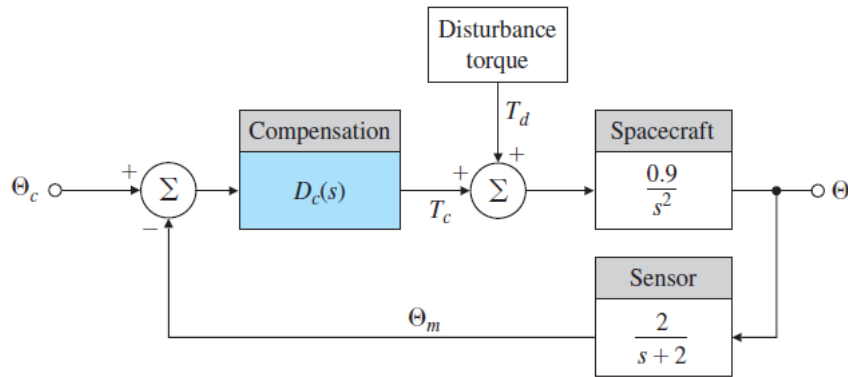
for $D_c(s)$ to contain an integral term.



Example

Example 6.20: PID Compensation

for Spacecraft Attitude Control

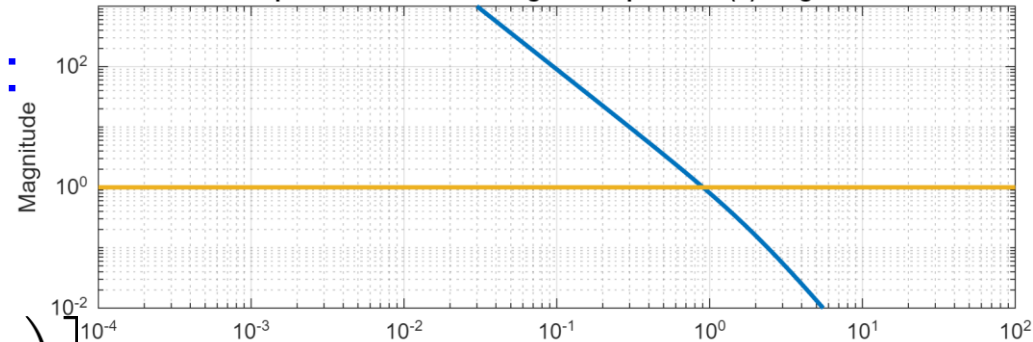


$$G(s) = \frac{0.9}{s^2}$$

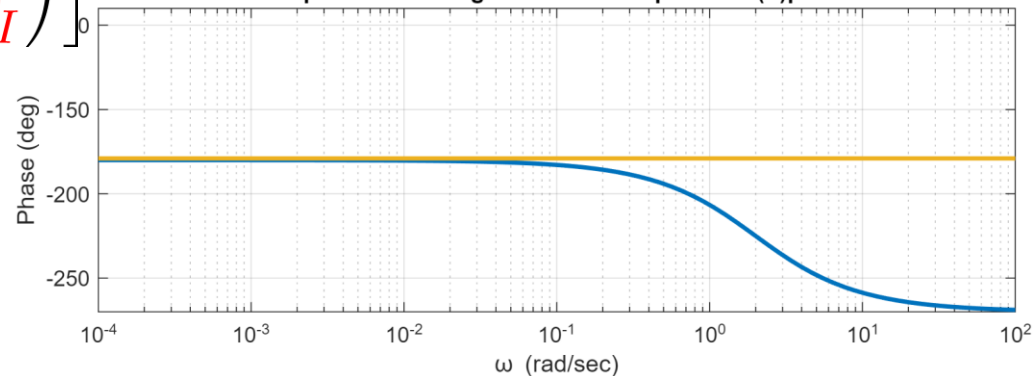
$$H(s) = \frac{2}{s+2}$$

- Frequency Response of GH:
- Unstable

Compensation for PID design Example 6.20: (a) magnitude.



Compensation design for PID Example 6.20: (b) phase.



$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right]$$

- Select: K , T_D , T_I
- PM = 65° at high frequency
 - By adjusting T_D

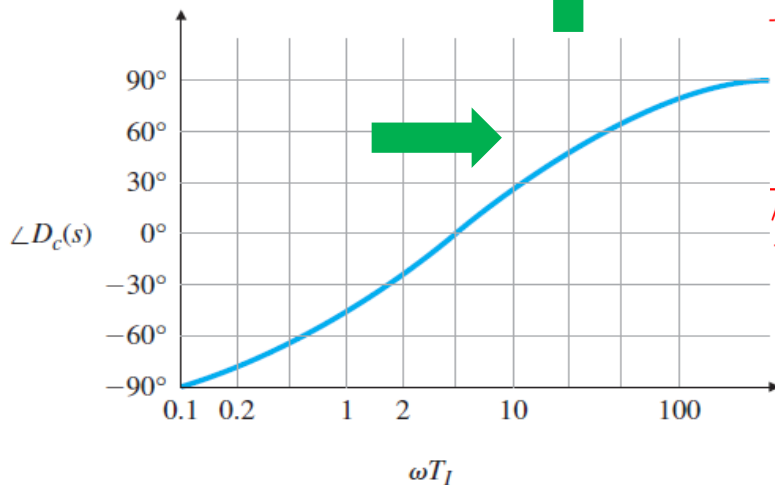
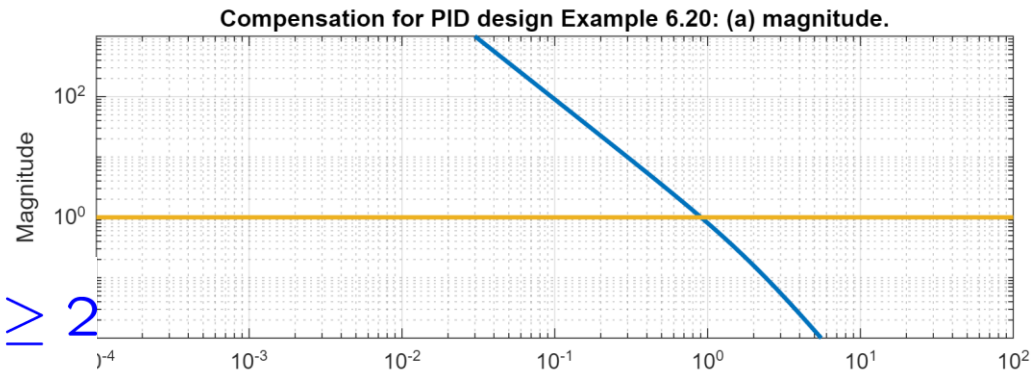
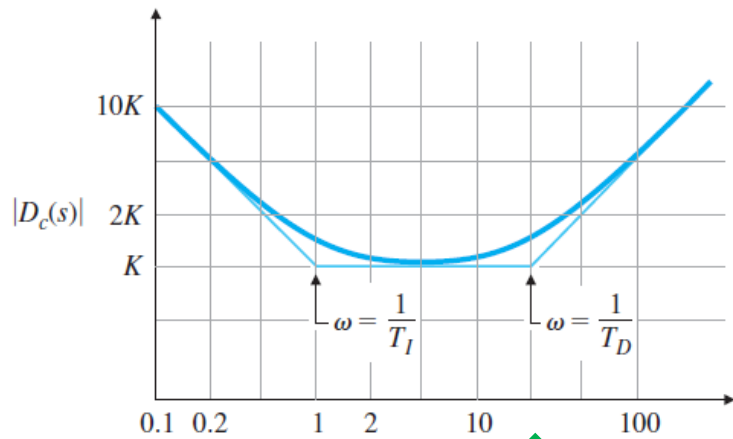
Example

Example 6.20: PID Compensation

$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

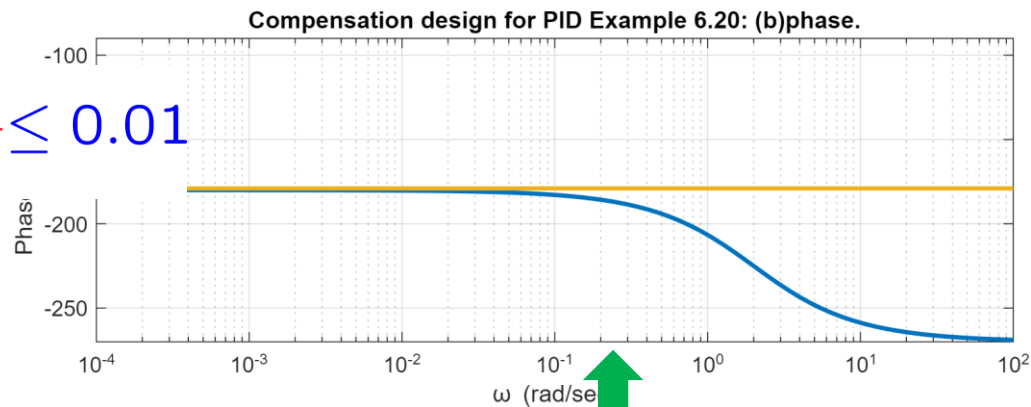
$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s + 2}$$



$$\frac{1}{T_D} \geq 2$$

$$\frac{1}{T_D} \leq 0.01$$



Example

Example 6.20: PID Compensation

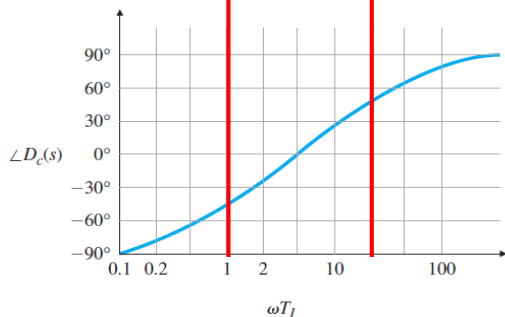
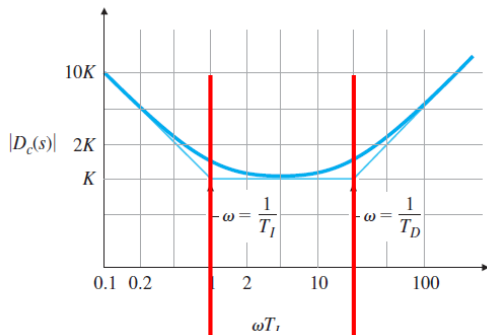
$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

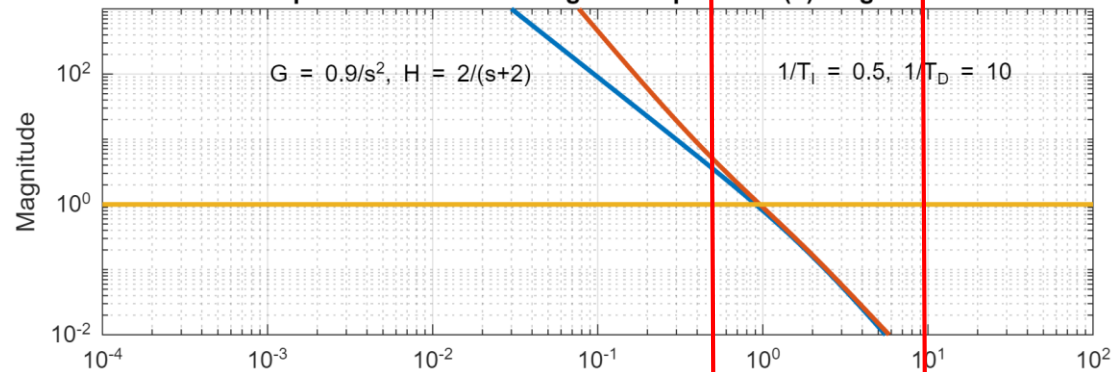
$$H(s) = \frac{2}{s + 2}$$

$$\frac{1}{T_I} = 0.5$$

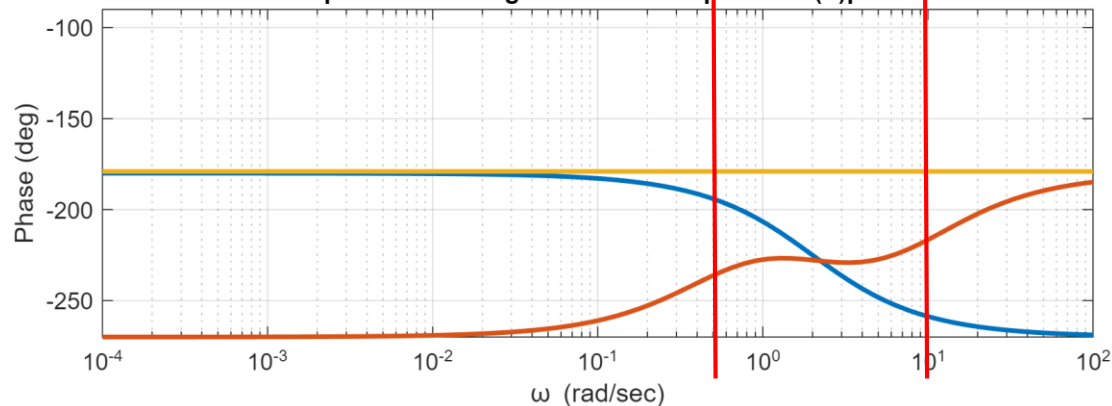
$$\frac{1}{T_D} = 10$$



Compensation for PID design Example 6.20: (a) magnitude.



Compensation design for PID Example 6.20: (b) phase



Example

Example 6.20: PID Compensation

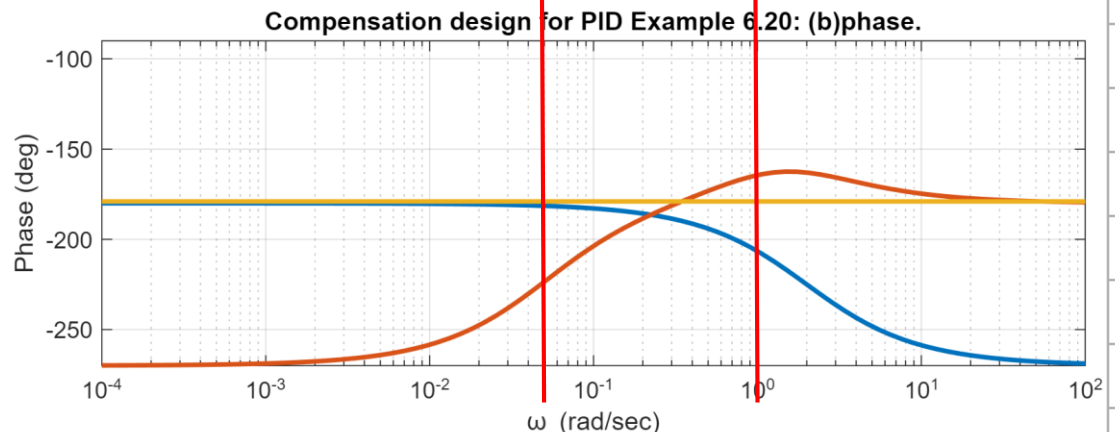
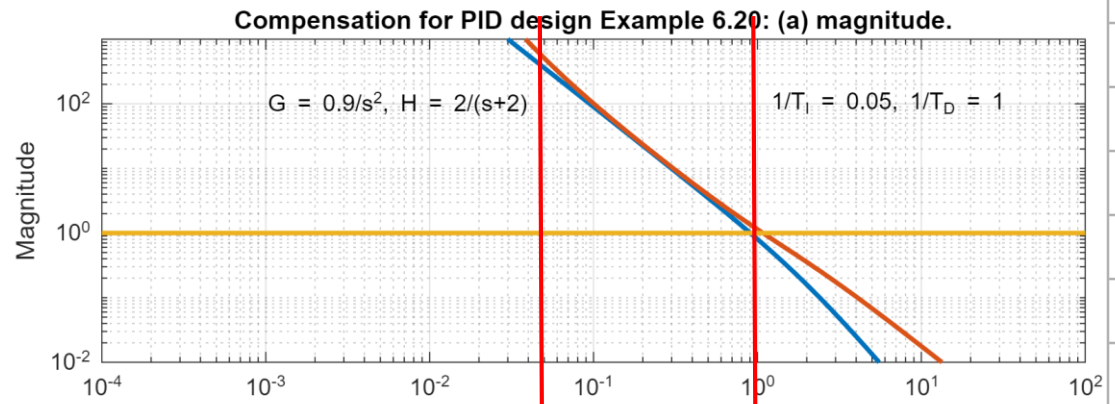
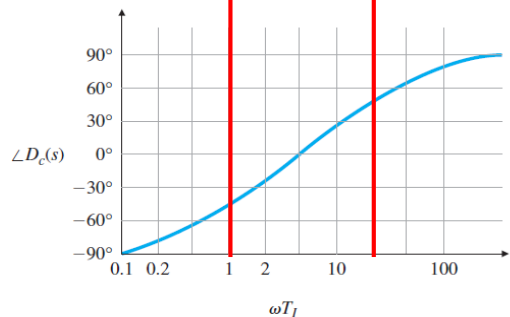
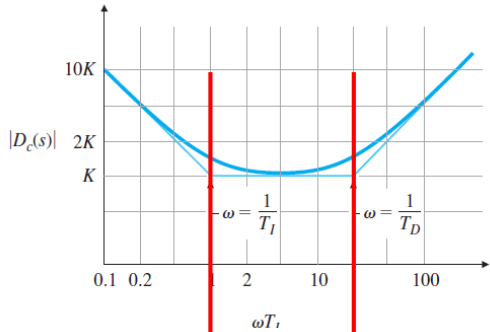
$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s + 2}$$

$$\frac{1}{T_I} = 0.05$$

$$\frac{1}{T_D} = 1$$



Example

Example 6.20: PID Compensation

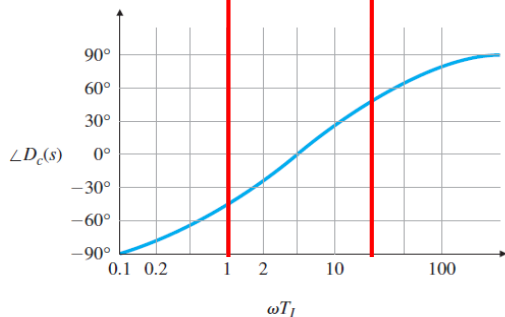
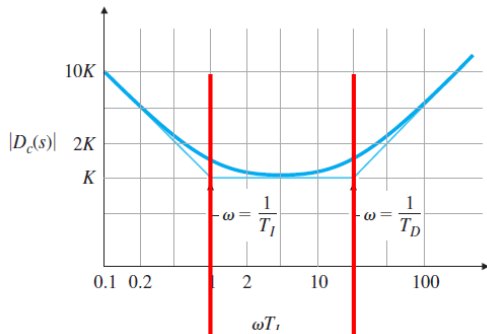
$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

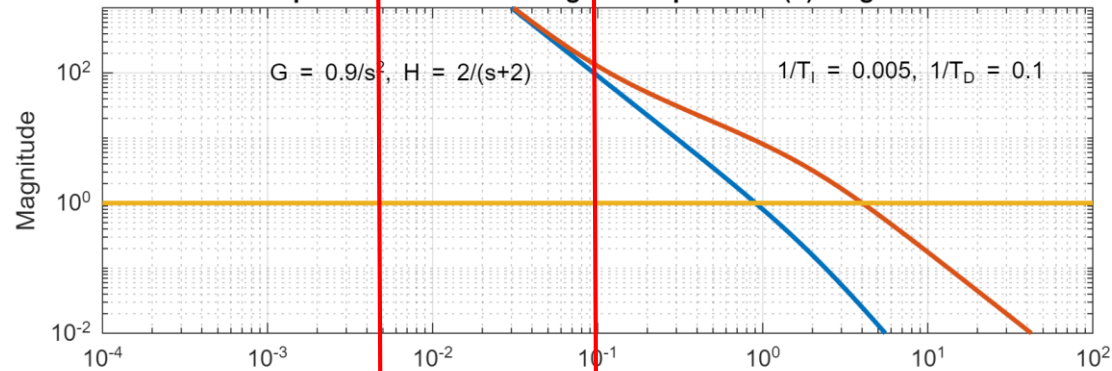
$$H(s) = \frac{2}{s + 2}$$

$$\frac{1}{T_I} = 0.005$$

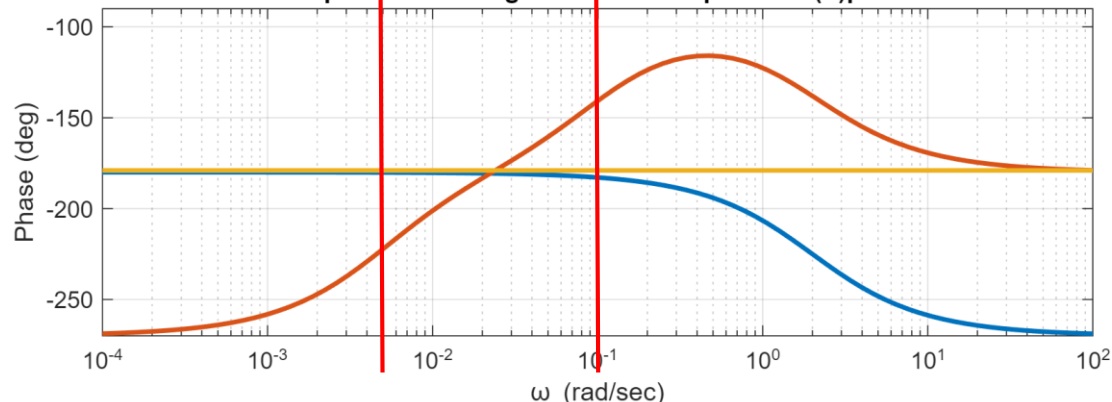
$$\frac{1}{T_D} = 0.1$$



Compensation for PID design Example 6.20: (a) magnitude.



Compensation design for PID Example 6.20: (b) phase.



Example

Example 6.20: PID Compensation

$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

$$H(s) = \frac{2}{s + 2}$$

$\frac{1}{T_I} = 0.5$	$\frac{1}{T_D} = 10$
$\frac{1}{T_I} = 0.05$	$\frac{1}{T_D} = 1$
$\frac{1}{T_I} = 0.005$	$\frac{1}{T_D} = 0.1$

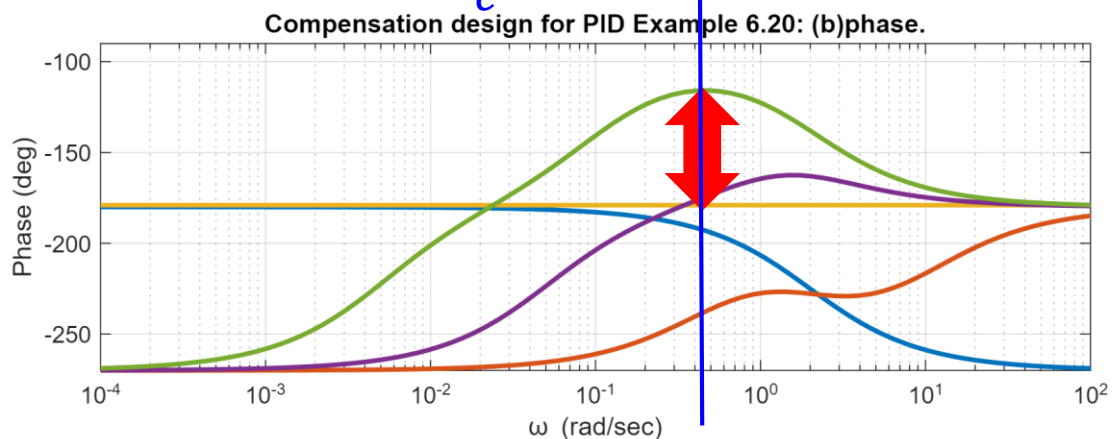
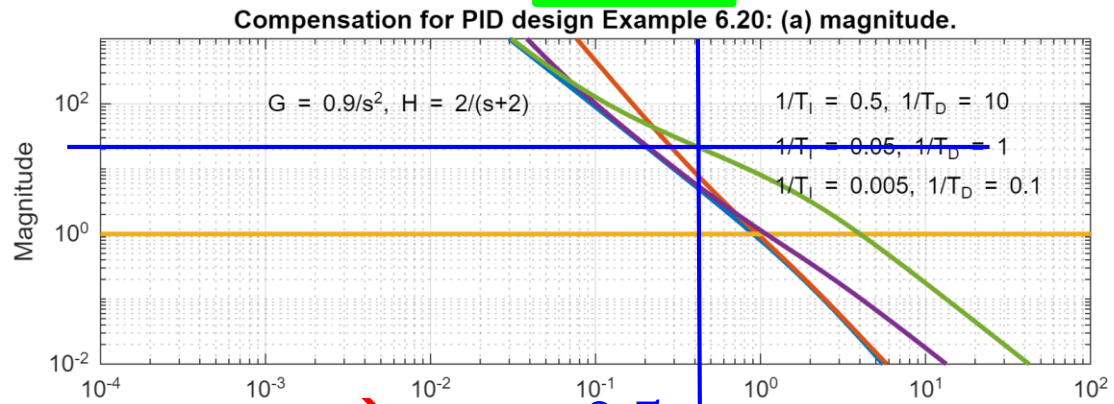
PM = 65°

→ Find K

| D_c(s) G(s) | = 20

1/K = 20

K = 0.05



Example 6.20: PID Compensation

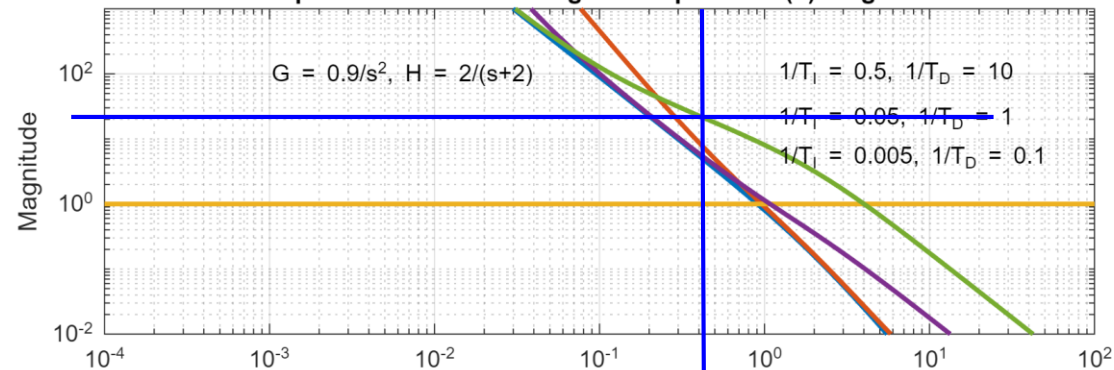
$$D_c(s) = \frac{K}{s} \left[(T_D s + 1) \left(s + \frac{1}{T_I} \right) \right] \quad \text{for Spacecraft Attitude Control}$$

$$G(s) = \frac{0.9}{s^2}$$

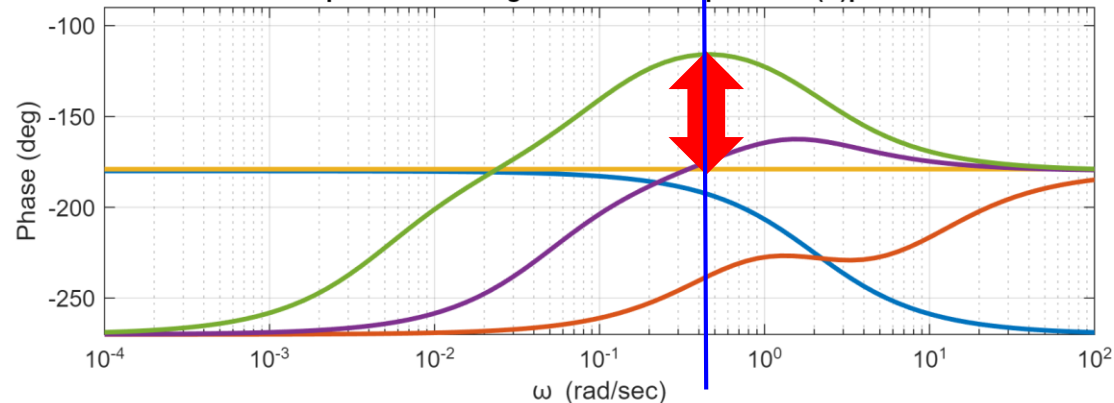
$$H(s) = \frac{2}{s + 2}$$

$$= \frac{0.05}{s} \left[(10s + 1) \left(s + \frac{1}{0.005} \right) \right]$$

Compensation for PID design Example 6.20: (a) magnitude.



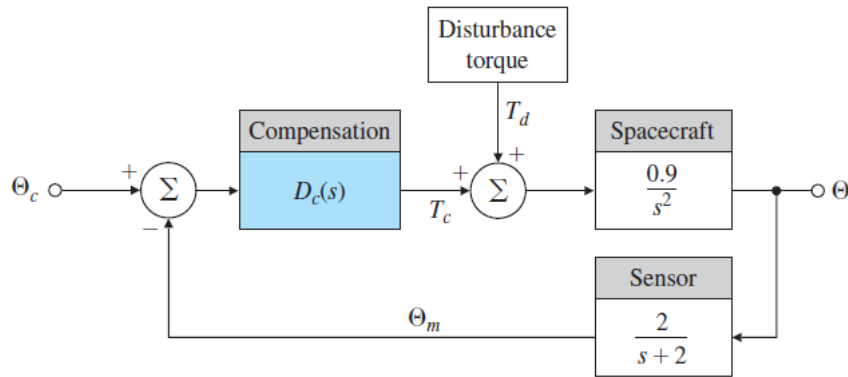
Compensation design for PID Example 6.20: (b) phase.



Example

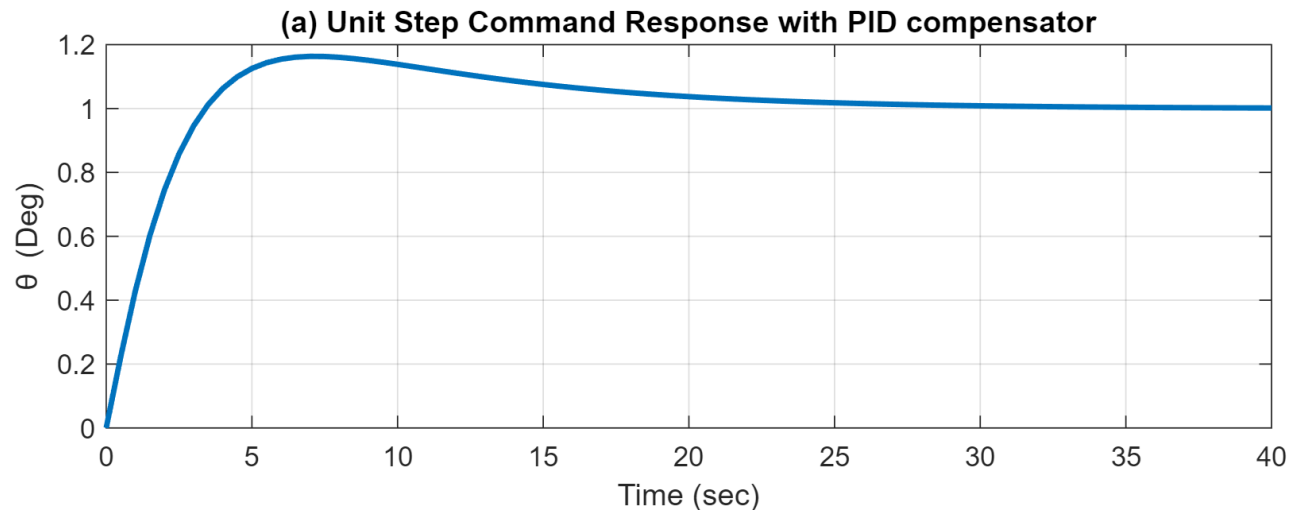
Example 6.20: PID Compensation

for Spacecraft Attitude Control



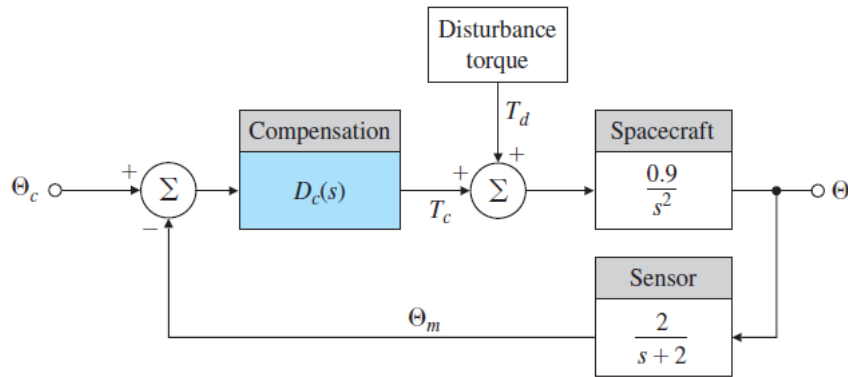
- Response of the system for a **unit step** θ_{com} is found from:

$$\mathcal{T}(s) = \frac{\Theta}{\Theta_{com}} = \frac{D_c G}{1 + D_c G H}$$



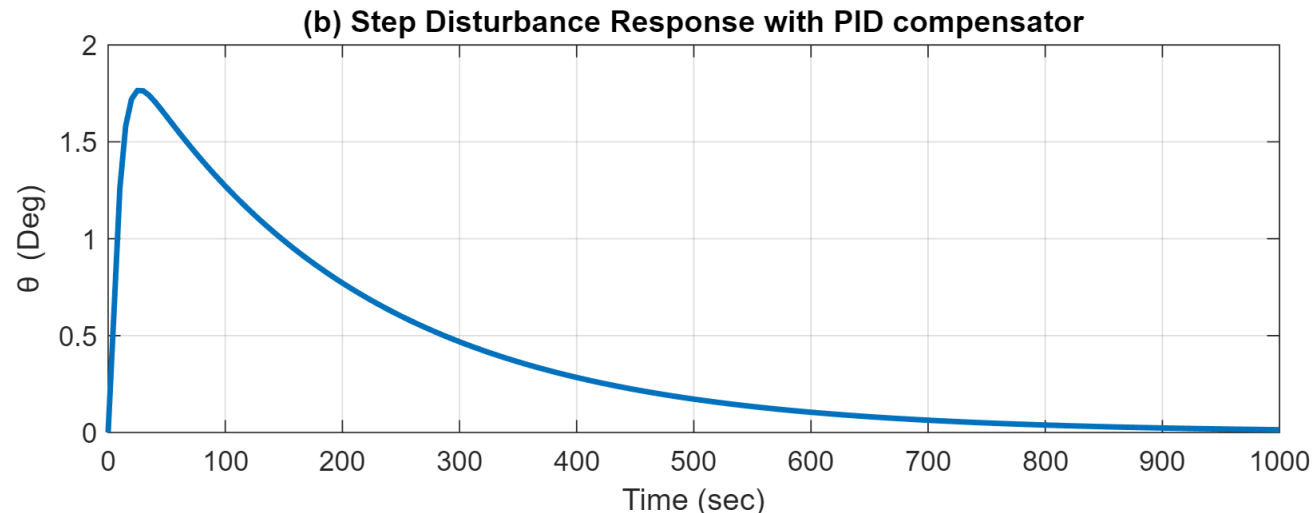
Example 6.20: PID Compensation

for Spacecraft Attitude Control



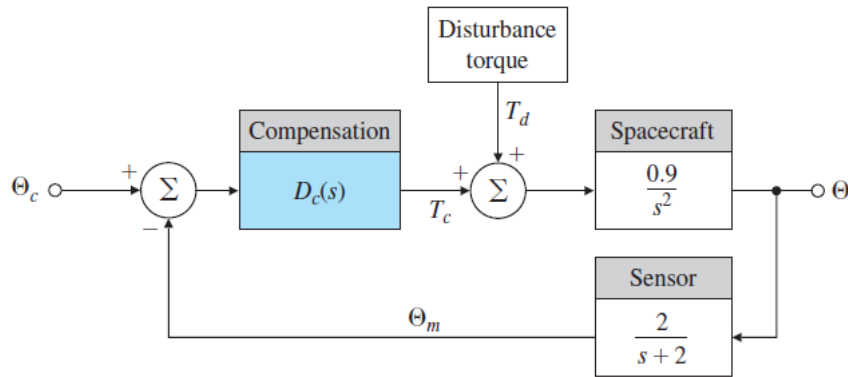
Response for a step disturbance torque of T_d is found from:

$$\frac{\Theta}{T_d} = \frac{G}{1 + D_c G H}$$



Example 6.20: PID Compensation

for Spacecraft Attitude Control



- Frequency Response of $T(s)$ and $S(s)$ are shown:

