

Spring 2021

控制系統
Control Systems

Unit 6G
Bode's Gain-Phase Relationship

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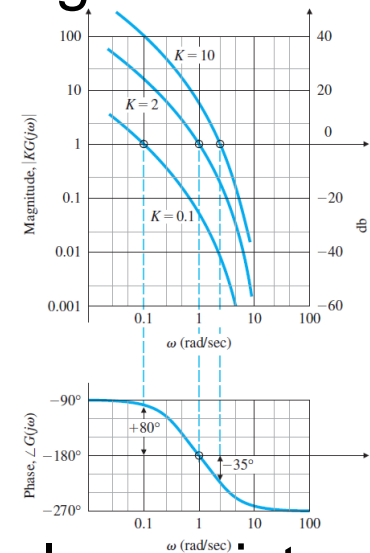
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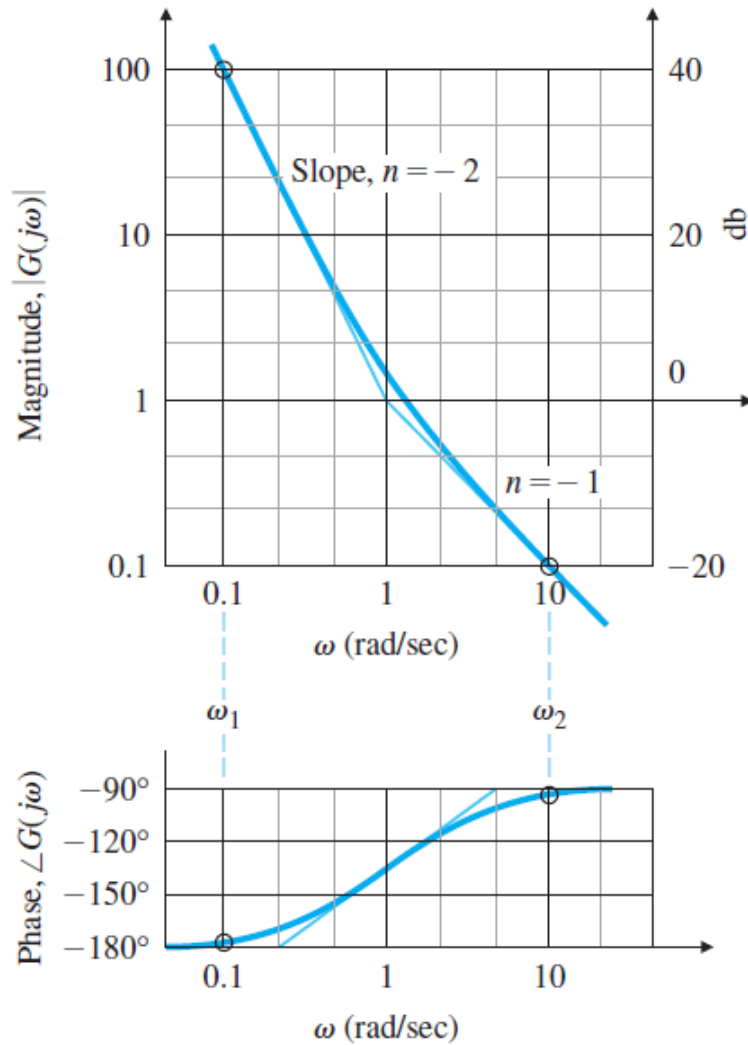
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- One of **Bode's important contributions** is the following theorem:
- For any **stable minimum-phase** system (no RHP zeros/poles),
- The **phase** of $G(j\omega)$ is uniquely related to the **magnitude** of $G(j\omega)$
- When the **slope** of $|G(j\omega)|$ versus ω on a log-log scale persists at a **constant value** for approximately **a decade of frequency**, the relationship is particularly simple and is given by:

$$\angle G(j\omega) \approx n \times 90^\circ$$

- n : the **slope** of $|G(j\omega)|$ in units of decade of amplitude **per decade** of frequency





■ Slope:

- At $\omega_1 = 0.1$, $(n = -2)$
- At $\omega_2 = 10$, $(n = -1)$

■ Phase:

- At $\omega_1 = 0.1$, -180°
- At $\omega_2 = 10$, -90°

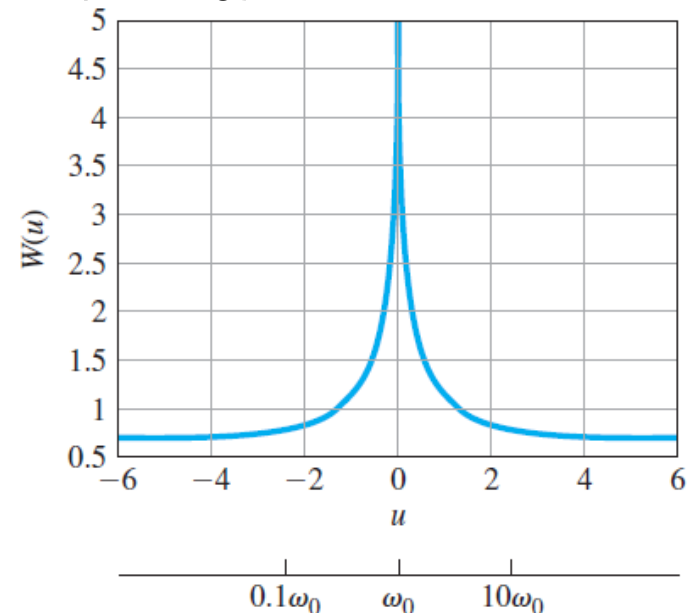
- An exact statement of **the Bode Gain-Phase Theorem** is:

$$\angle G(j\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \left(\frac{dM}{du} \right) W(u) du \quad \text{in radians}$$

- Where

- M = log magnitude = $\ln |G(j\omega)|$
- u = normalized frequency = $\ln(\omega / \omega_0)$
- $dM/du \sim$ slope n
- $W(u)$ = weighting function
= $\ln(\coth|u|/2)$

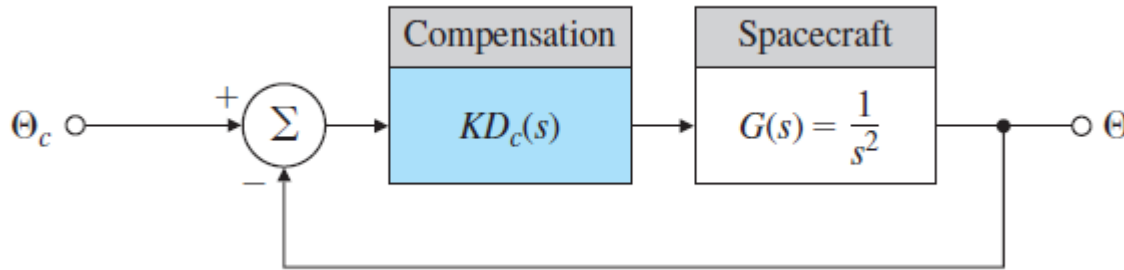
$$W(u) \approx \frac{\pi^2}{2} \delta(u)$$



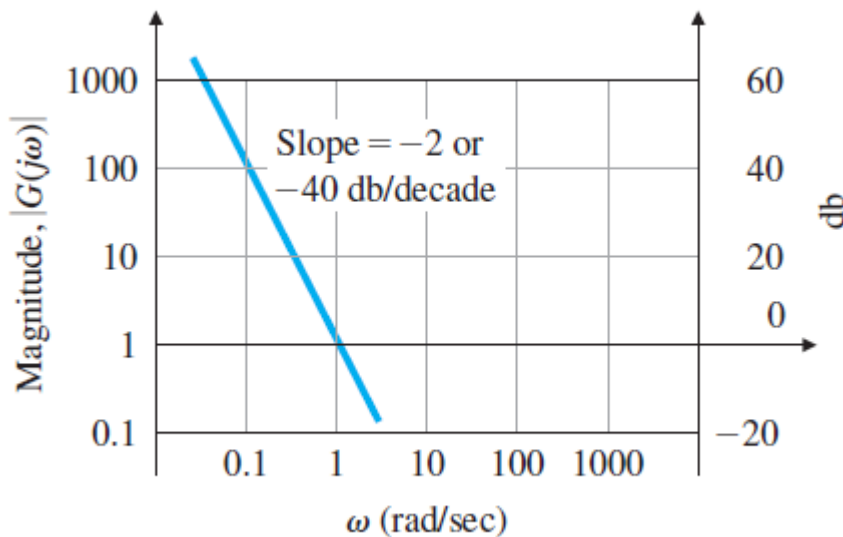
- But, usually use $\angle G(j\omega) \approx n \times 90^\circ$
- When $|KG(j\omega)| = 1$
 - $\angle G(j\omega) \approx -90^\circ$ if $n = -1$
 - $\angle G(j\omega) \approx -180^\circ$ if $n = -2$
- For **stability**, we want:
 - $\angle G(j\omega) > -180^\circ$ for **PM** to be > 0
- Therefore, **adjust the $|KG(j\omega)|$ curve**
- So that it has a **slope of -1** at the **crossover frequency ω_c**
- If the **slope = -1** for a decade above/below ω_c , then **PM $\approx 90^\circ$**
- However, to ensure **a reasonable PM**,
it is usually necessary only to insist that
a -1 slope persist for a decade in frequency centered **at ω_c**

- A very **simple design criterion**:
- Adjust the **slope** of the **magnitude curve** $|KG(j\omega)|$
- So that it crosses over **magnitude 1** with **a slope of -1**
for a decade **around** ω_c
- This criterion will usually be sufficient
to provide **an acceptable PM** and **adequate system damping**.
- To achieve the desired **speed of response**,
- the **system gain** is adjusted
- so that the **crossover point** is at a frequency
that will yield the **desired bandwidth** or **speed of response**.
- **Natural Freq** $\omega_n \sim$ **Bandwidth** $\omega_{BW} \sim$ **Crossover Freq** ω_c

Example 6.14: Use of Simple Design Criterion for Spacecraft Attitude Control



$$K D_c(s) = K (T_D s + 1)$$

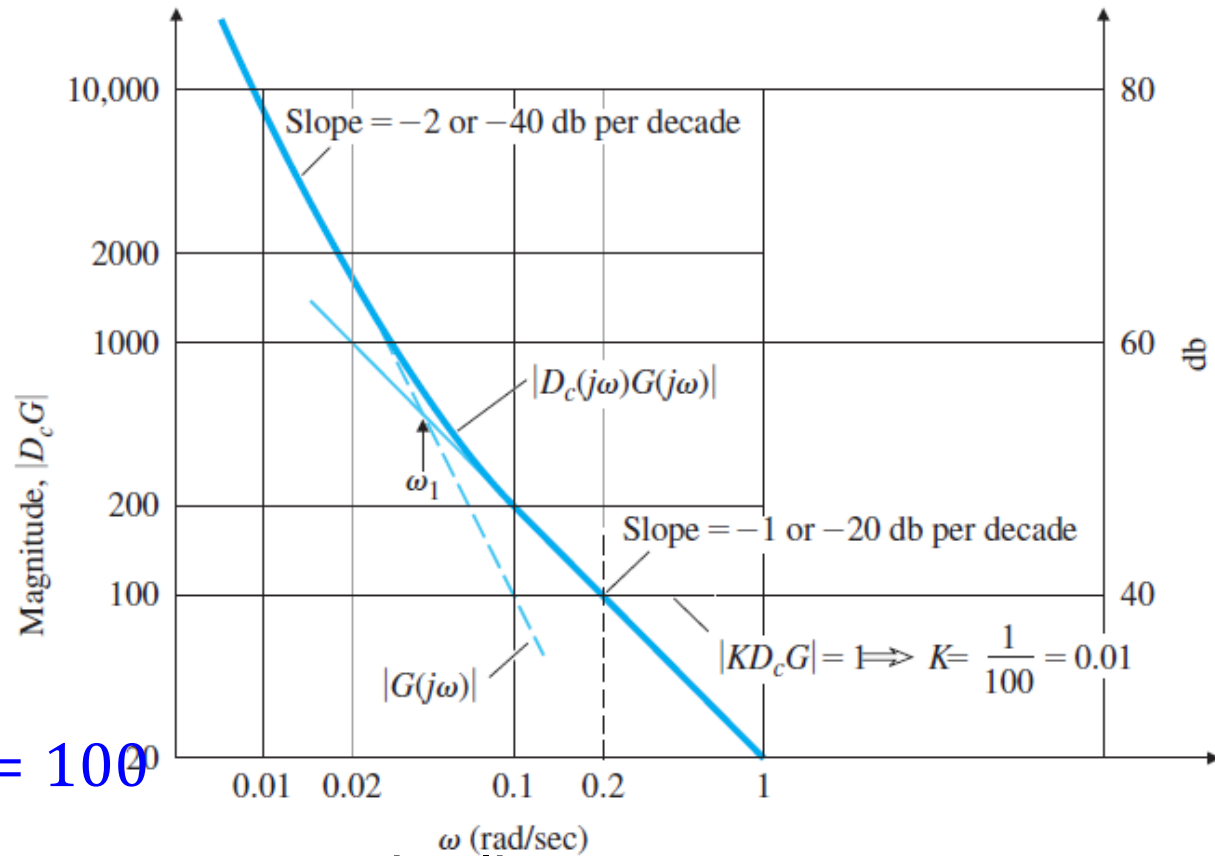


- Adjust K
- to provide desired bandwidth
- Adjust break point $\omega_1 = 1/T_D$
- to provide the -1 slope at the crossover frequency

0.2 rad/sec

Example

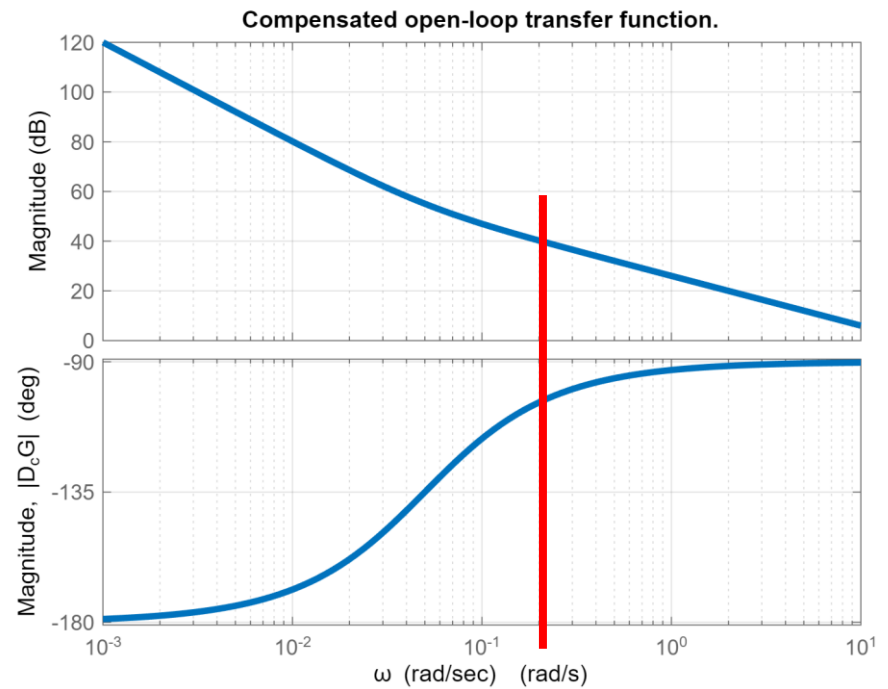
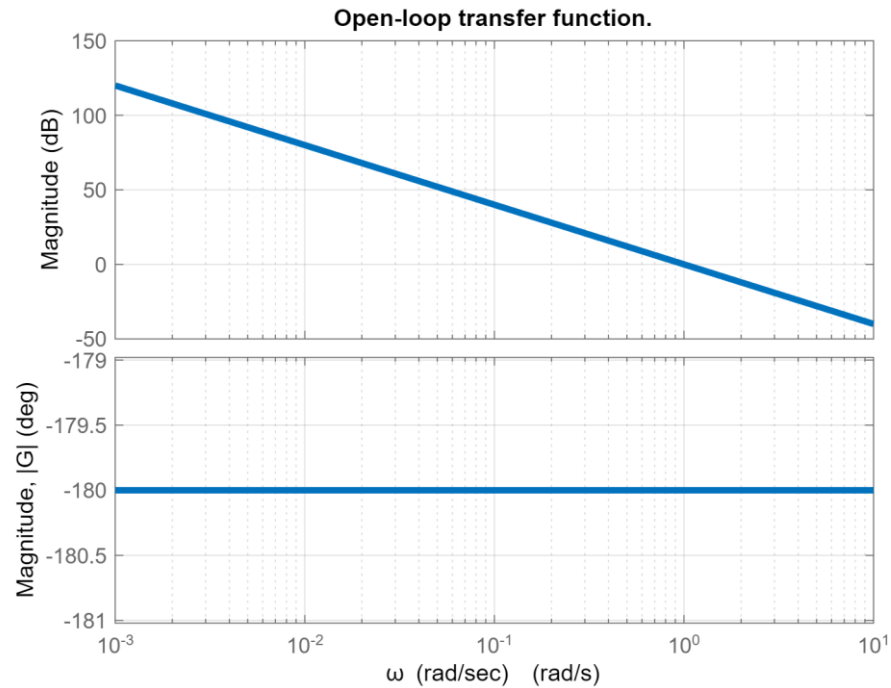
1. Plot $|G(j\omega)|$
2. Modify the plot to include $|D_c(j\omega)|$ with $\omega_1 = 0.05$
 $T_D = 20$
→ slope=-1 at 0.2



3. Determine $|D_c G| = 100$ where the $|D_c G|$ curve crosses the line $\omega = 0.2$

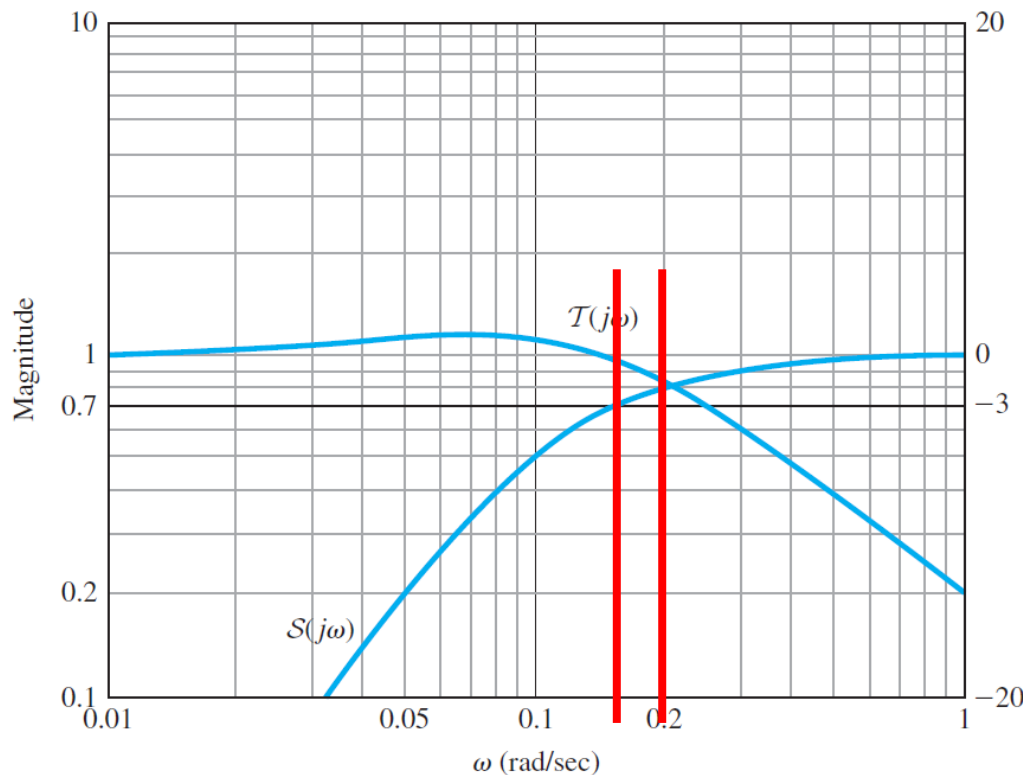
4. Compute
$$K = \frac{1}{|D_c G|_{\omega=0.2}} = \frac{1}{100} = 0.01$$

$$\Rightarrow K D_c(s) = 0.01 (20s + 1)$$



Example

- The closed-loop frequency-response magnitude $T(j\omega)$ and the sensitivity function $S(j\omega)$
- Desired Bandwidth = 0.2 rad/sec
- Disturbance Rejection (-3db) at 0.15 rad/sec



- Overshoot = 14%

