Spring 2021

控制系統
Control Systems

Unit 6F
Stability Margins

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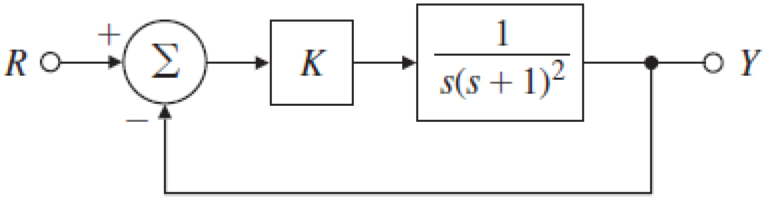
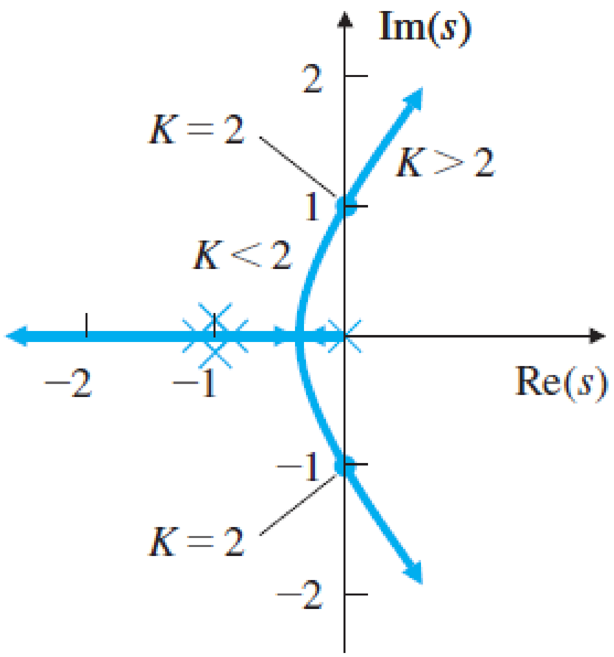
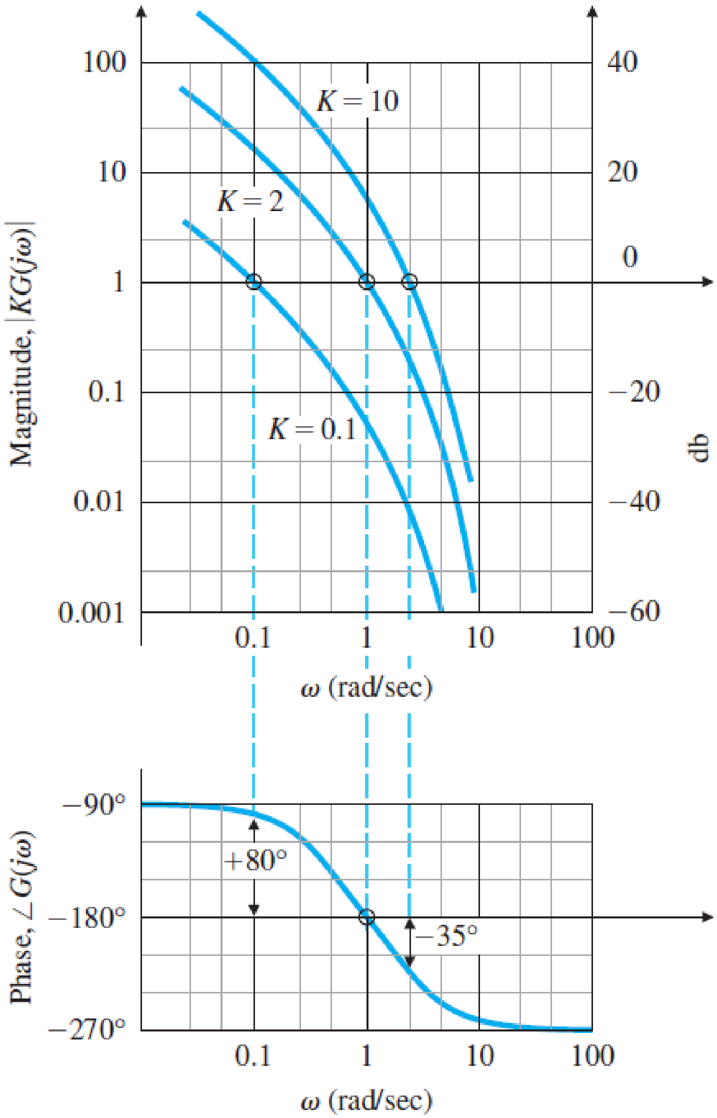
Stability Margins

- In U6D





(b)



$$| K\, G(jw) | < 1$$

$$\angle G(jw) = -180^o$$

- Gain Margin (GM) & Phase Margin (PM)
- Another measure of stability,

  originally defined by Smith (1958)
- Combine the two margins into

  Vector Margin / Complex Margin

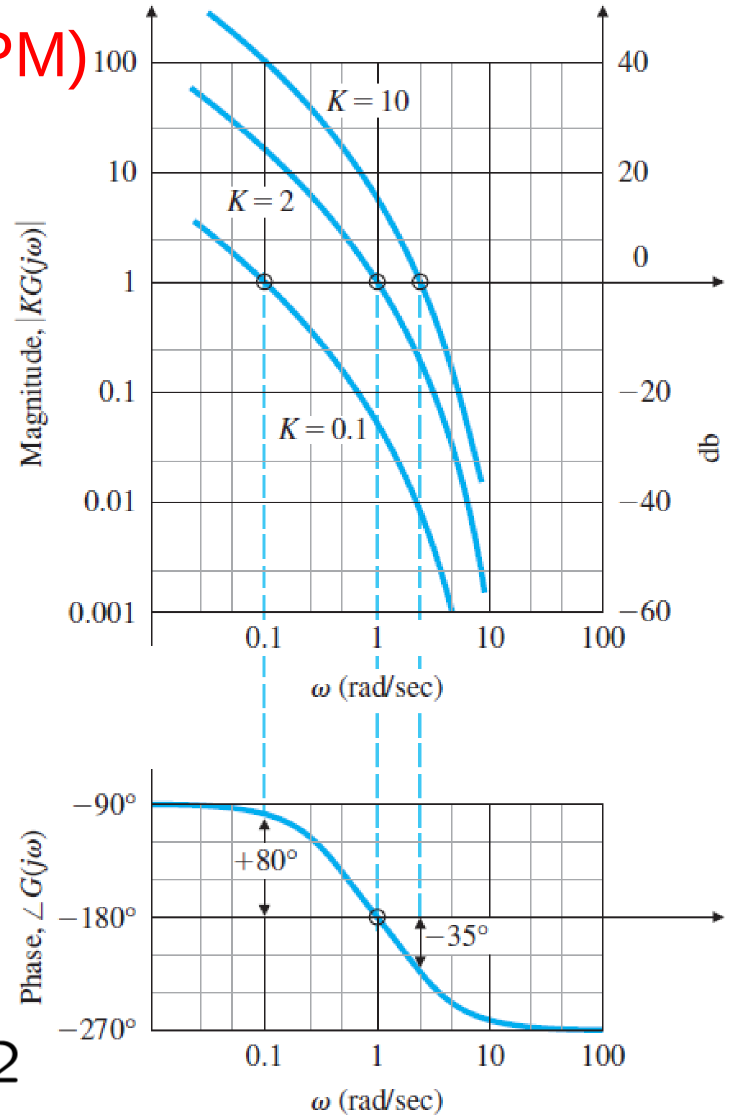
$$\angle G(jw) = -180^o$$

$$|\ K\ G(jw)\ | = 1$$

$\Rightarrow K{=}2 \quad |KG(jw)| = 1 \quad$ GM $=1$

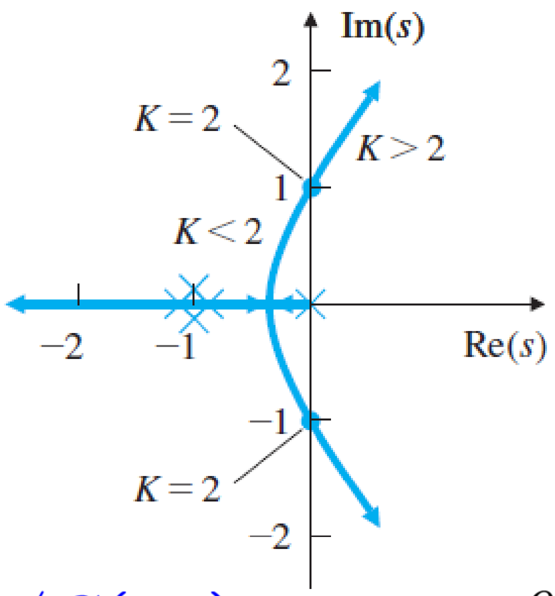
$\Rightarrow K{=}0.1 \quad |KG(jw)| = 0.05 \quad$ GM $=20$

$\Rightarrow K{=}10 \quad |KG(jw)| = 5 \quad$ GM $=0.2$



- Unstable

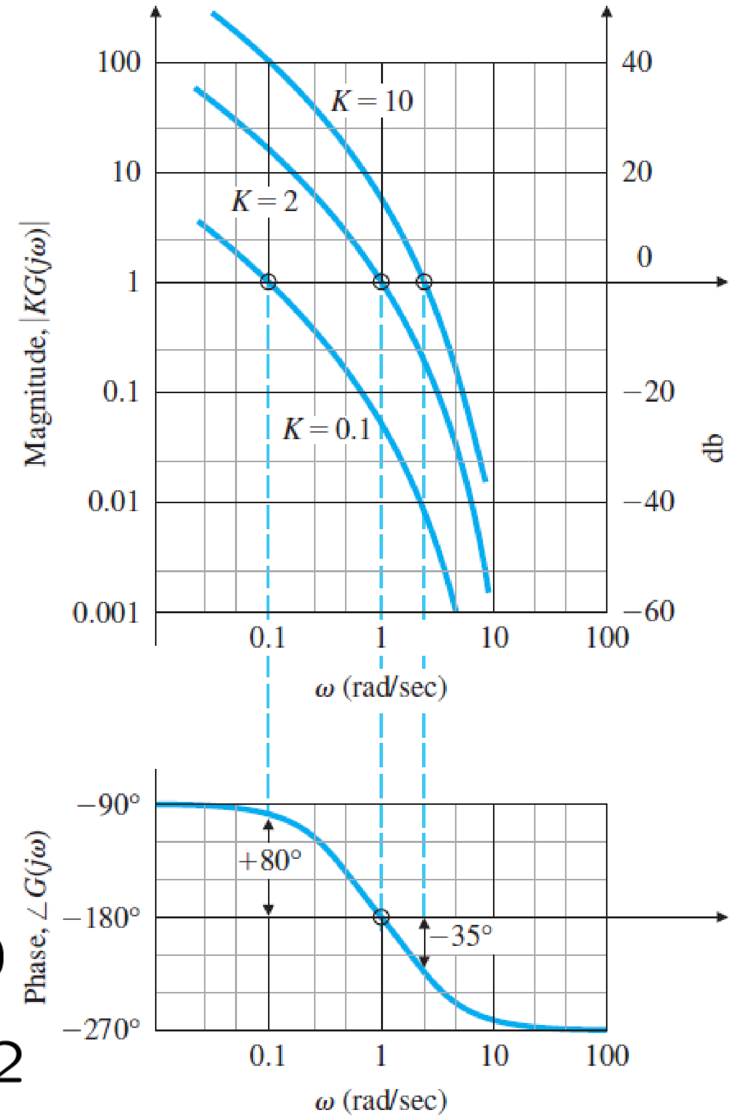
Stability Margins



(b)

$$\angle G(jw) = -180^o$$

$$|\ K\ G(jw)\ | = 1$$



$\Rightarrow K=2 \quad |KG(jw)| = 1 \quad$ GM $=1$

$\Rightarrow K=0.1 \quad |KG(jw)| = 0.05 \quad$ GM $=20$

$\Rightarrow K=10 \quad |KG(jw)| = 5 \quad$ GM $=0.2$

- Unstable

$$\mathcal{T}(s) = \frac{Y(s)}{R(s)} = \frac{K\ G(s)}{1 + K\ G(s)}$$

- Gain Margin (GM) & Phase Margin (PM)

$$| K\, G(jw) | \;=\; 1$$

$$\Rightarrow\; \angle G(jw) \;=\; ?$$

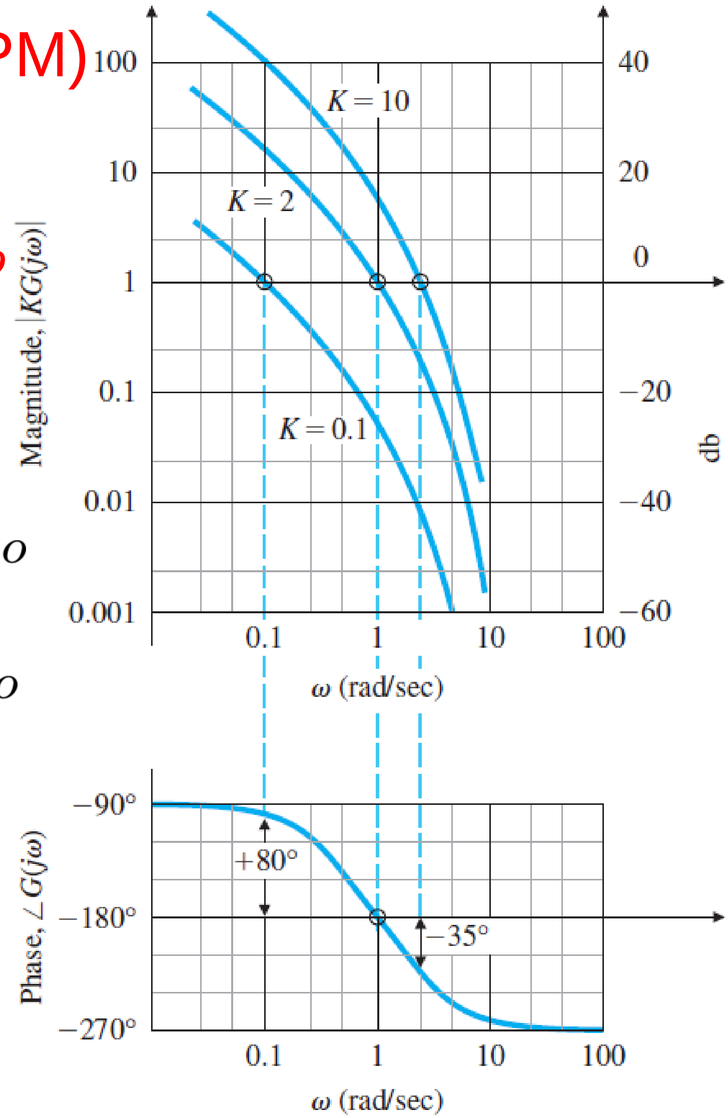
- Exceeds -180^o

$$\Rightarrow K{=}2 \quad |KG(jw)| = 1 \quad \text{PM} = 0^o$$

$$\Rightarrow K{=}0.1 \quad |KG(jw)| = 1 \quad \text{PM} = +80^o$$

$$\Rightarrow K{=}10 \quad |KG(jw)| = 1 \quad \text{PM} = -35^o$$

- Unstable



- Gain Margin (GM) & Phase Margin (PM)

$\Rightarrow K = 0.1 \quad |KG(jw)| = 0.05 \quad$ GM $= 20$

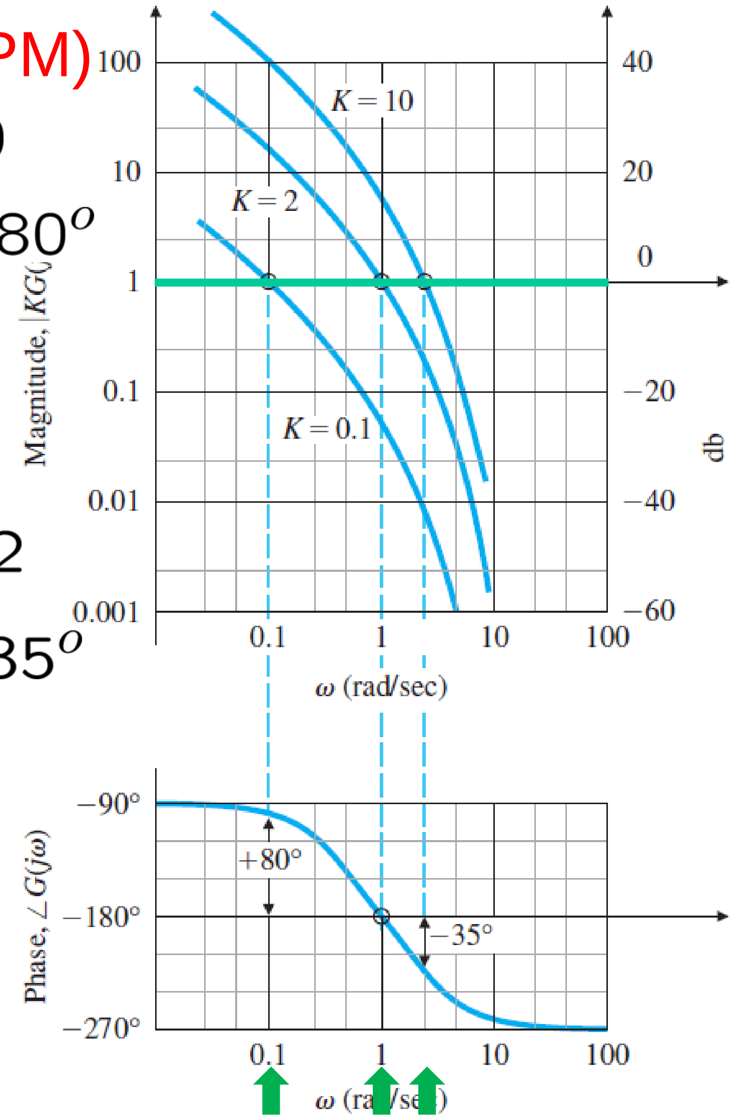
$|KG(jw)| = 1 \quad$ PM $= +80^o$

$\Rightarrow K = 2 \quad |KG(jw)| = 1 \quad$ GM $= 1$

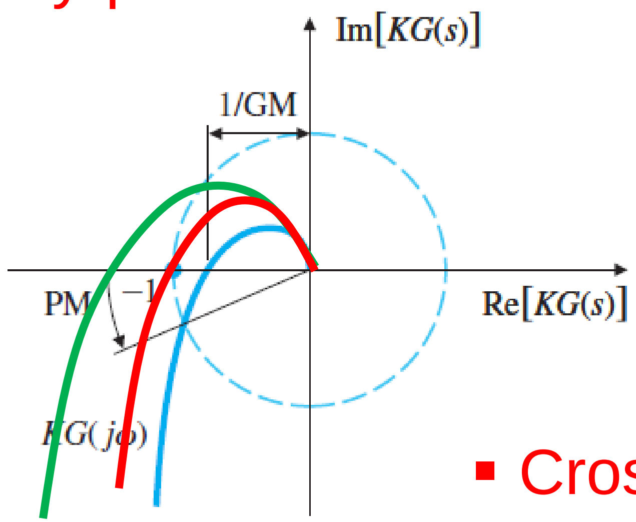
$|KG(jw)| = 1 \quad$ PM $= 0^o$

$\Rightarrow K = 10 \quad |KG(jw)| = 5 \quad$ GM $= 0.2$

$|KG(jw)| = 1 \quad$ PM $= -35^o$



- Nyquist Plot



$K = 0.1$

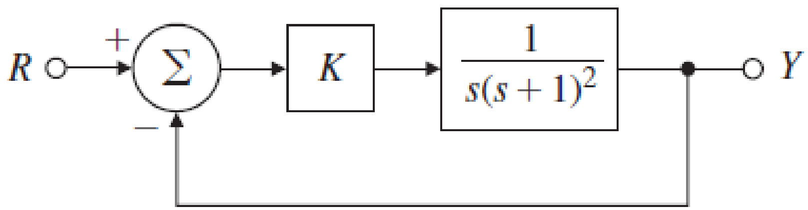
$K = 2$

$K = 10$

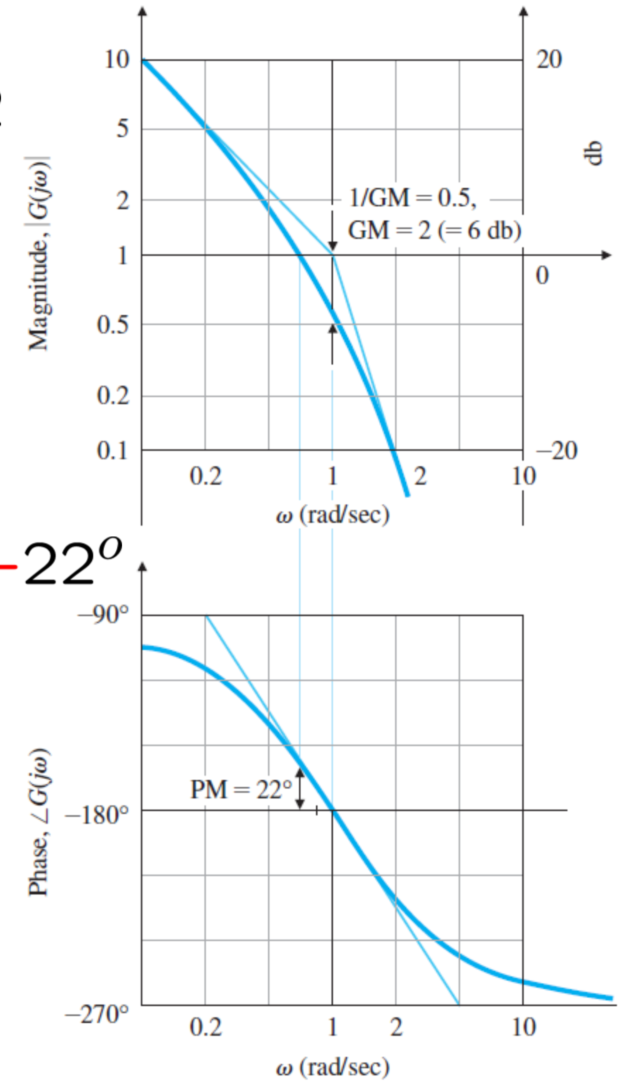
$|K\,G(jw)| = 1$ OR 0 db

- Crossover Frequency, ω_c

- In Example 6.9:

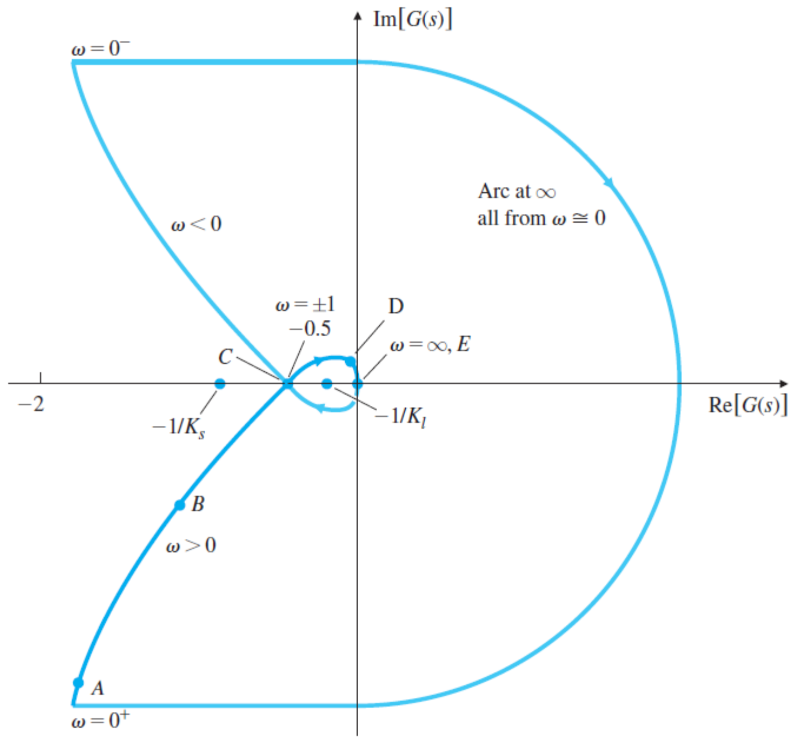


$GM = 2$

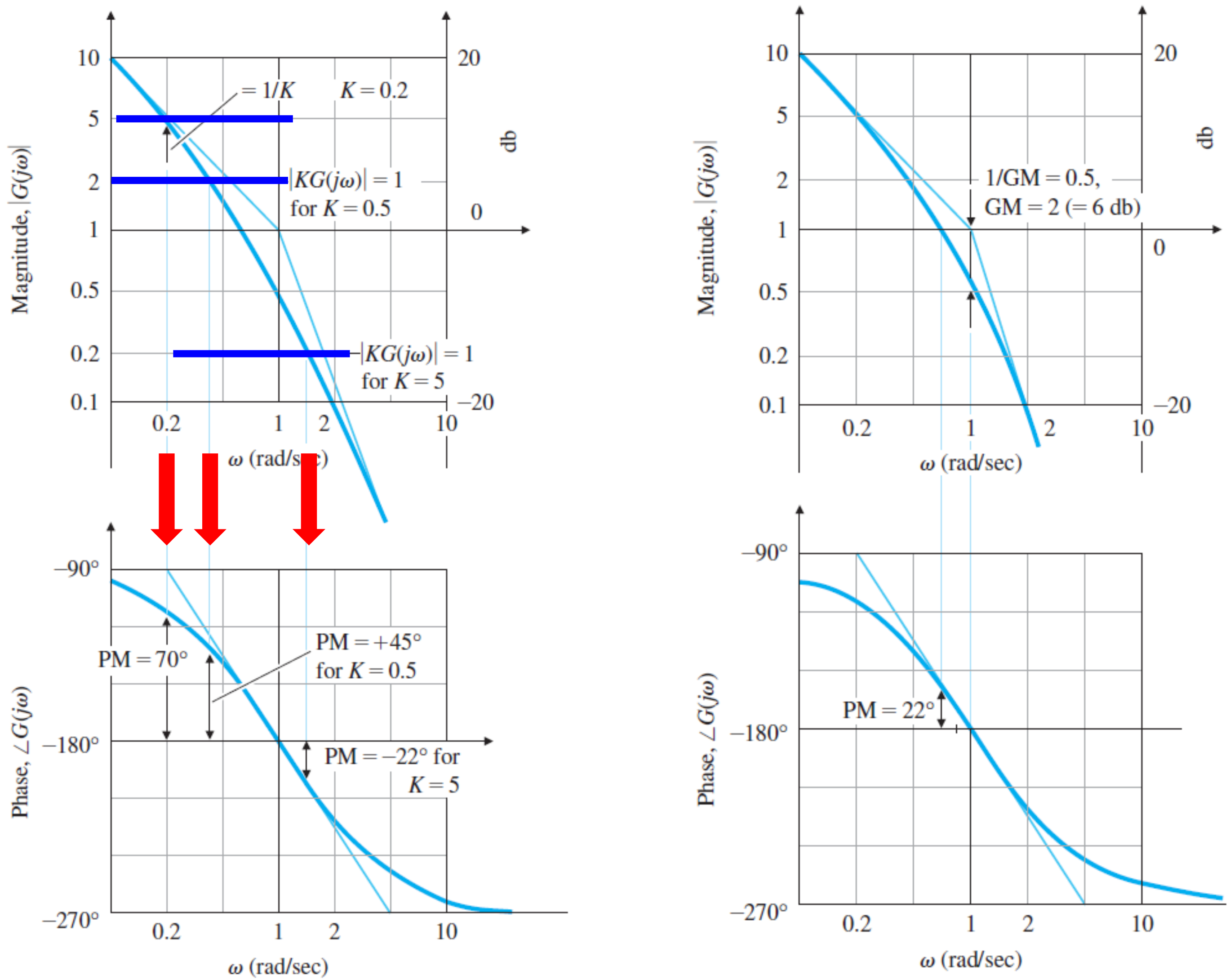


$PM = +22^{o}$

- Nyquist Plot



- PM vs K
- K = 5
- | KG(jw) | = 1
- PM = -22^o
- K = 0.5
- | KG(jw) | = 1
- PM = $+45^o$
- K = 0.2
- | KG(jw) | = 1
- PM = $+70^o$



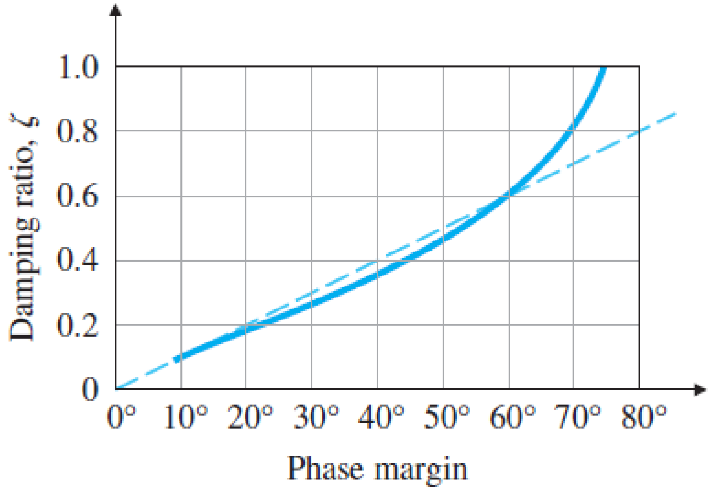
- The PM is more commonly used

  to specify control system performance

  because it is more closely to the damping ratio of the system.
- For the open-loop 2nd-order system: $G(s) = \dfrac{w_n^2}{s\,(\,s + 2\,\zeta\,w_n\,)}$
- With unity feedback, produces the closed-loop system:

$$T(s) = \frac{w_n^2}{s^2 + 2\,\zeta\,w_n\,s + w_n^2}$$

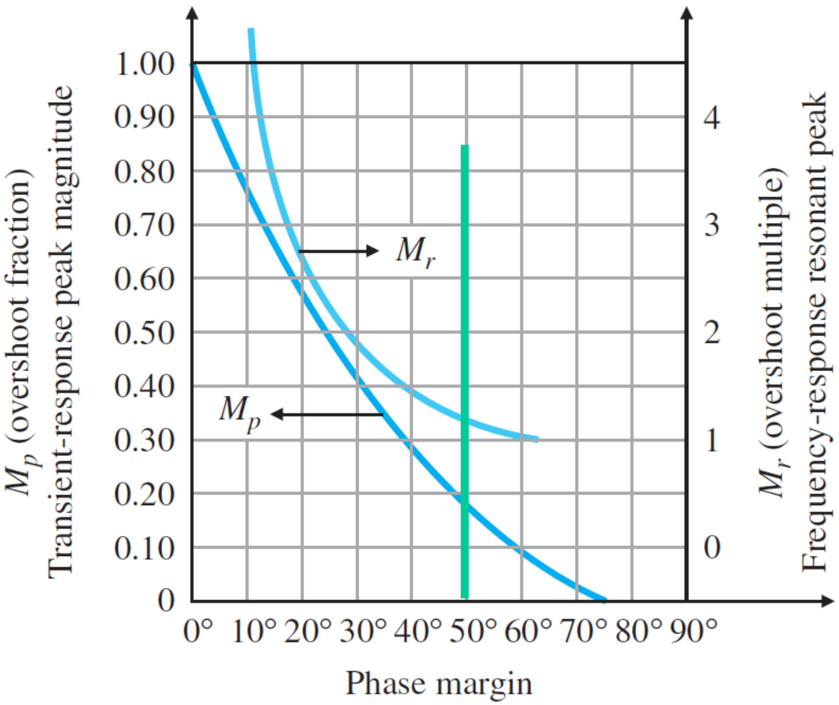
- The relationship between the PM and ζ is:



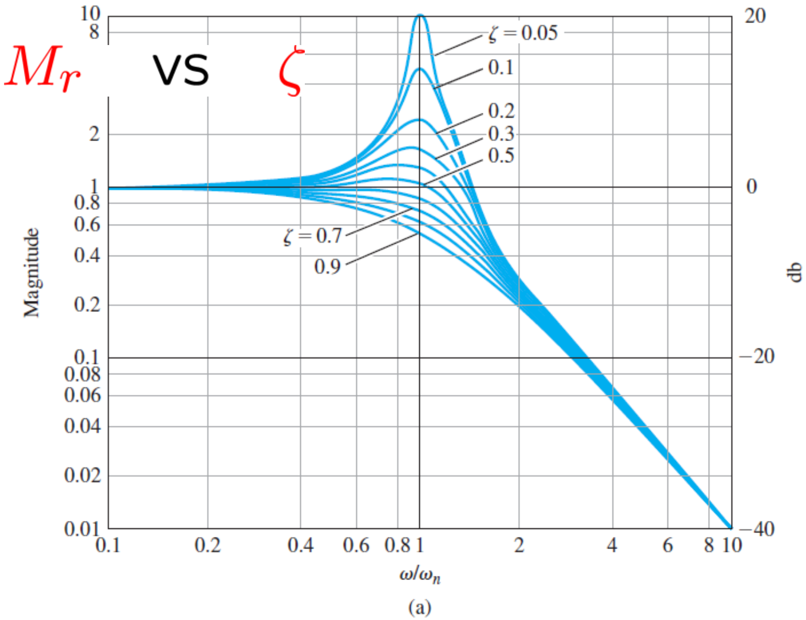
$$PM = \tan^{-1}\left[\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}\right]$$

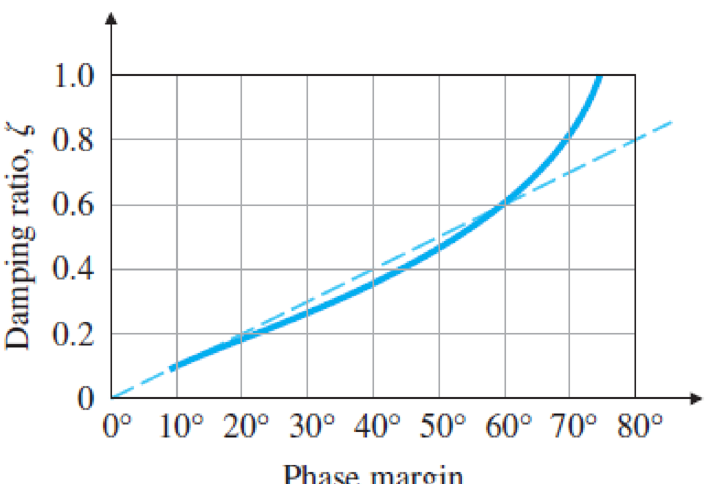
$$\zeta \approx \frac{PM}{100} \qquad \text{for} \quad PM < 60^o$$

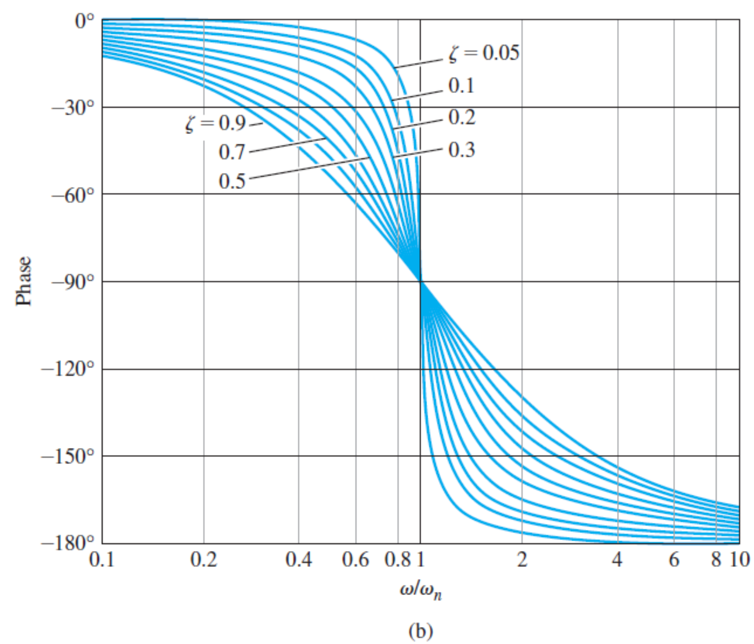
Stability Margins



M_r vs ζ







- In some cases,

  the PM and GM are not useful indicators of stability.

- For 1st- and 2nd-order systems,

  the phase never crosses the 180^o line;

- Hence, the GM is always ∞ and not a useful design parameter.

- For higher-order systems,

  it is possible to have more than one frequency

  where $| K\, G(jw) | = 1$ or where $\angle KG(jw) = 180^o$

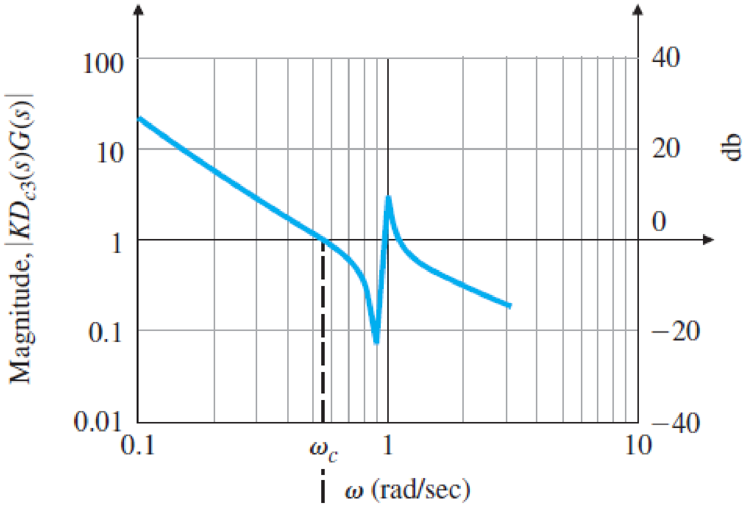
- And the margins as previously defined need clarification.

- An example as follows:

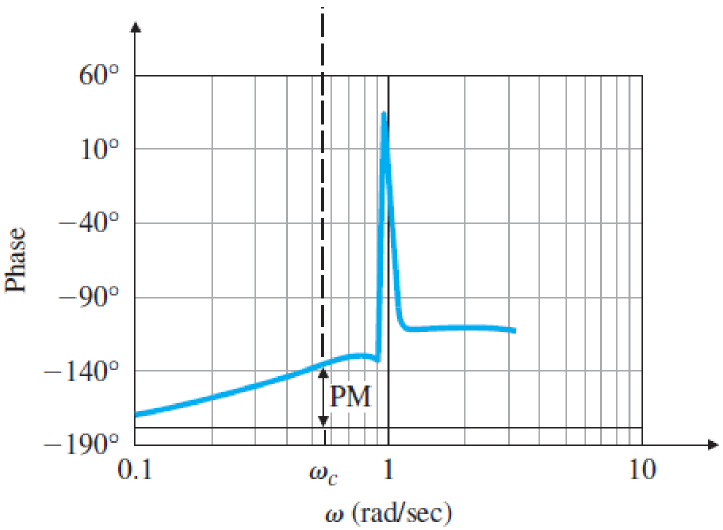
- In Chapter 10
- The magnitude crosses 1 three times
- Define PM by the first crossing
- Because the PM at this crossing

  was the smallest of these 3 values

and thus the most conservative assessment of stability

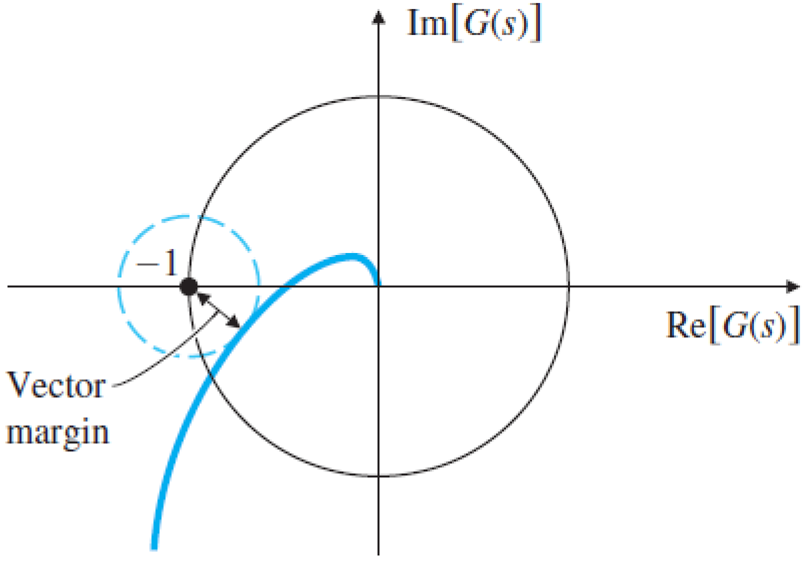


(a)



(b)

- Vector Margin (or Complex Margin)
- The distance to the -1 point
  from the closet approach
  of the Nyquist Plot

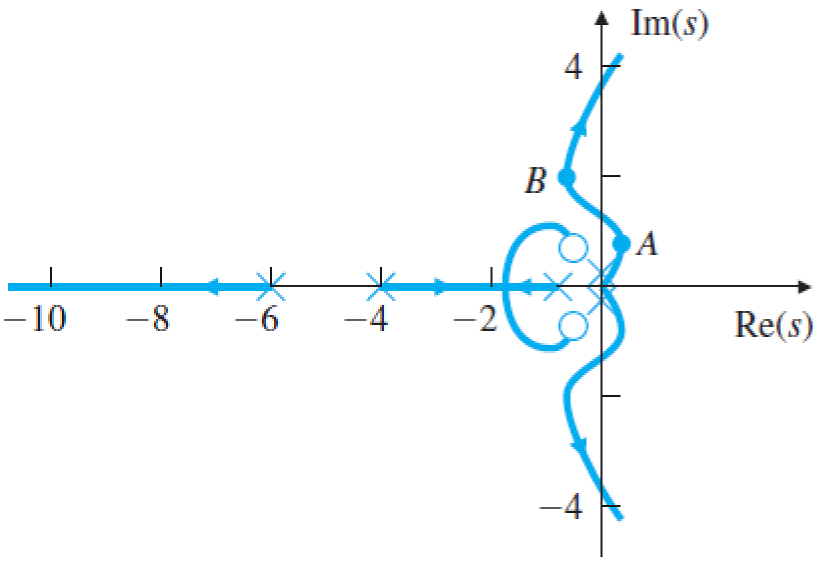


- Vector margin is
  a single margin parameter,
- It removes all the ambiguities
  in assessing stability
  that come with using GM and PM in combination.

- Conditionally Stable Systems
- Point A:
- Increase gain

  → make stable
- Point B:
- Increase/decrease gain

  → make unstable



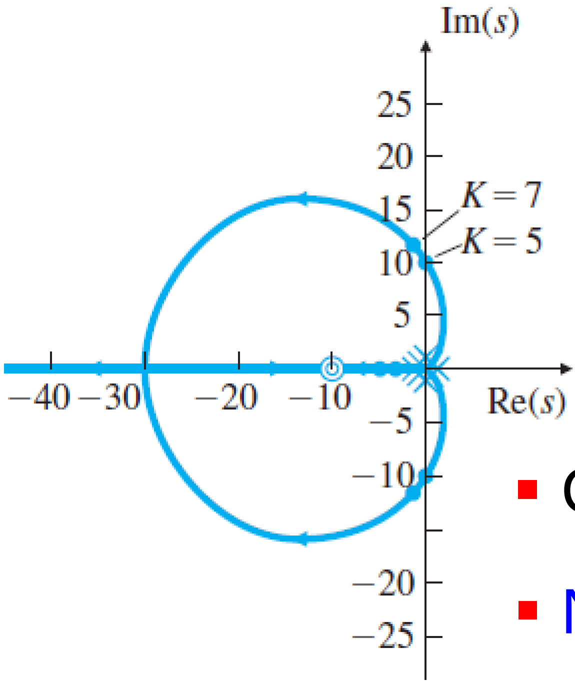
- Example 6.12: Stability Properties for a Conditionally Stable System

$$K\,G(s) = \frac{K\,(s+10)^2}{s^3}$$

- Conflicting !!!

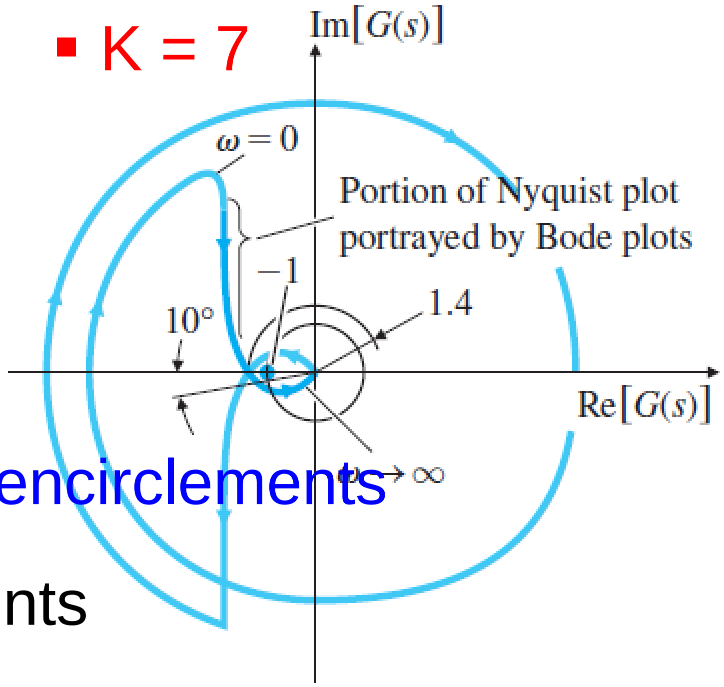
- Unstable: K < 5
- Stable: K > 5

- PM = $+10^o$ (Stable)
- GM = 0.6 (Unstable)



(a)

- K = 7



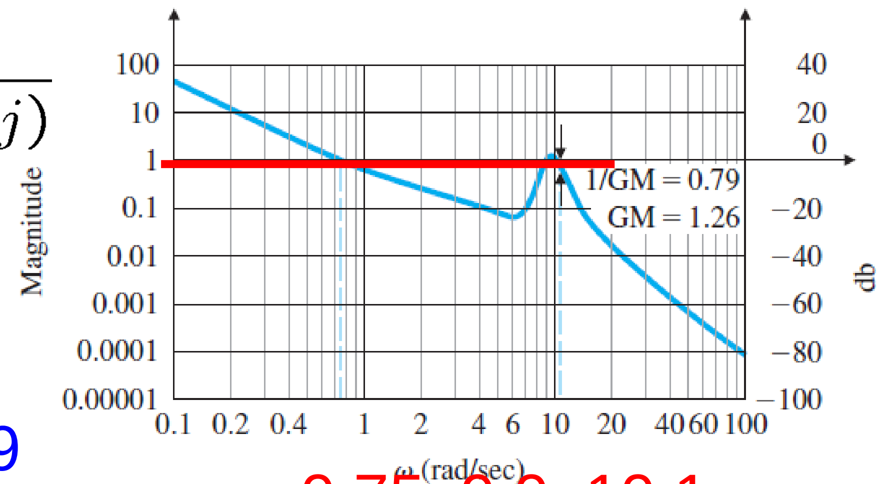
(b)

- Counting Nyquist encirclements
- No net encirclements
- → Stable for K = 7

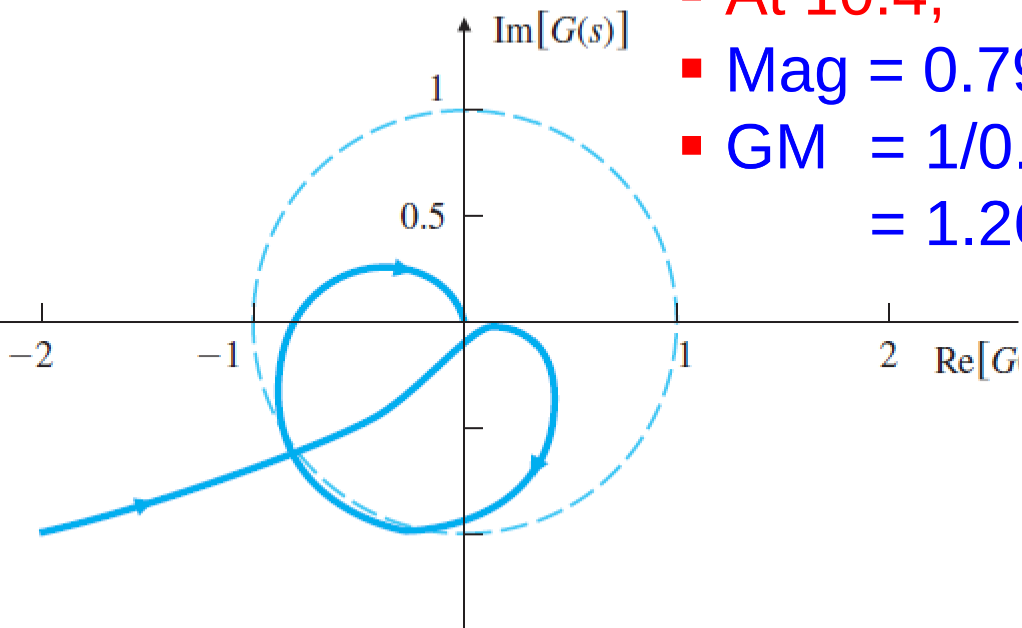
- Example 6.13: Nyquist Plot for a System with Multiple Crossover Frequencies

$$G(s) = \frac{85\,(s+1)\,(s^2+2s+43.25)}{s^2\,(s^2+2s+82)\,(s^2+2s+101)}$$

$$= \frac{85\,(s+1)\,(s+1\pm 6.5j)}{s^2\,(s+1\pm 9j)\,(s+1\pm 10j)}$$



- 3 crossover frequencies



- At 10.4,
- Mag = 0.79
- GM = 1/0.79 = 1.26

- 0.75, 9.0, 10.1
- PM = $37^o, 80^o, 40^o$