控制系統
Control Systems

Unit 6I
PD Compensation and Lead Compensation

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- **Dynamic elements** (or **compensation**)

  are typically added to feedback controllers

- To improve the system's stability and error characteristics

- Because the process itself cannot be made

  to have acceptable characteristics

  with proportional feedback alone.

- Unit 43:

  - 3 basic types of feedback: P, I, D.

- Unit 54:

  - 3 kinds of dynamic compensation: Lead-PD, Lag-PI, Notch.

▪ The closed-loop system:
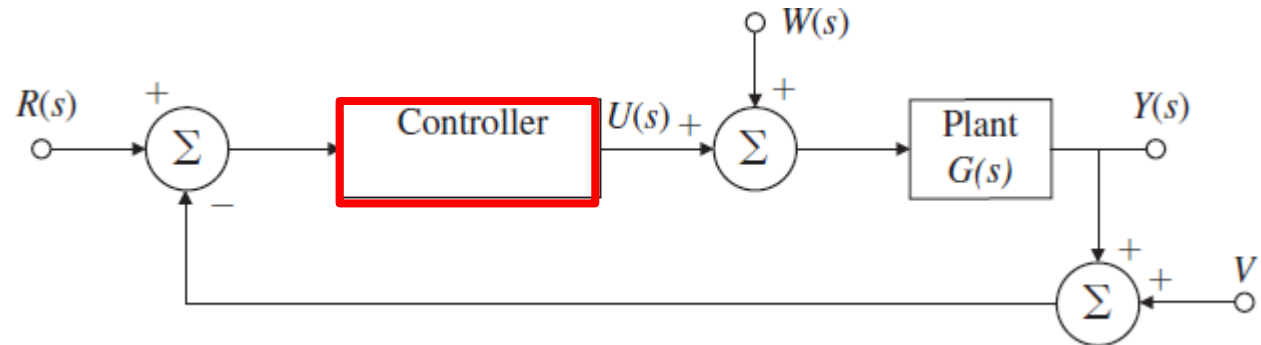
▪ Controller   = K $1 + K\,G(s) = 0$

▪ Controller   = $K\,D_c(s)$ $1 + K\,D_c(s)\,G(s) = 0$

▪ PD Control or PD Compensation $D_c(s) = (\,T_D\,s + 1\,)$

PD Compensation

Root Locus of PD Compensation

$$D_c(s) = K$$

$$G(s) = \frac{1}{s(s+1)}$$

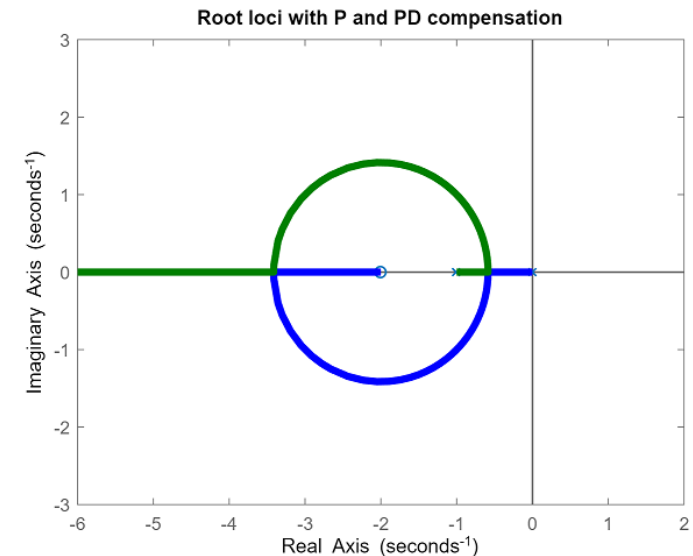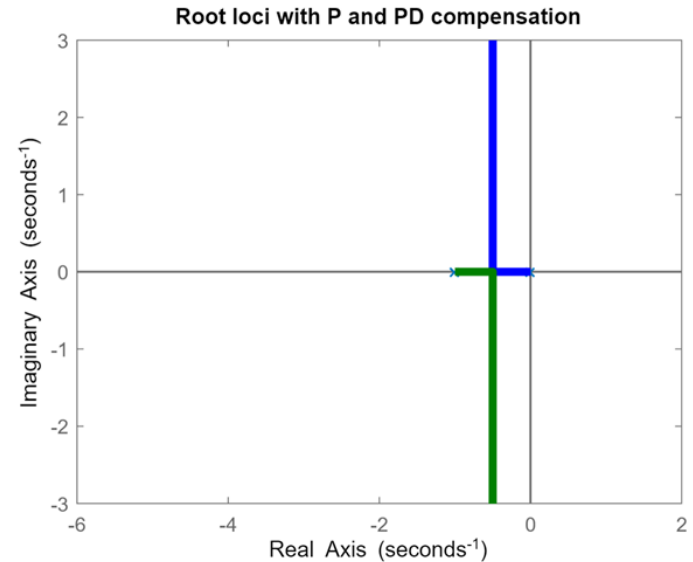$$\Rightarrow 1 + K\frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K = 0$$

Root loci with P and PD compensation

$$D_c(s) = K\,(s+2)$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K\,(s+2)\frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K(s+2) = 0$$

Root loci with P and PD compensation

PD Compensation

- ## PD Control or PD Compensation $D_c(s) = (T_D s + 1)$
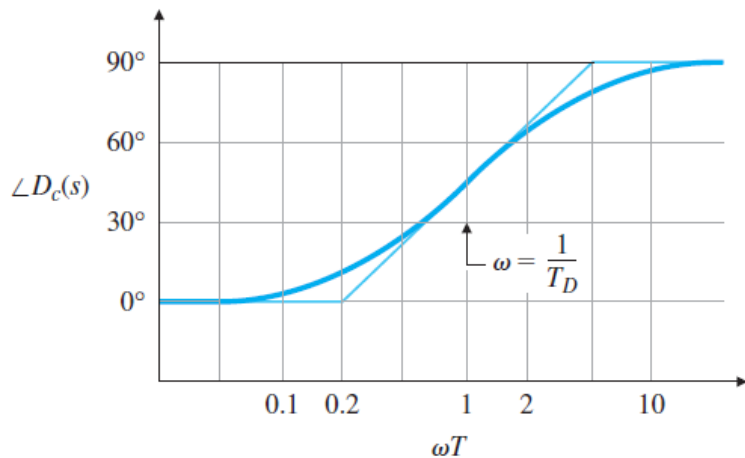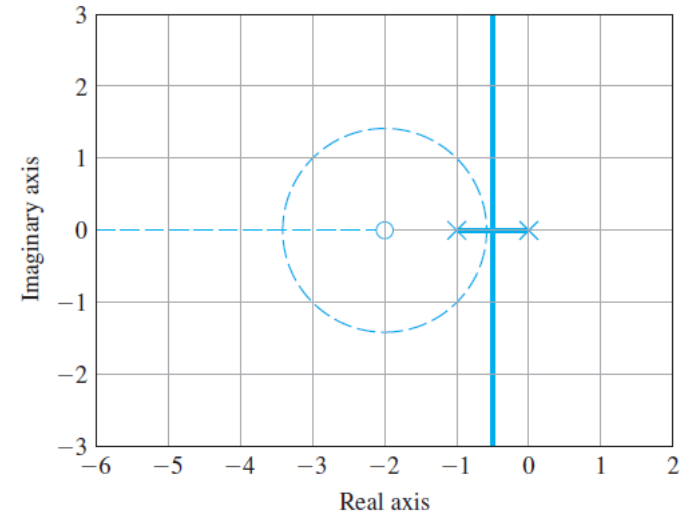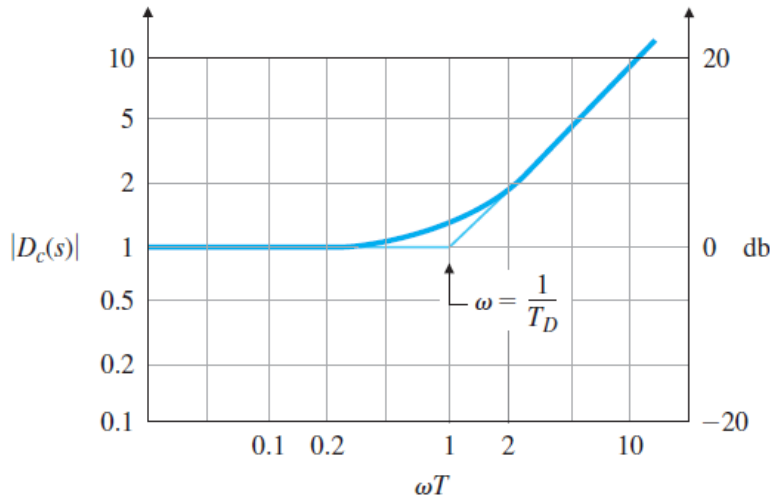
- **Increase** in phase

- **+1 slope above** $\omega = 1/T_D$

- **Gain increased** with frequency
  - Undesirable
  - Amplify high-frequency noise

- To alleviate the high-frequency amplification

   of the PD compensation,

- A first-order pole is added in the denominator

   at frequencies substantially higher than

   the break point of PD compensator.

- Thus, the phase increase (or lead) still occurs,

- But, the amplification at high frequencies is limited.

- The resulting lead compensation has a transfer function of:

$$D_c(s) = \frac{T_D\,s\,+\,1}{\alpha\,T_D\,s\,+\,1},\quad \alpha < 1$$

$$\frac{1}{\alpha} = 10$$

$$D_c(s) = \frac{T_D\, s + 1}{\alpha\, T_D\, s + 1}, \quad \alpha < 1$$
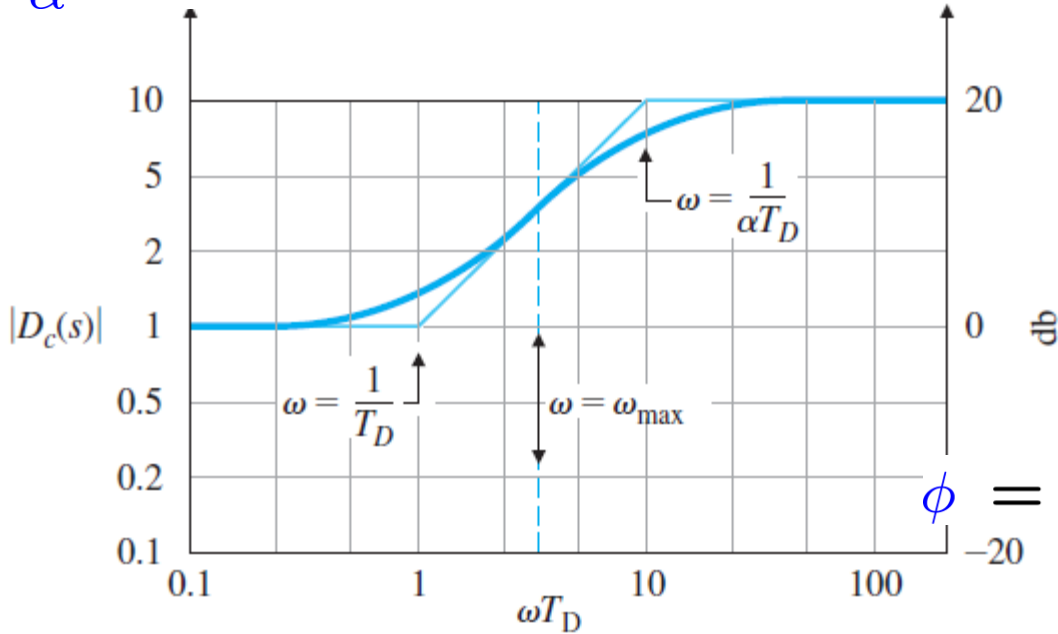
- **Phase lead**

- **Much less amplification** at high frequencies

- **Phase Contribution**
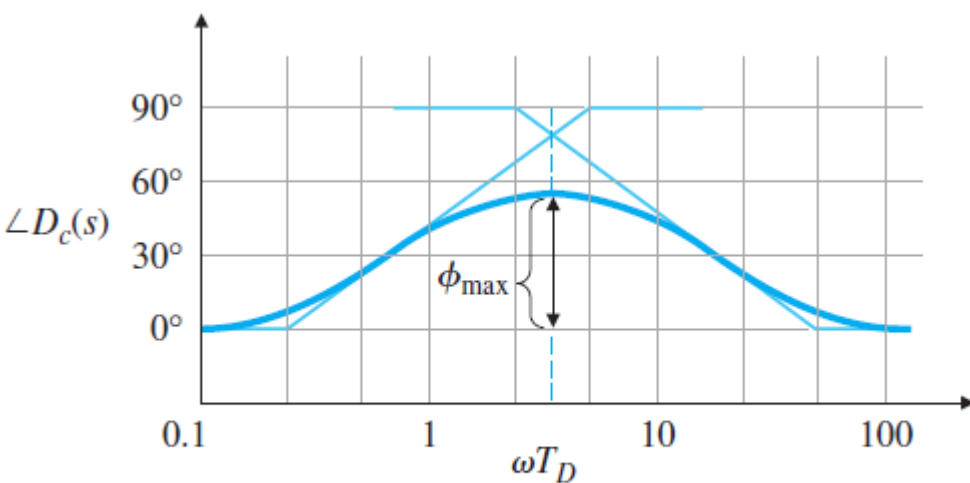
$$\phi = \tan^{-1}(T_D w) - \tan^{-1}(\alpha T_D w)$$

- **Max Phase**

$$w_{\max} = \frac{1}{T_D \sqrt{\alpha}}$$

$$\sin \phi_{\max} = \frac{1 - \alpha}{1 + \alpha}$$

$$\alpha = \frac{1 - \sin \phi_{\max}}{1 + \sin \phi_{\max}}$$

- The maximum phase occurs at a frequency

  that lie midway between the two break-point frequencies

  (sometimes called corner frequencies)

  on a logarithmic scale

$$\log w_{\max} = \log \frac{\frac{1}{\sqrt{T_D}}}{\sqrt{\alpha T_D}} = \log \frac{1}{\sqrt{T_D}} + \log \frac{1}{\sqrt{\alpha T_D}}$$

$$D_c(s) = \frac{s + z}{s + p} = \frac{1}{2} \left[\log \left(\frac{1}{T_D} \right) + \log \left(\frac{1}{\alpha T_D} \right) \right]$$

$$\Rightarrow w_{\max} = \sqrt{|z| \, |p|}$$

$$\log w_{\max} = \frac{1}{2} \left(\log|z| + \log|p| \right)$$

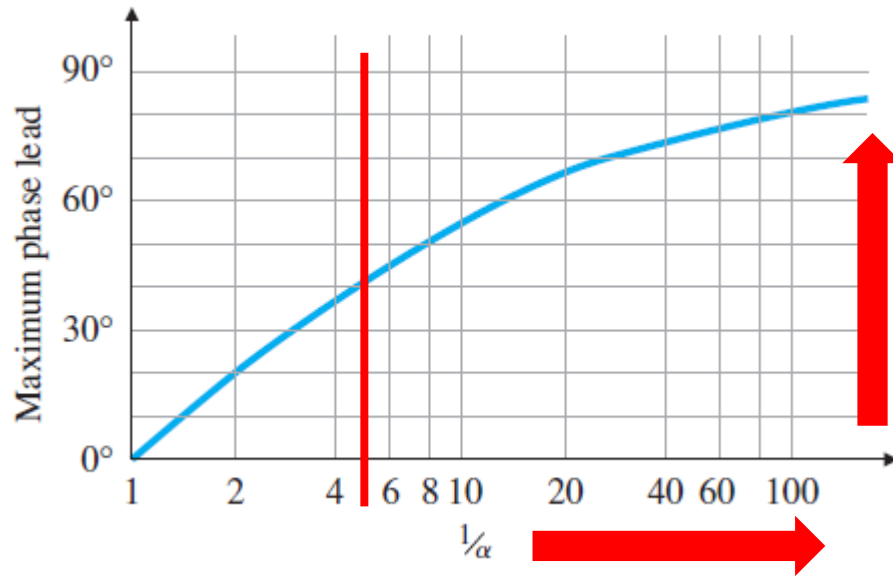$$z = \frac{-1}{T_D}$$

$$p = \frac{-1}{\alpha T_D}$$

- For example, a lead compensator:

a zero at $s = -2$ $(T_D = 0.5)$

a pole at $s = -10$ $(\alpha T_D = 0.1)$ $\alpha = \dfrac{1}{5}$

$$\Rightarrow w_{max} = \sqrt{|z|\,|p|}$$
$$= \sqrt{2 \times 10}$$
$$= 4.47 \text{ rad/sec}$$

$$\Rightarrow \phi_{max} = 40^o$$

- Larger $1/\alpha$

- Produce higher amplification

  at higher frequencies

- Select $1/\alpha \rightarrow$ a good compromise
  between acceptable PM & noise sensitivity at high frequencies

$$\Rightarrow \phi_{max} = 70^o$$

- Double-lead compensation
    for greater phase lead:

$$D_c(s) = \left(\frac{T_D\, s + 1}{\alpha\, T_D\, s + 1} \right)^2$$
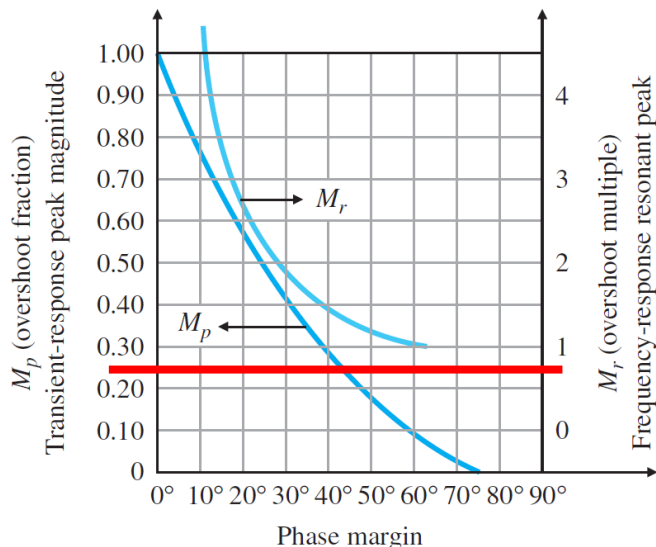
- Example 6.15: Lead Compensation for a DC Motor

$$G(s) = \frac{1}{s(s+1)}$$

- Steady-state error <= 0.1 for a unit-ramp input
- Overshoot $M_p < 25\%$

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{1 + K\,D_c(s)\,G} \right] R(s) \qquad R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \left[\frac{1}{s + K\,D_c(s)\left(\frac{1}{s+1}\right)} \right] = \frac{1}{K\,D_c(0)}$$
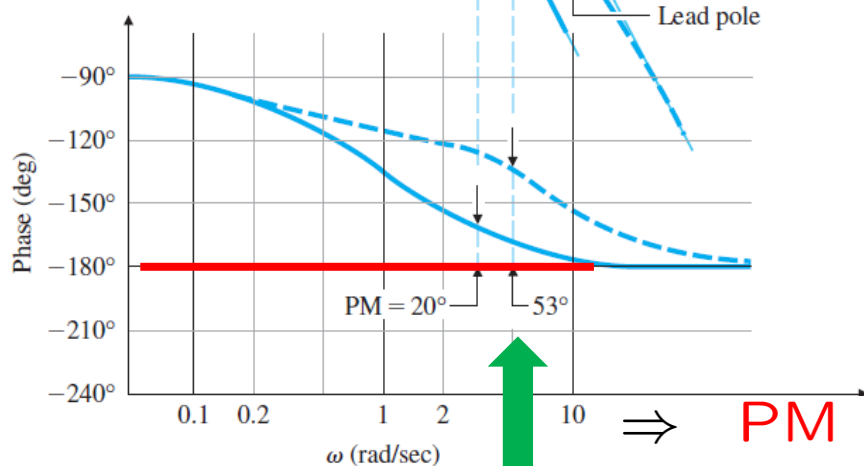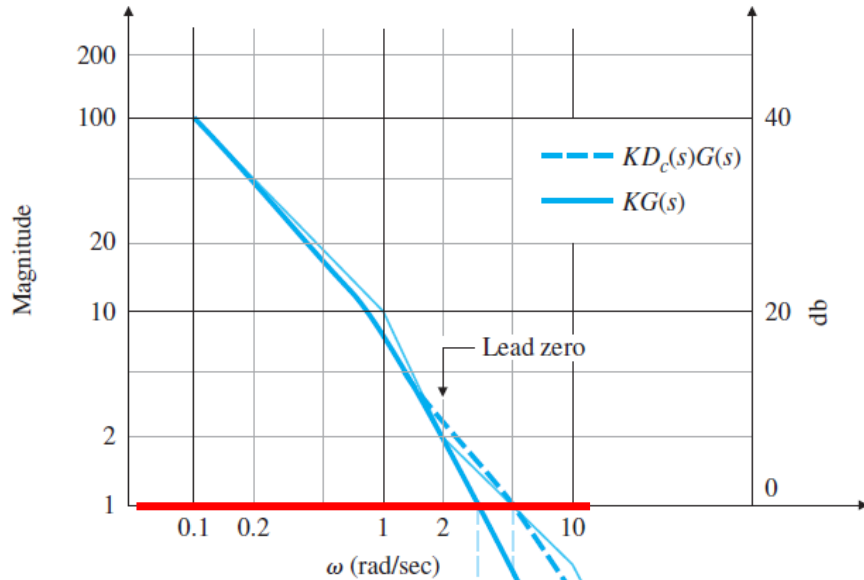
$$\Rightarrow \ |K\,D_c(0)| \ \geq \ 10$$

$$\Rightarrow \quad \text{PM} \ \geq \ 45^o$$

- ## Example 6.15: Lead Compensation for a DC Motor

$K\ G(s)$ $K\ D_c(s)\ G(s)$

$\Rightarrow$ PM $\geq 25^o$,   at $w_c = 3$

$\Rightarrow$ Phase lead $= 40^o$,

$\Rightarrow \dfrac{1}{\alpha} = 5$
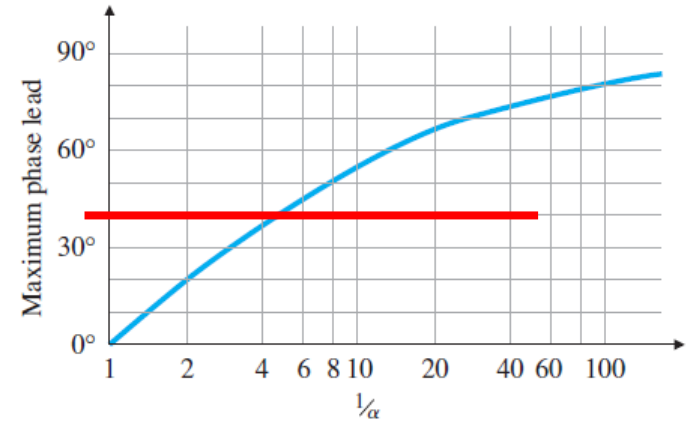
a zero at $w = 2$ rad/sec

a pole at $w = 10$ rad/sec

$K\ D_c(s) = 10\ \dfrac{\dfrac{s}{2}+1}{\dfrac{s}{10}+1}$

$\Rightarrow$ PM $= 53^o$,   at $w_c = 5$

Example 6.15: Lead Compensation for a DC Motor

$$K\, D_c(s) \;=\; 10\,\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$$KD_c(s) = K\;\frac{\frac{s}{2}+1}{\frac{s}{10}+1}$$

$$G(s) = \frac{1}{s(s+1)}$$

$K = 10$
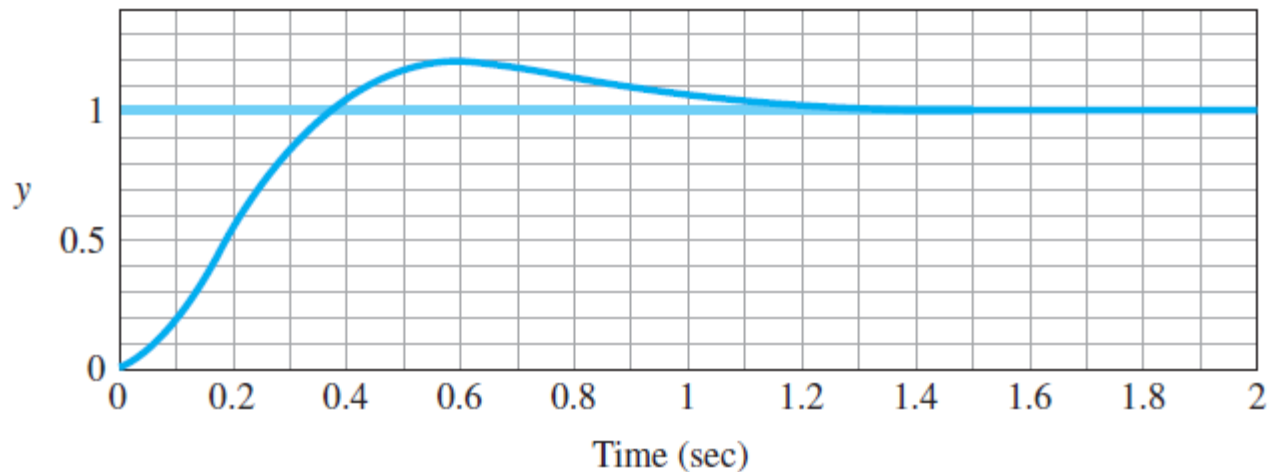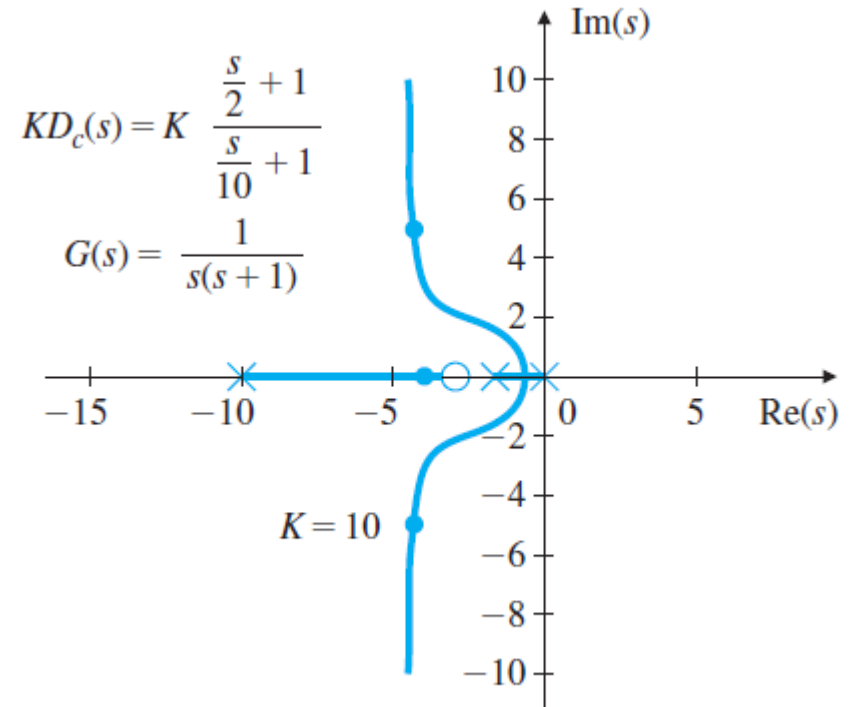
1. Crossover frequency ω_c,

    which determines

    bandwidth ω_{BW},   rise time t_r,   settling time t_s

2. Phase Margin (PM),

    which determines

    damping coefficient ζ,   overshoot M_p

3. The low-frequency gain,

    which determines

    the steady-state error characteristics

1. Determine gain K to satisfy error or bandwidth requirements:

    a) To meet error requirements, pick K

        to satisfy error constants (K_P, K_v, K_a), so that e_{ss} is met.

    b) To meet bandwidth requirements, pick K

        so that the OL crossover frequency

                is a factor of two below the desired CL bandwidth.

2. Evaluate the PM of the uncompensated system

        using the value of K obtained from Step 1

3. Allow for extra margin (about 10°) and

        determine the needed phase lead ϕ_{max}

4. Determine α

5. Pick ω_{max} to be the crossover frequency;
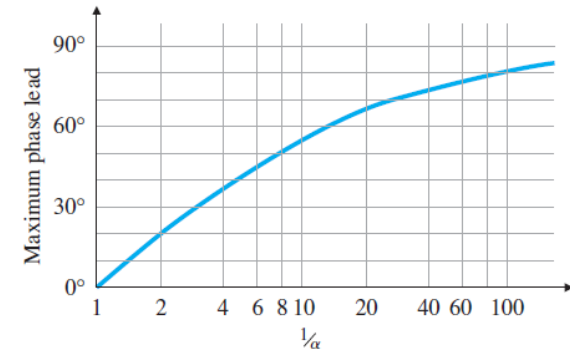
   a zero at $1/T_D = w_{max}\sqrt{\alpha}$

   a pole at $1/(\alpha T_D) = w_{max}/\sqrt{\alpha}$

6. Draw the compensated frequency response and check PM

7. Iterate on the design.

   • Adjust compensator parameters (poles, zeros, gain)

      until all specification are met.

   • Add an additional lead compensator if necessary.

Example 6.16: Lead Compensation
for Temperature Control System

$$K\,G(s) \;=\; \frac{K}{(\frac{s}{0.5}+1)\,(\frac{s}{1}+1)(\frac{s}{2}+1)}$$

- $K_p = 9$

- PM $> 25^o$

1. Determine gain K:

$$K_p \;=\; \lim_{s\to 0}\; K\,G(s)$$

$$=\; K \;=\; 9$$

2. Bode plot

   of un-uncompensated system:

   KG(s), with K = 9
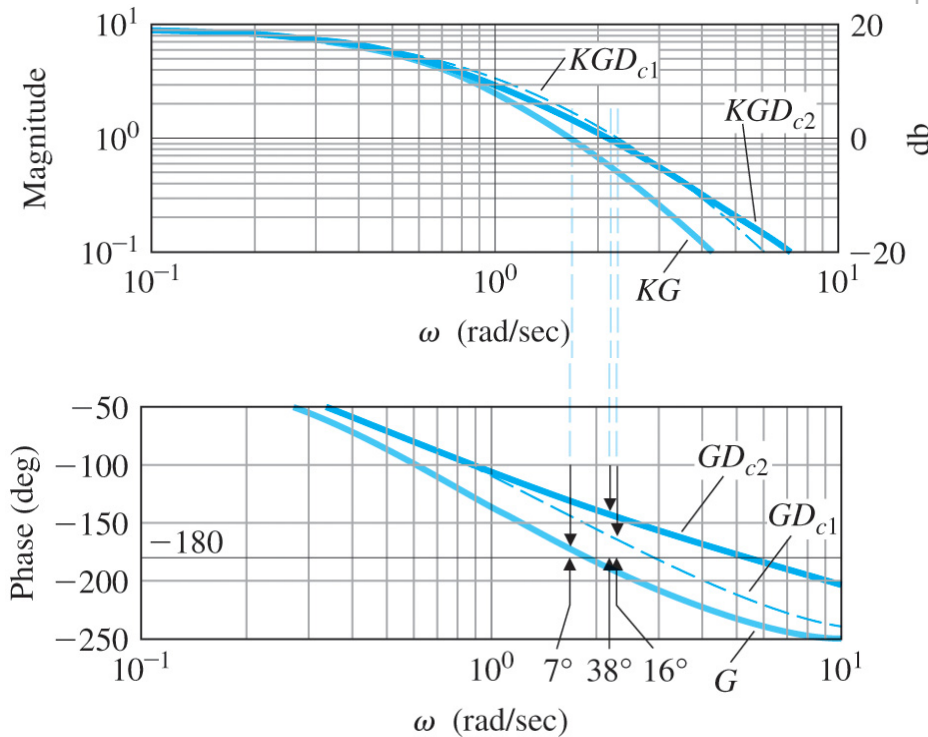
(b)

→ GM = 1.25, PM = 7.14, Wcg = 1.87, Wcp = 1.68

Example 6.16: Lead Compensation
for Temperature Control System

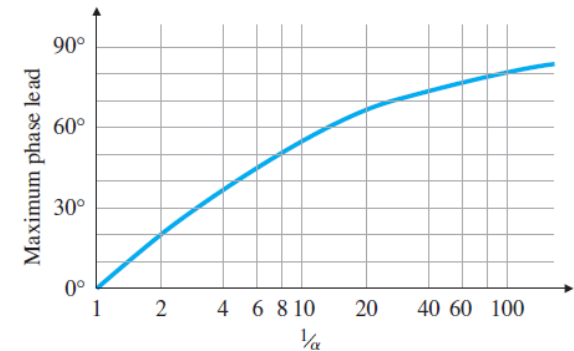3. Allow for 10° of extra margin → $25^o + 10^o - 7^o = 28^o$

4. Pick α → $1/\alpha = 3$

5. Zero & Pole

a zero at 1 $T_D = 1$

a pole at 3 $\alpha T_D = 1/3$
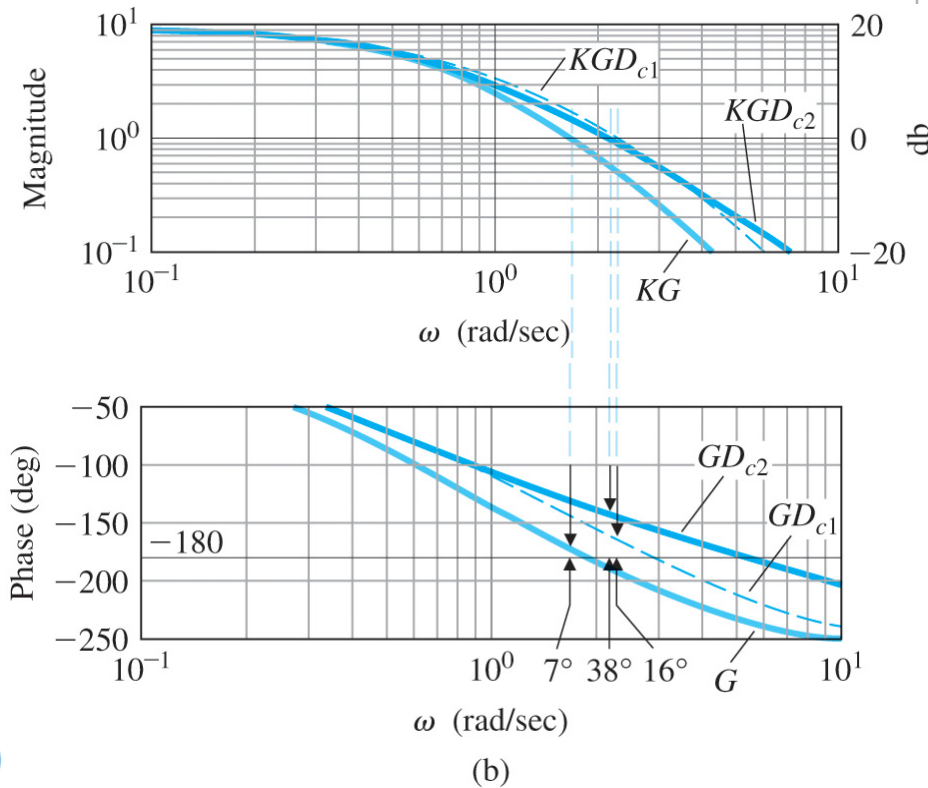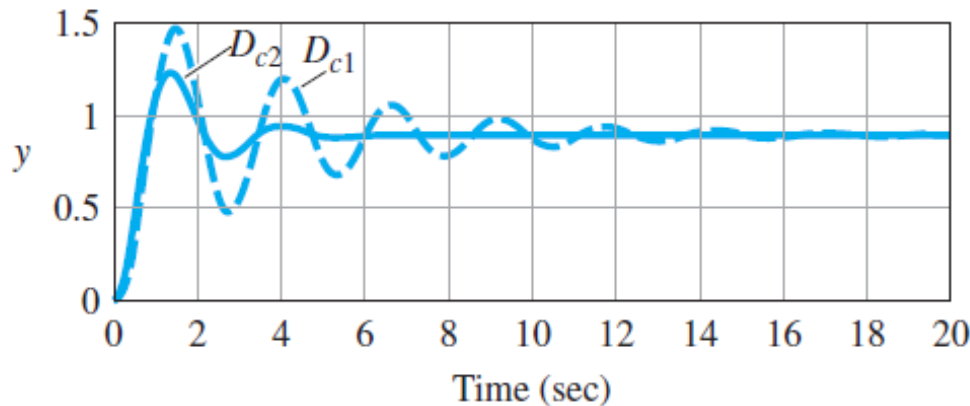
$$D_1(s) = \frac{(\frac{s}{1} + 1)}{(\frac{s}{3} + 1)} = \frac{1}{0.333}\left(\frac{s + 1}{s + 3}\right)$$

Example 6.16: Lead Compensation
for Temperature Control System

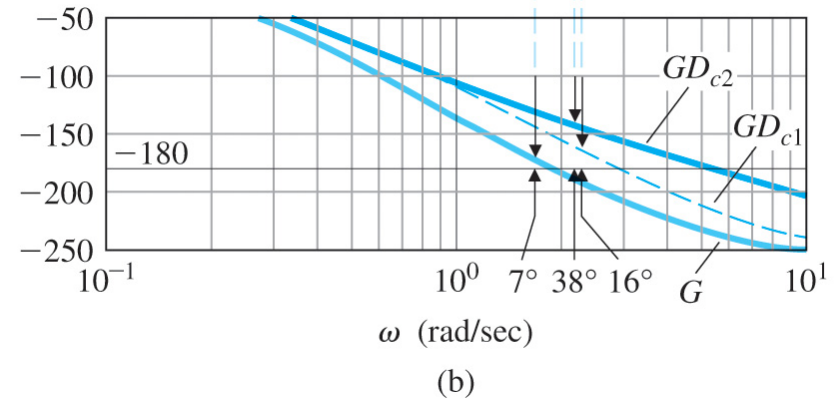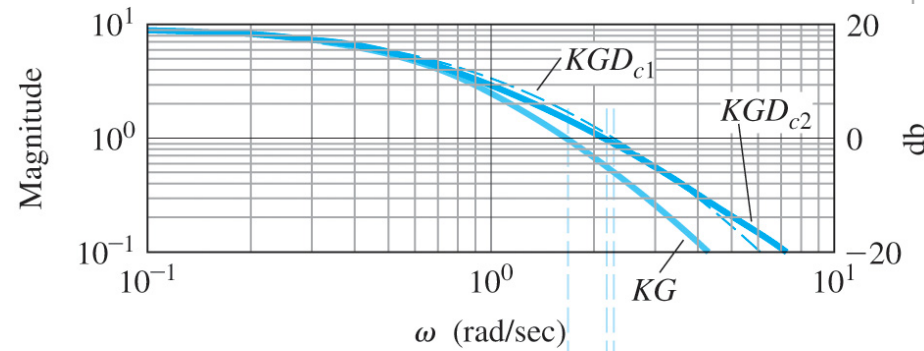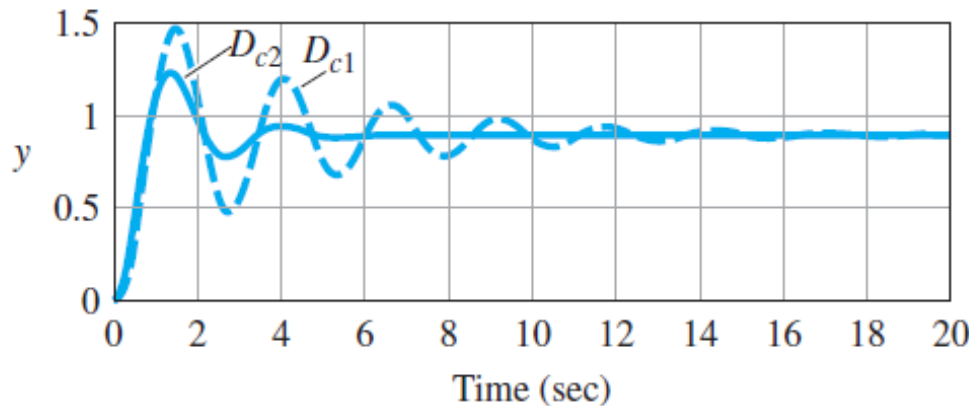$$D_1(s) = \frac{(\frac{s}{1} + 1)}{(\frac{s}{3} + 1)} = \frac{1}{0.333}\left(\frac{s + 1}{s + 3}\right)$$

- PM = 16°

(b)

- **Example 6.16: Lead Compensation**
  for Temperature Control System

7. Move zero:

a zero at $s = -1.5$ $\alpha = 1/10$

$$D_2(s) = \frac{(\frac{s}{1.5} + 1)}{(\frac{s}{15} + 1)}$$

$$= \frac{1}{0.1}\left(\frac{s + 1.5}{s + 15}\right)$$

- **PM = 38°**

- Example 6.17: Lead-Compensation Design
                            for Type 1 Servomechanism System

$$K\,G(s) \;=\; K\,\frac{10}{s\left(\frac{s}{2.5}+1\right)\left(\frac{s}{6}+1\right)}$$
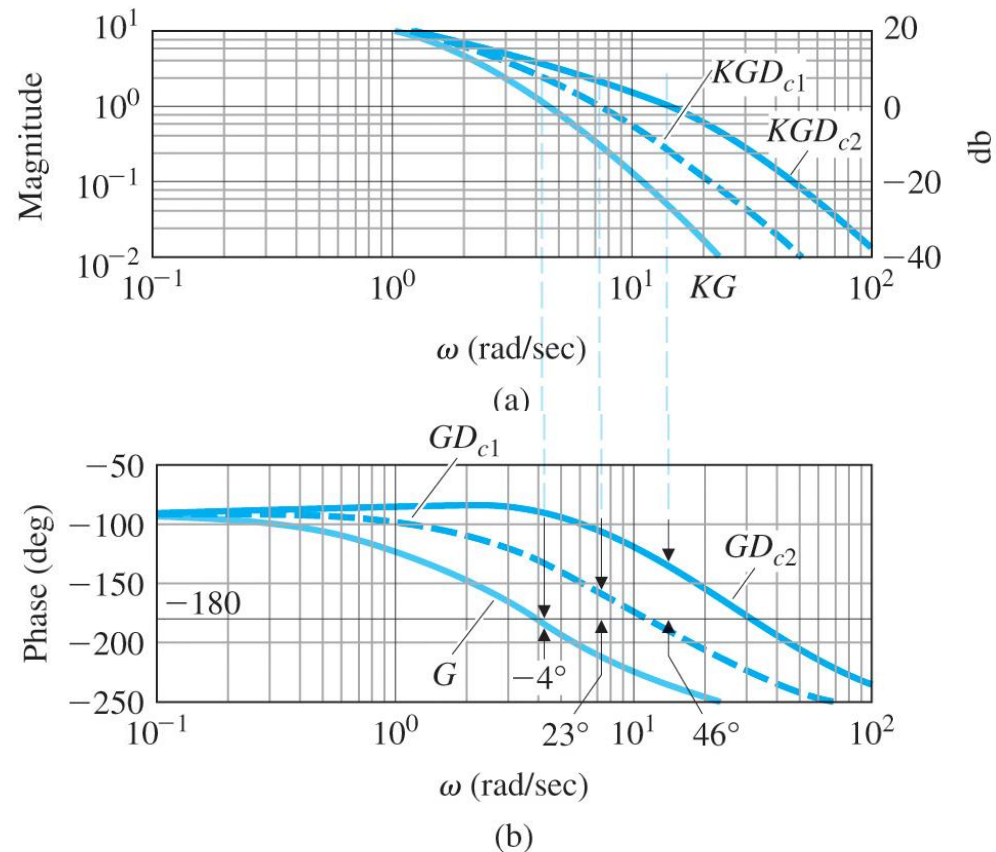
- $K_v = 10$

- PM $= 45^o$

1. Determine gain K:

$$K_v \;=\; \lim_{s\to 0}\; s\,K\,G(s)$$

$$=\; K \times 10 \;=\; 10$$

$$\Rightarrow\; K \;=\; 1$$

2. Bode plot of KG(s), K = 1

→ PM ~= - 4, Wcp ~= 4

(a)

(b)

Example 6.17: Lead-Compensation Design
for Type 1 Servomechanism System

3. Allow for 5° of extra margin

→ $45^o + 5^o - (-4^o) = 54^o$

4. Pick α → $1/\alpha = 10$

5. Zero & Pole

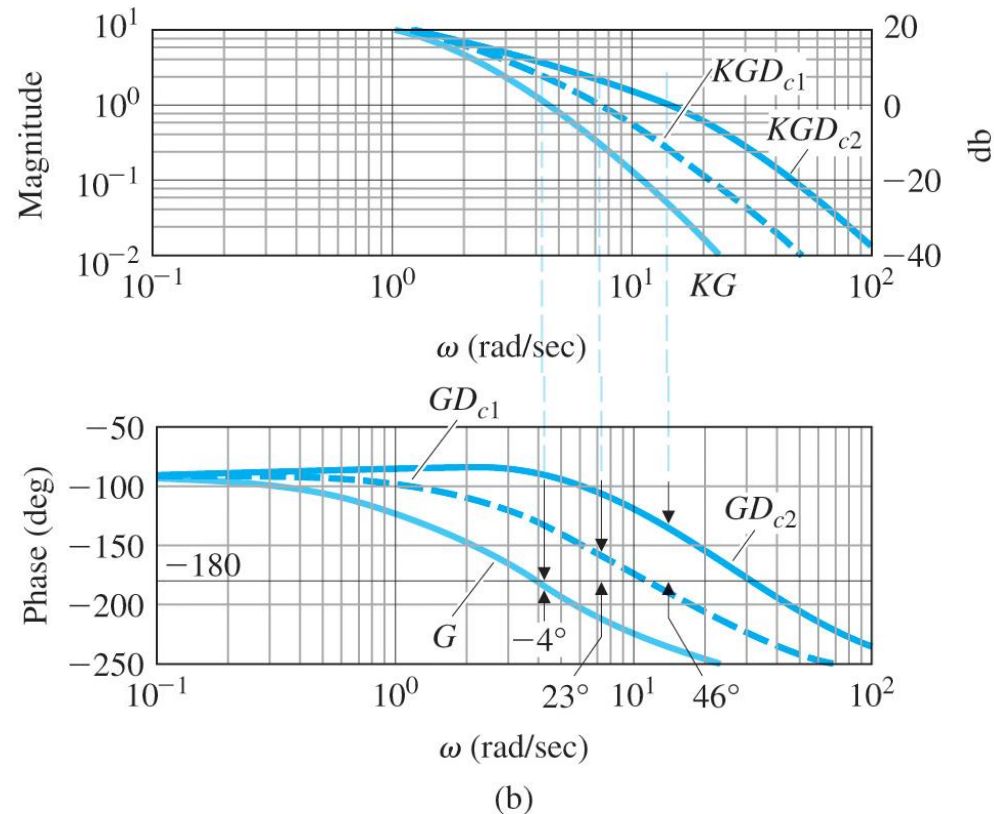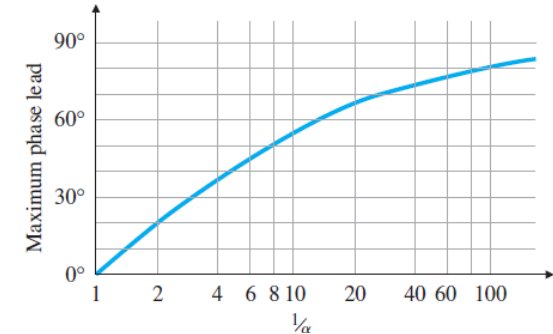a zero at 2

a pole at 20

$$D_1(s) = \frac{(\frac{s}{2}+1)}{(\frac{s}{20}+1)}$$

$$= \frac{1}{0.1}\left(\frac{s+2}{s+20}\right)$$

→ PM ~= 23, Wcp ~= 7

- Example 6.17: Lead-Compensation Design
                        for Type 1 Servomechanism System

7. A double-lead compensator:

$$D_2(s) = \frac{(\frac{s}{2} + 1)(\frac{s}{4} + 1)}{(\frac{s}{20} + 1)(\frac{s}{40} + 1)} = \frac{1}{(0.1)^2}\frac{(s + 2)(s + 4)}{(s + 20)(s + 40)}$$

- PM = 46°

(b)