

Spring 2021

控制系統
Control Systems

Unit 6J
PI Compensation and Lag Compensation

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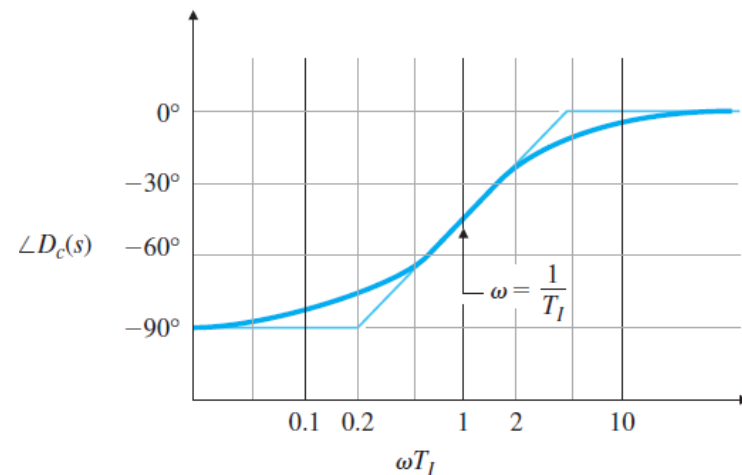
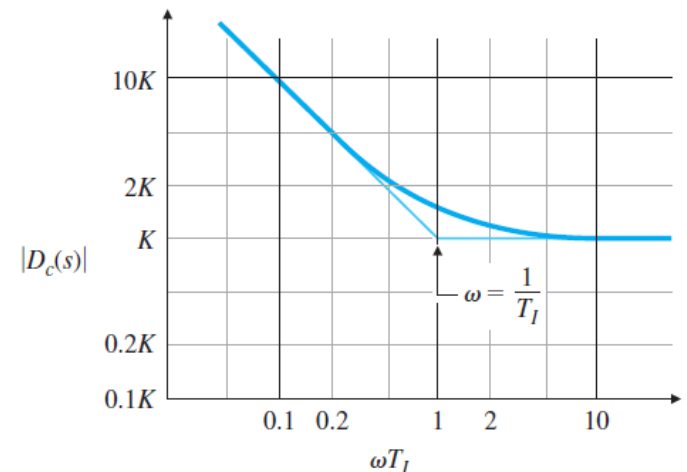
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- In many problems,
 - it is important to **keep the bandwidth low**
 - and also to **reduce the steady-state error**
- For this purpose,
 - a **PI Controller**
 - or **Lag Compensator** is useful
- **PI Controller:**

$$D_c(s) = K \left(1 + \frac{1}{T_I} \frac{1}{s} \right)$$

$$= \frac{K}{s} \left(s + \frac{1}{T_I} \right)$$



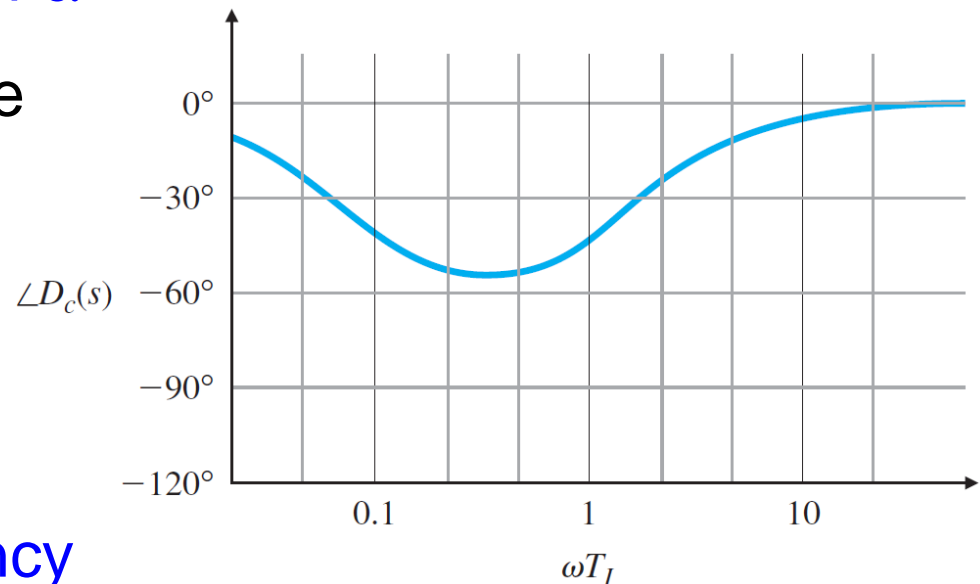
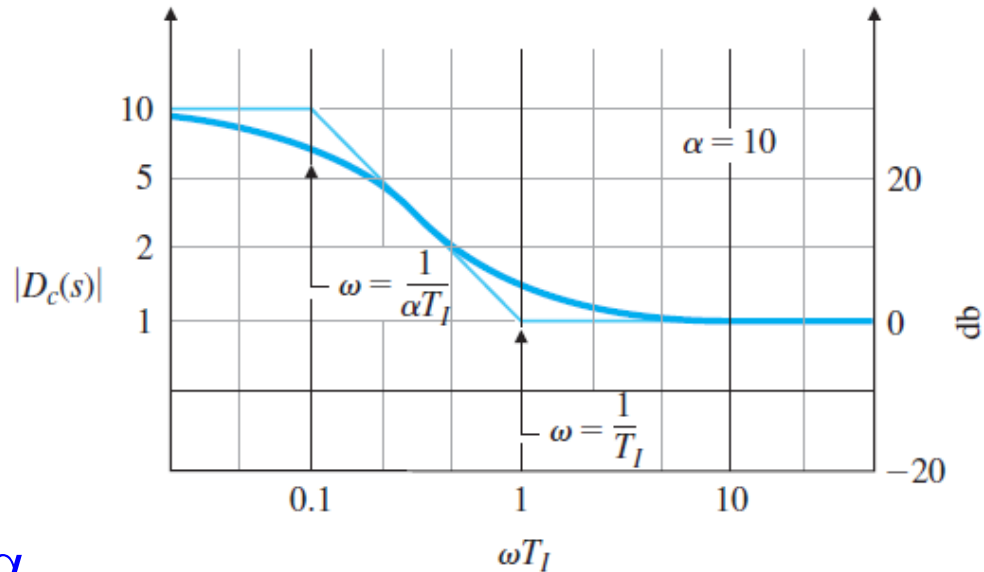
$$D_c(s) = \alpha \frac{T_I s + 1}{\alpha T_I s + 1}, \quad \alpha > 1$$

■ Low frequency:

- Amplitude: **increase**
- Phase: **decrease**

■ Features:

- Provide **additional gain of α**
in low-frequency range
- Leave the system
sufficient PM
- Put pole/zero
at **much lower frequency**



1. Determine **OL gain K** that meet the **PM** requirements
2. Draw the **Bode Plot** of the uncompensated system with **crossover frequency** from Step 1, and evaluate the **low-frequency gain**
3. Determine **α** to meet **low-frequency gain error** requirement
4. Choose the corner frequency **$\omega = 1/T_I$** , (the zero) to be **one octave to one decade** below new **ω_c**
5. The other corner frequency **$\omega = 1/\alpha T_I$** , (the pole)
6. Iterate on the design. Adjust poles/zeros/gain.

Example 6.18: Lag-Compensation Design for Temperature Control System

$$K G(s) = \frac{K}{\left(\frac{s}{0.5} + 1\right) \left(\frac{s}{1} + 1\right) \left(\frac{s}{2} + 1\right)}$$

- $K_p = 9$
- $PM > 40^\circ$

▪ (1)

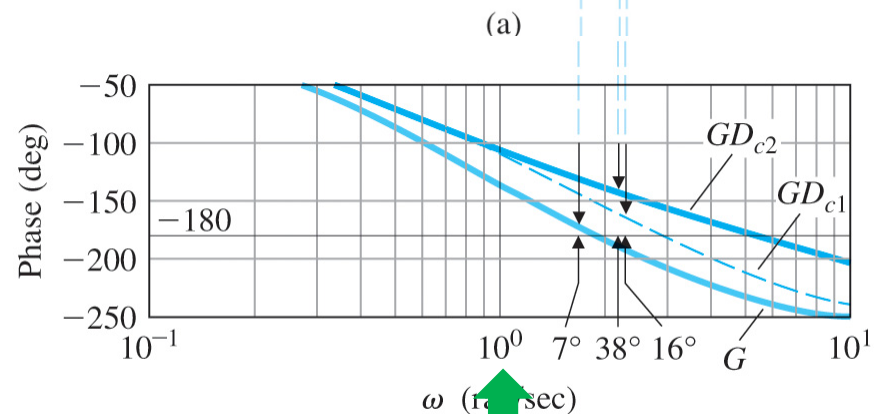
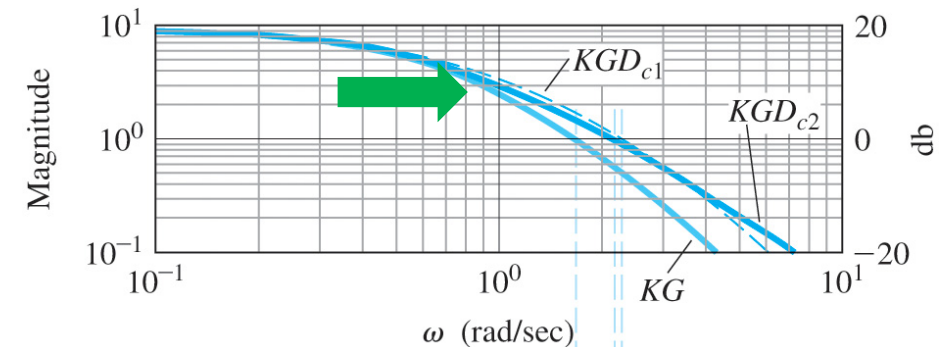
In Ex. 6.16, $K = 9$

▪ For $PM > 40^\circ$,

at $\omega_c \sim < 1$

▪ $Mag \sim = 3$

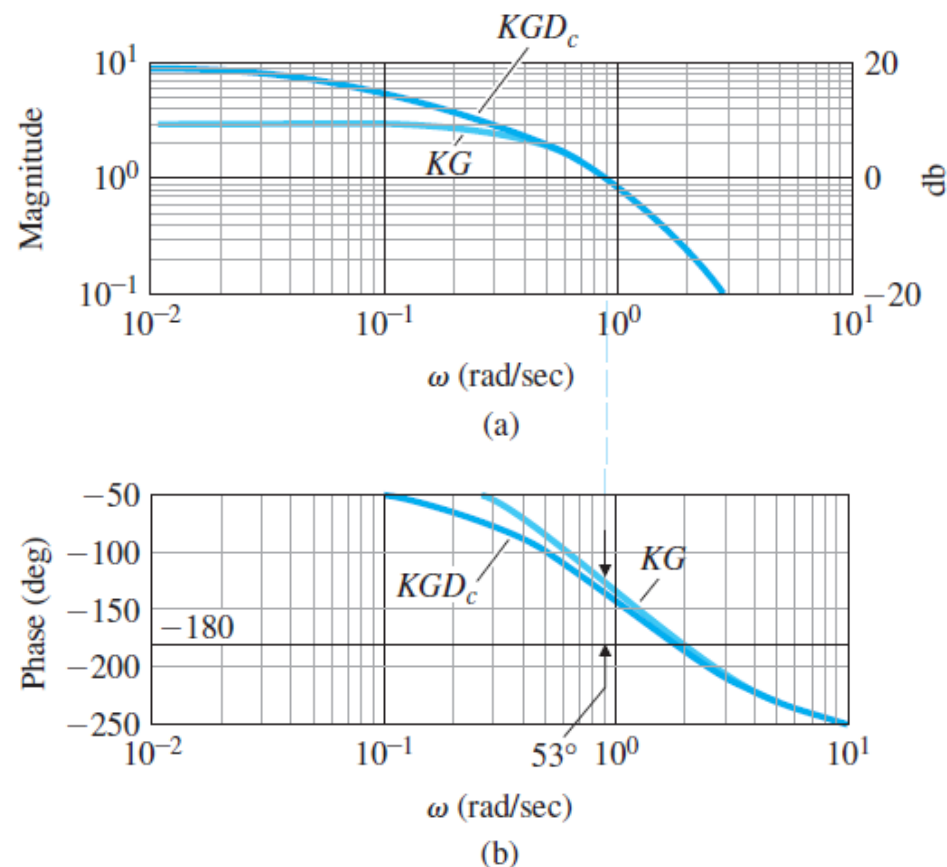
▪ $K = 3$



Examples

Example 6.18: Lag-Compensation Design for Temperature Control System

- (2)
- $K = 3$
- $PM \approx 50^\circ$
- Low-frequency gain = 3
- (3)
- The low-frequency gain should be raised by a factor of 3,
- \rightarrow the lag compensation needs to have $\alpha = 3$



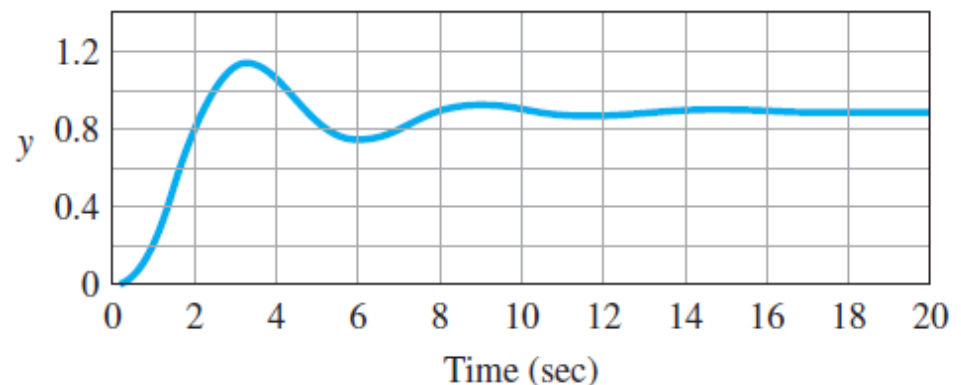
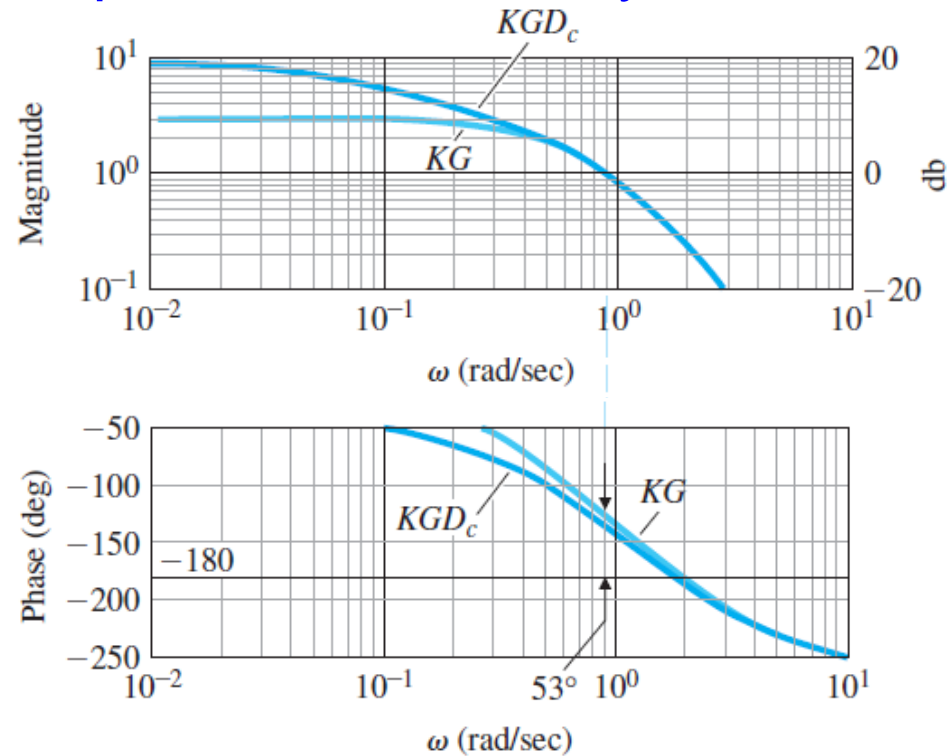
Examples

Example 6.18: Lag-Compensation Design for Temperature Control System

- (4)
- Zero: $1/5 = 0.2$
- $\rightarrow 1/T_I = 0.2$, or $T_I = 5$
- (5)
- $\omega = 1/\alpha T_I = 1 / (3 \times 5) = 1/15$

$$D_c(s) = 3 \frac{5s + 1}{15s + 1}$$

- $K D_I(0)G(0) = 3K = 9$
- $K_p = 9$, $PM = 44^\circ$



Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

▪ $K = 10$

▪ $PM = 20^\circ$ at $\omega_c \approx 3$

▪ Select break points

✓ ω_c is lowered

✓ more favorable PM results

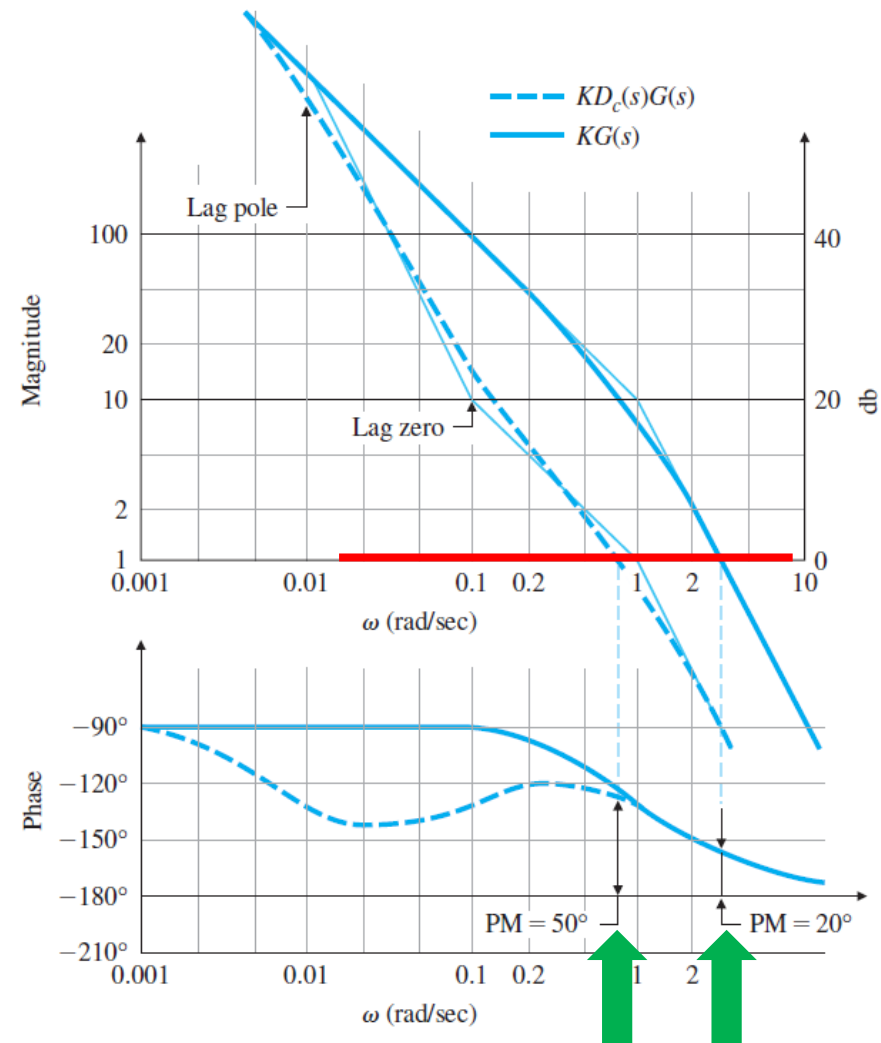
➤ Lag zero = 0.10

➤ Lag pole = 0.01

✓ $PM = 50^\circ$

▪ Error constant: $K_v = 10$

▪ $PM = 45^\circ$

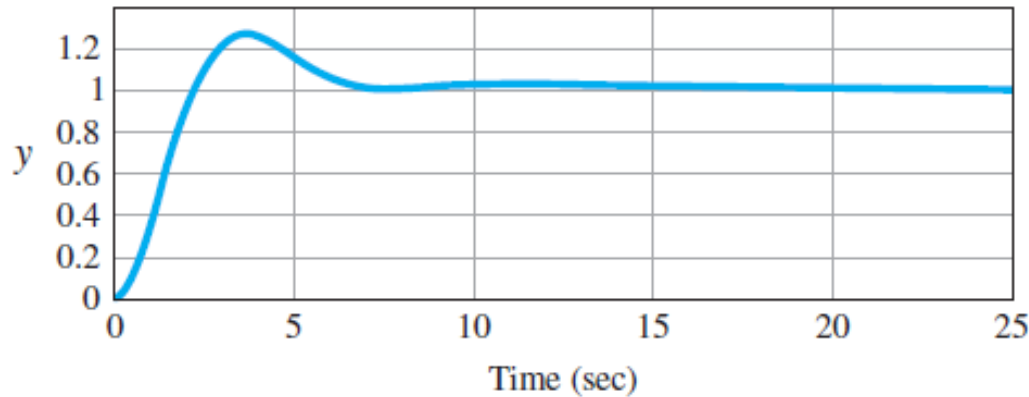


- Example 6.19: Lag Compensation for the DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- Error constant: $K_v = 10$

- PM = 45°



- No steady-state error
 - ✓ a Type 1 system
- Settling time ≈ 25 sec
- Rise time ≈ 2 sec

- Example 6.15: Lead Compensation

