Spring 2021

控制系統
Control Systems

Unit 6E
The Nyquist Stability Criterion

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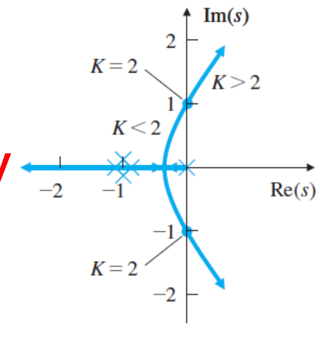
- For most systems,

  an increasing gain eventually causes instability

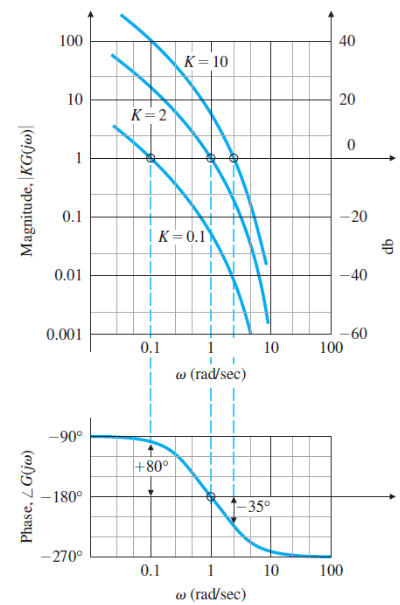


(b)

- In very early days of feedback control design,

  this relationship between gain and stability margins

  was assumed to be universal.



- However, designers found occasionally that

  the relationship reversed itself;

- That is, the amplifier would become unstable

  when the gain was decreased.

- The confusion motivated Harry Nyquist of Bell Tele Lab in 1932

- The Nyquist Stability Criterion

- Nyquist Stability Criterion:
- Based on the Argument Principle in complex variable theory.
- Relate OL frequency response

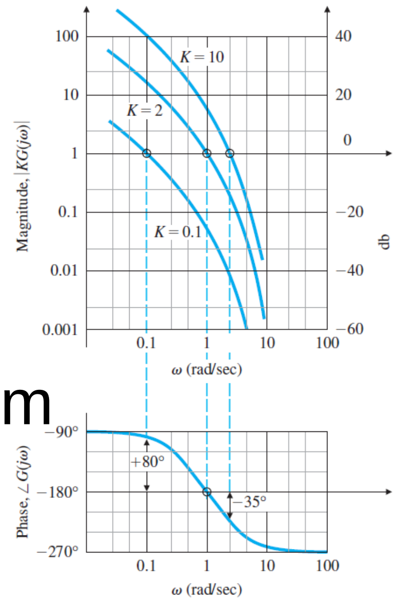
  to the number of CL poles in the RHP
- Determine stability

  from frequency response of a complex system
- The magnitude curve crosses 1 several times

  and/or the phase curve crosses 180^o several times.
- Deal with (a) OL unstable systems,

  (b) non-minimum-phase systems,

  (c) systems with pure delays

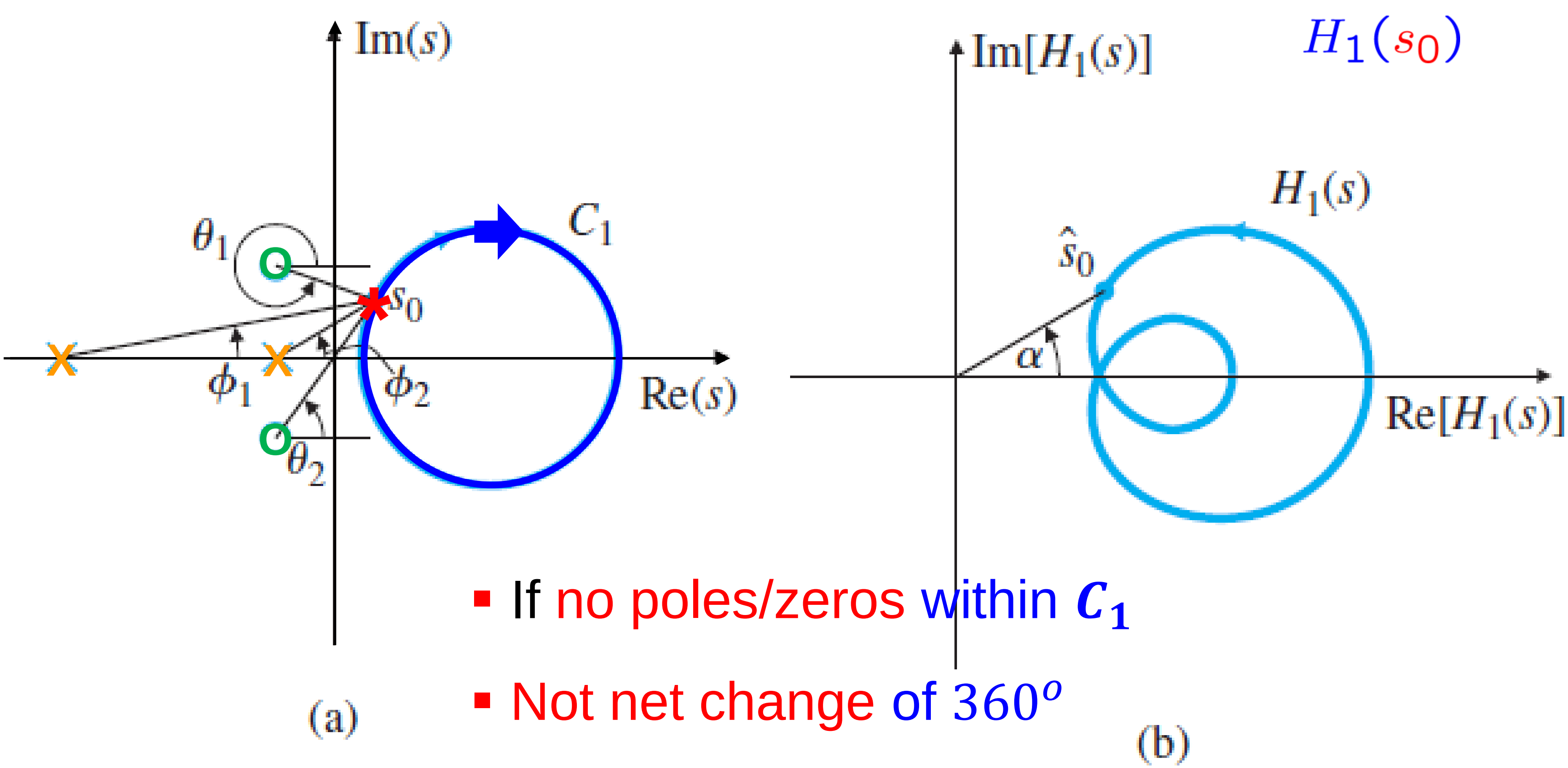


The Argument Principle

- $H_1(s)$
- s_0
- Contour Evaluation

$$H_1(s_0) = \vec{v} = |\vec{v}|\, e^{j\,\alpha}$$

argument: $\alpha = \theta_1 + \theta_2 - (\phi_1 + \phi_2)$

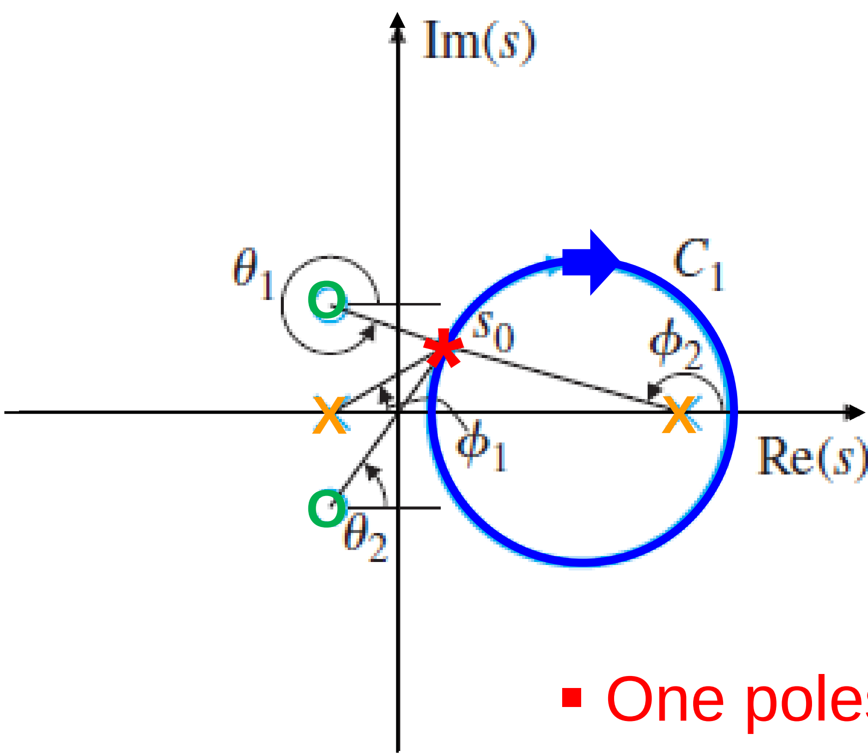


- If no poles/zeros within C_1
- Not net change of 360^o

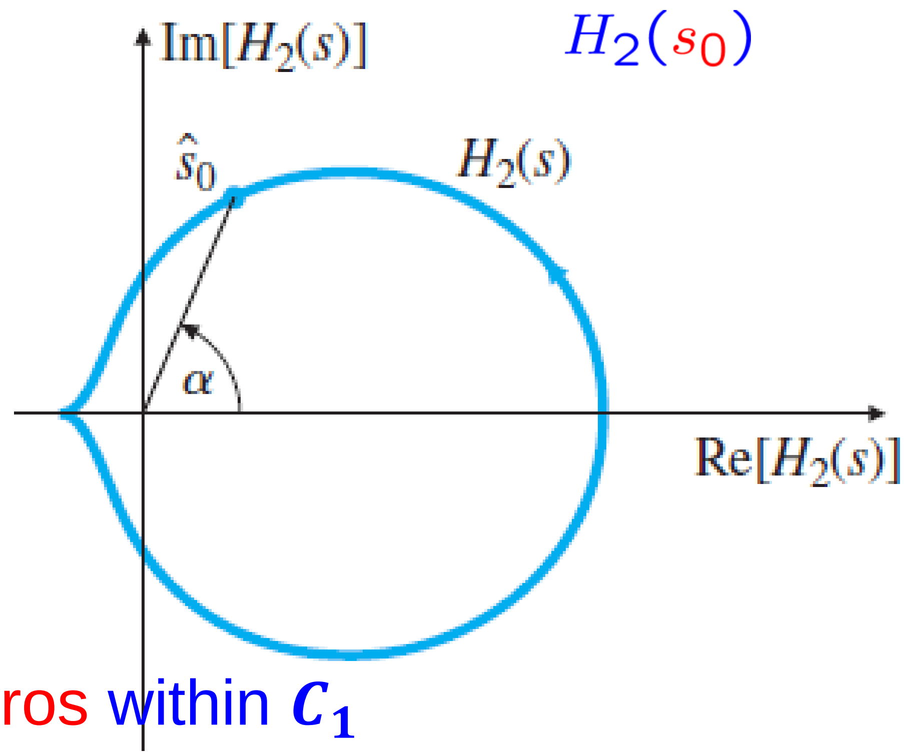
- $H_2(s)$
- s_0
- Contour Evaluation

$$H_2(s_0) = \vec{v} = |\vec{v}|\, e^{j\,\alpha}$$

argument: $\alpha = \theta_1 + \theta_2 - (\phi_1 + \phi_2)$



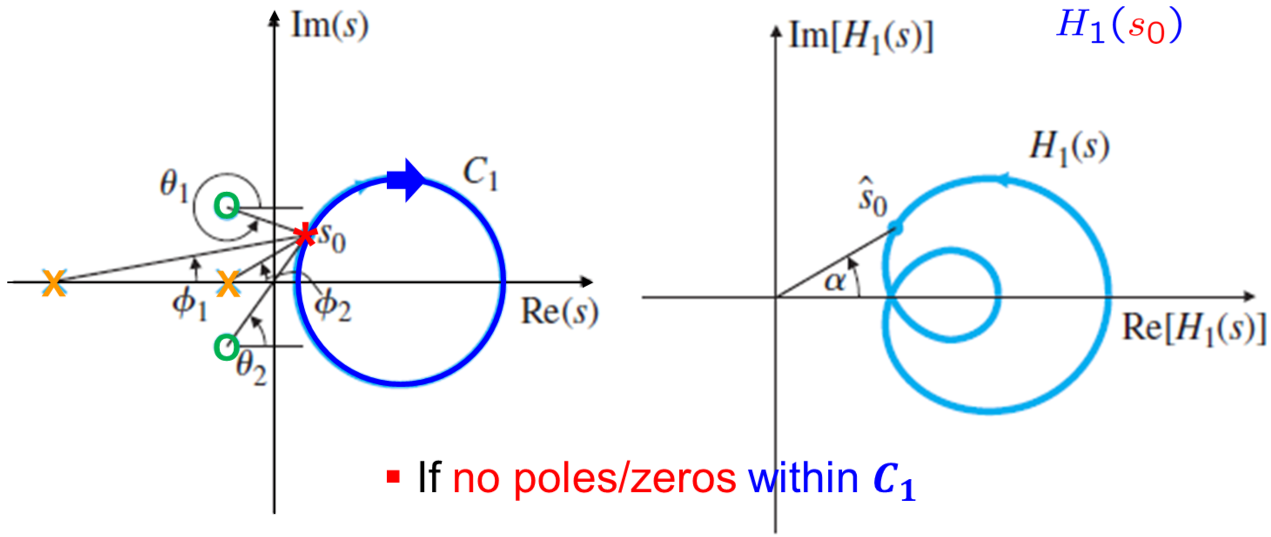
(c)



(d)

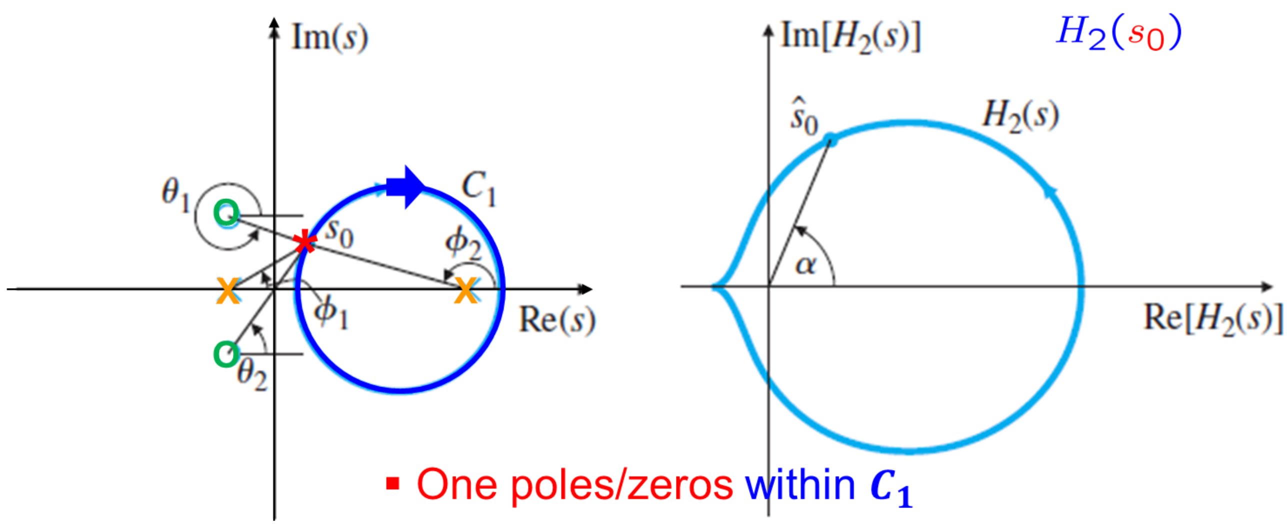
- One poles/zeros within C_1
- Net change of -360^o

The Argument Principle



- If no poles/zeros within C_1
- Not net change of 360^o

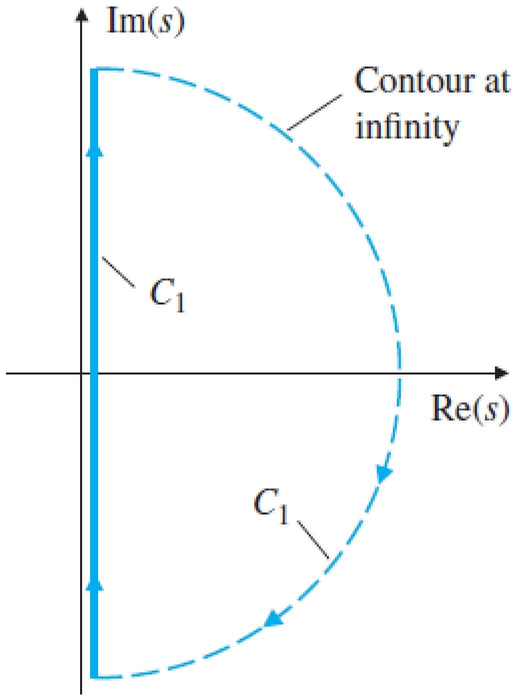
(a) (b)



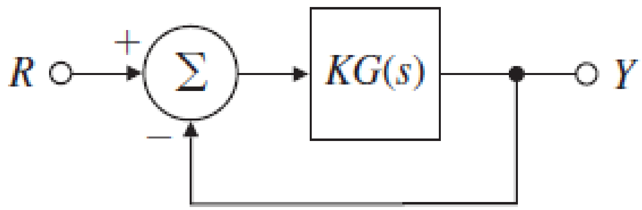
- One poles/zeros within C_1
- Net change of -360^o

(c) (d)

- The essence of the Argument Principle
- A contour map of a complex function
  will encircle the origin Z – P times,
- where Z is the number of zeros
  and P is the number of poles
  of the function inside the contour.
- For controller design,
  let the C_1 contour encircle entire RHP,
  where a pole would cause an unstable system.

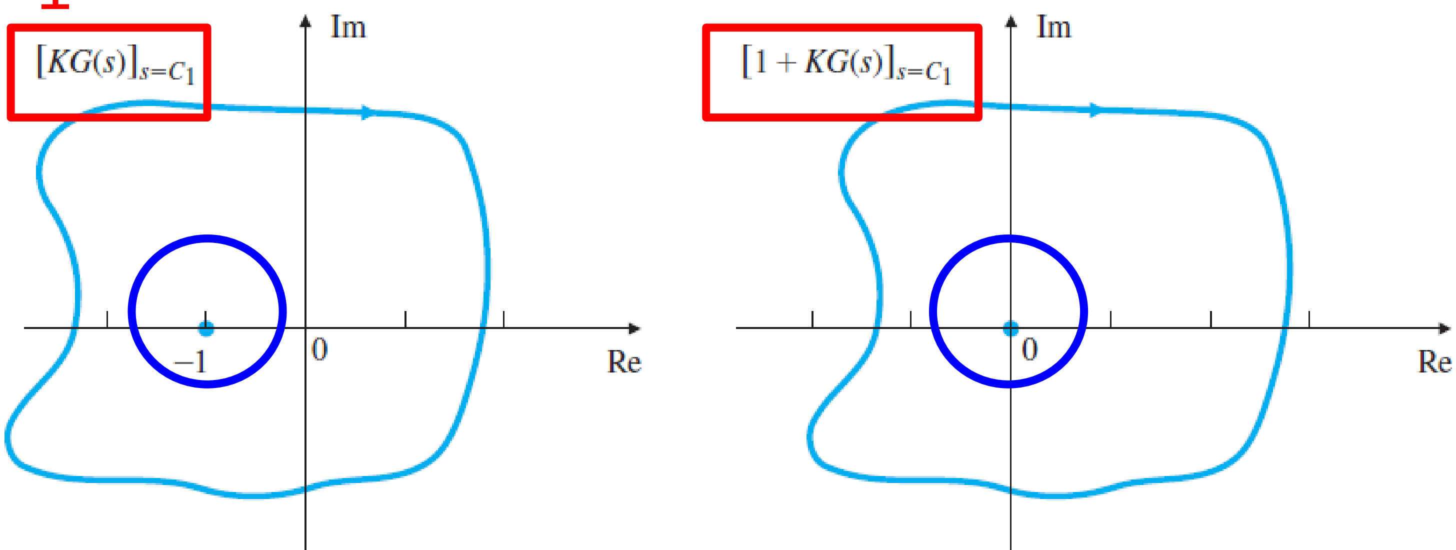


The Argument Principle



$$T(s) = \frac{Y(s)}{R(s)} = \frac{K\,G(s)}{1 + K\,G(s)}$$

- CL roots: $1 + K\,G(s) = 0$
- The contour evaluation for $1 + K\,G(s) = 0$
- → Encircling the origin!
- Equivalently, the contour evaluation for $K\,G(s) = 0$
- → Encircling -1
- Nyquist Plot
- Polar Plot



The Argument Principle

$$1 + K\,G(s) = 1 + K\,\frac{b(s)}{a(s)} = \frac{a(s) + K\,b(s)}{a(s)}$$

$$\text{poles of } 1 + K\,G(s) = \text{poles of } G(s)$$

- A clockwise contour C_1 enclosing a zero of 1 + KG(s)

  result in KG(s) encircling the -1 point in a clockwise direction

- Likewise, C_1 enclose a pole of 1 + KG(s)

  (if there is an unstable OL pole)

  there will be a counterclockwise encirclement of the -1 point.

- Furthermore, two poles or zeros are in the RHP,

  KG(s) will encircle the -1 point twice, and so on.

- Net number of CW encirclements N = Z - P

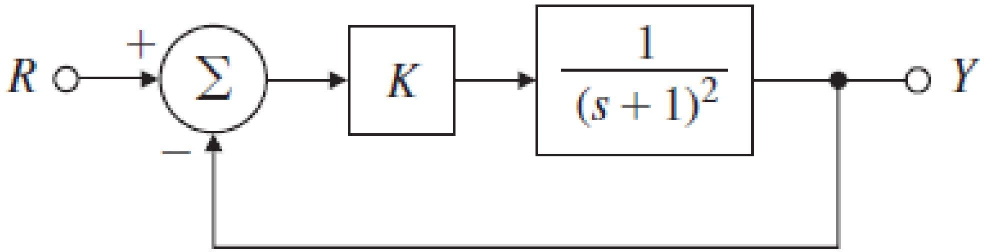
  Z = zeros in RHP, P = poles in RHP

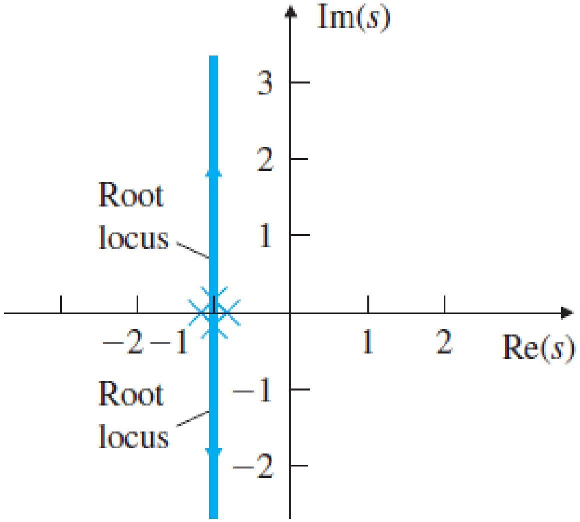
Procedure for Determining Nyquist Stability

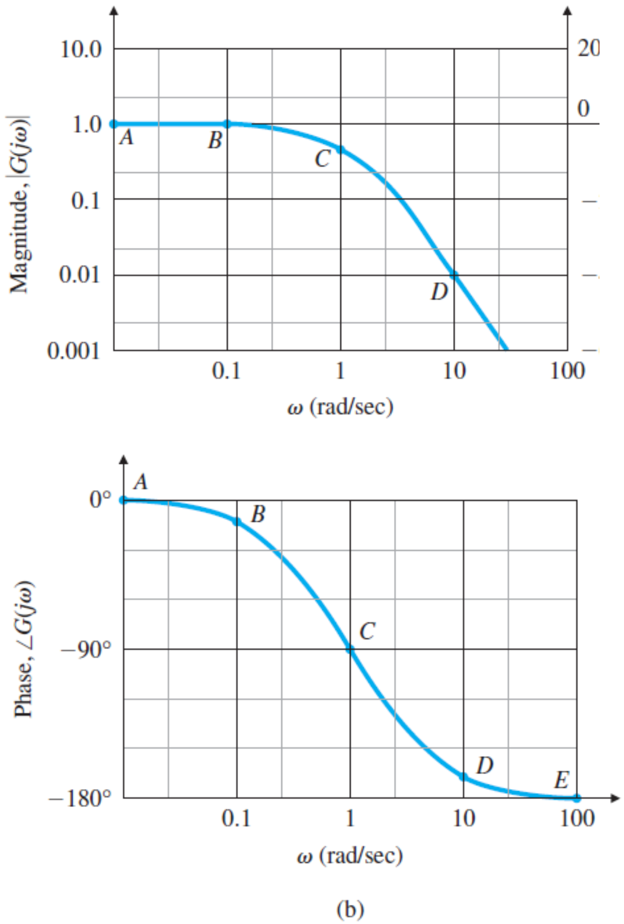
1. Plot $KG(s)$ for $-j\infty \leq s \leq +j\infty$. Do this by first evaluating $KG(j\omega)$ for $\omega = 0$ to ω_h, where ω_h is so large that the magnitude of $KG(j\omega)$ is negligibly small for $\omega > \omega_h$, then reflecting the image about the real axis and adding it to the preceding image. The magnitude of $KG(j\omega)$ will be small at high frequencies for any physical system. The Nyquist plot will always be symmetric with respect to the real axis. The plot is normally created by the NYQUIST Matlab function.
2. Evaluate the number of clockwise encirclements of -1, and call that number N. Do this by drawing a straight line in any direction from -1 to ∞. Then count the net number of left-to-right crossings of the straight line by $KG(s)$. If encirclements are in the counterclockwise direction, N is negative.
3. Determine the number of unstable (RHP) poles of $G(s)$, and call that number P.
4. Calculate the number of unstable closed-loop roots Z:

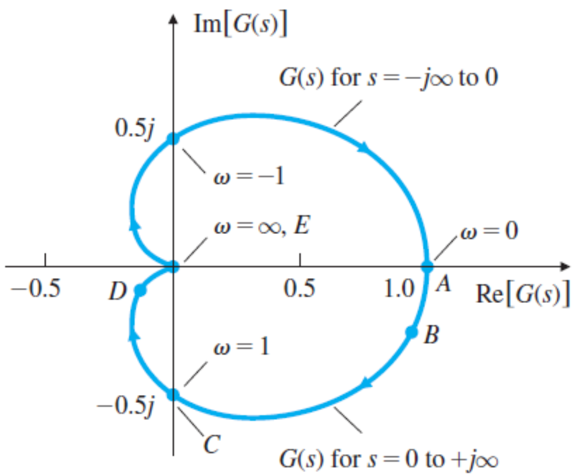
$$Z = N + P. \tag{6.28}$$

- Example 6.8: Nyquist Plot for a Second-Order System



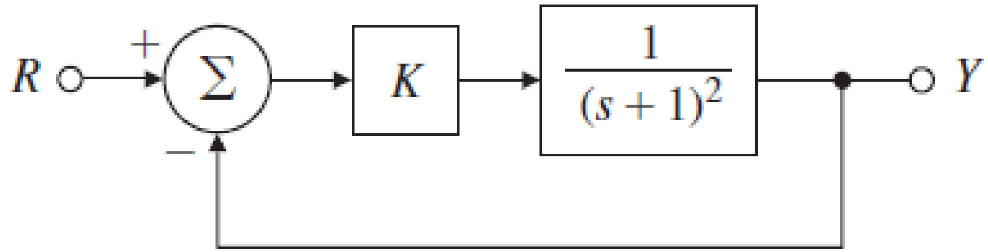


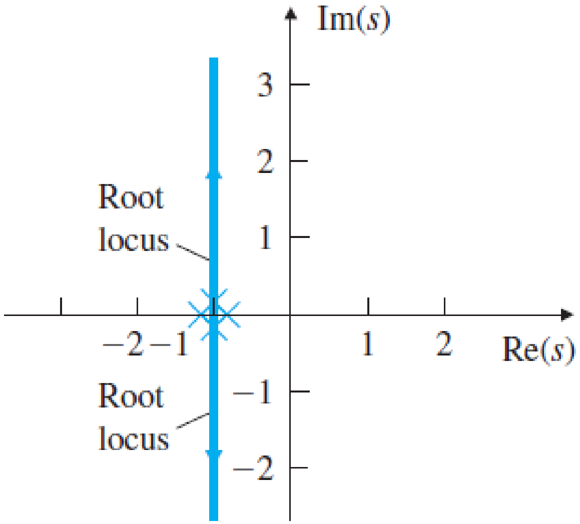


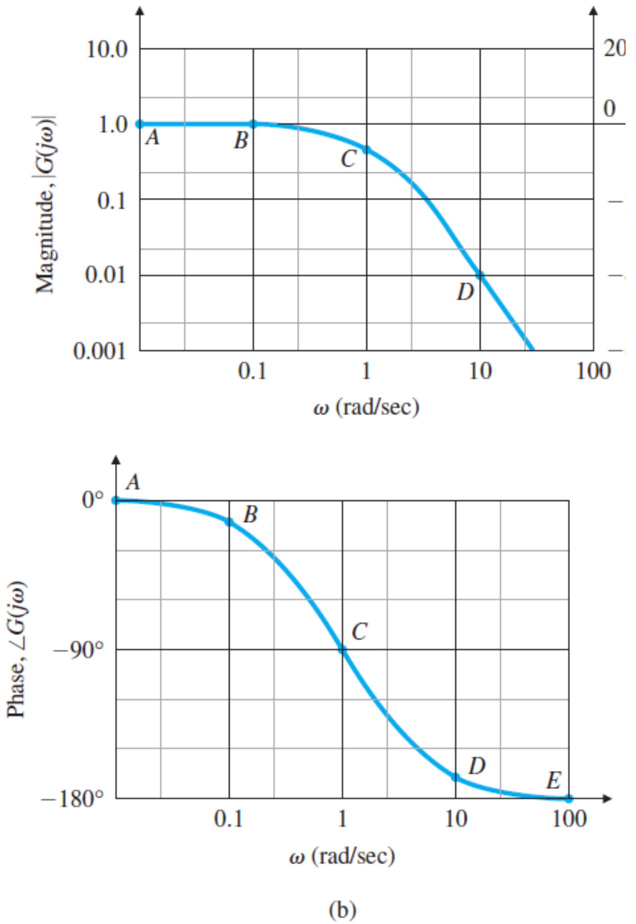


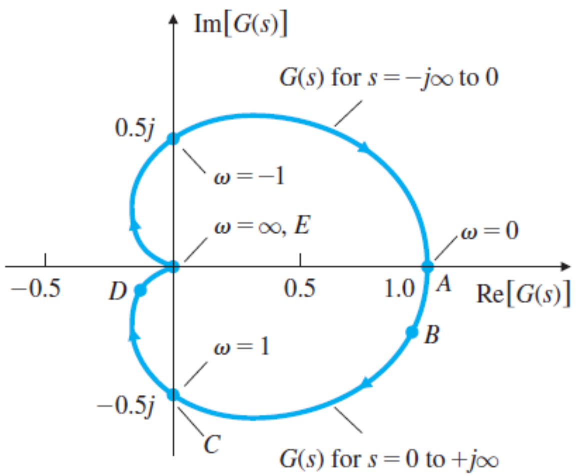

```
w = logspace(-2,3,100);
num = 1;
den = [1 2 1];

[re,im] = nyquist( num, den, w );
plot( re, im, re, -im );
```

- Example 6.8: Nyquist Plot for a Second-Order System





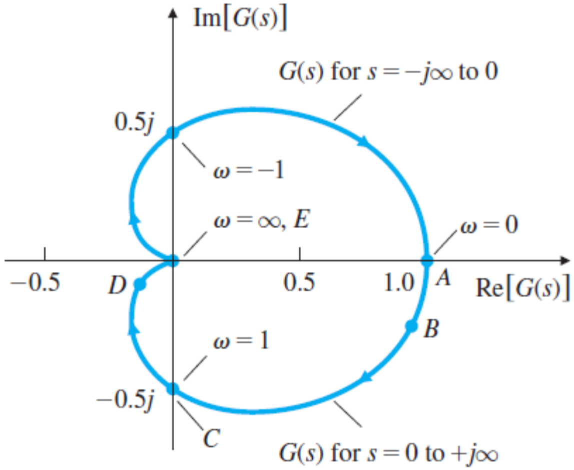




- N = 0: not encircle -1
- P = 0: no poles of G(s) in RHP
- Z = N + P → Z = 0,
  no unstable roots for K = 1
- K > 0 also holds

- Example 6.8: Nyquist Plot for a Second-Order System

- Another viewpoint:
  - 1 + KG(s) for the origin point
  - KG(s) for the -1 point
  - G(s) for the -1/K point

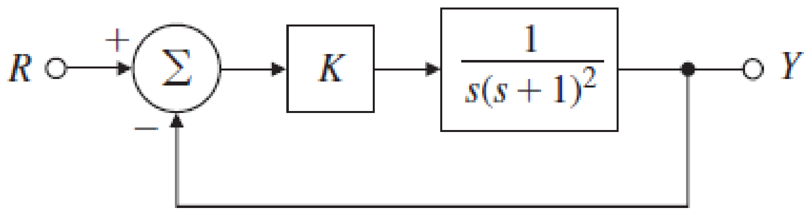


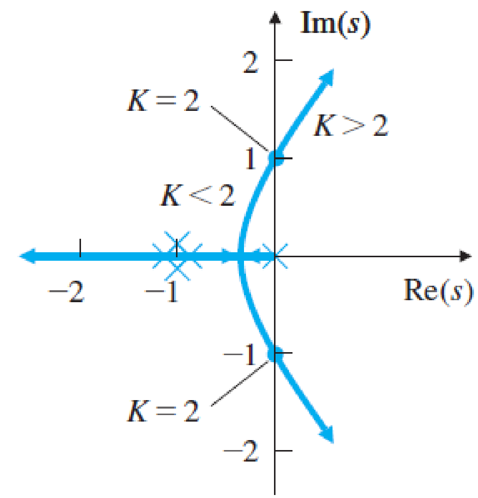
- No encirclement of G(s) on -1/K

  for any K > 0

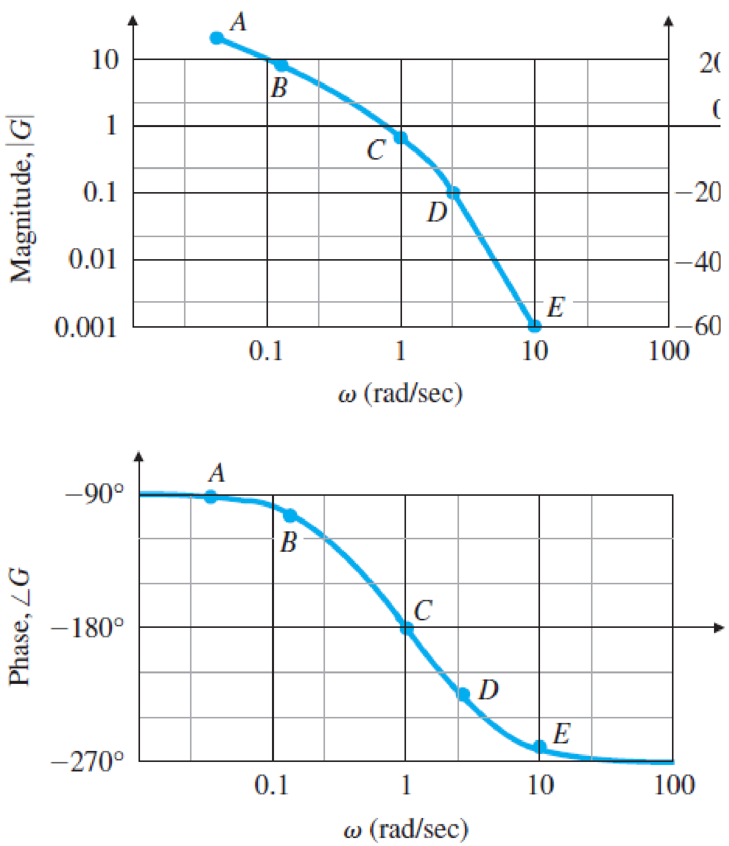
- Hence, K > 0 is stable

Example 6.9: Nyquist Plot for a Third-Order System





(b)



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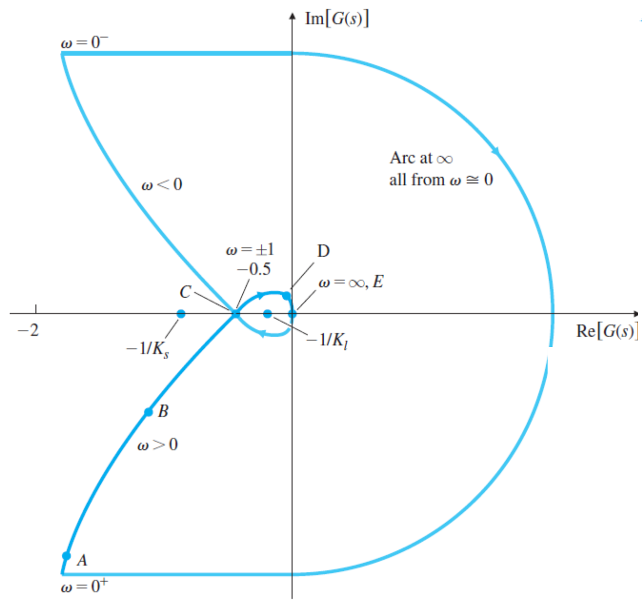
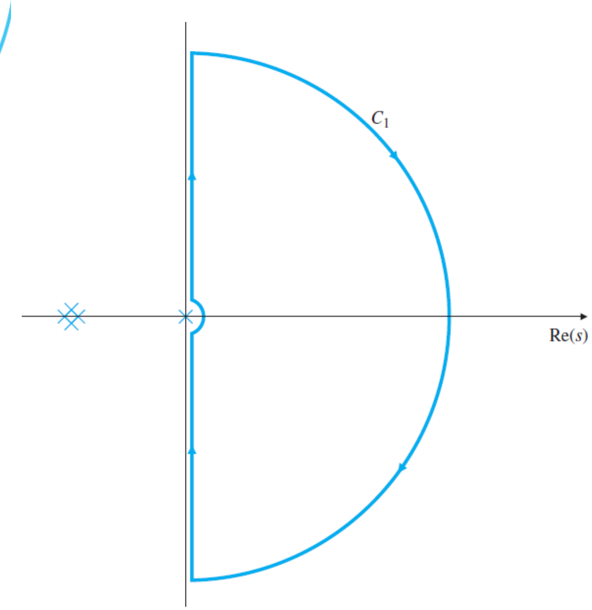
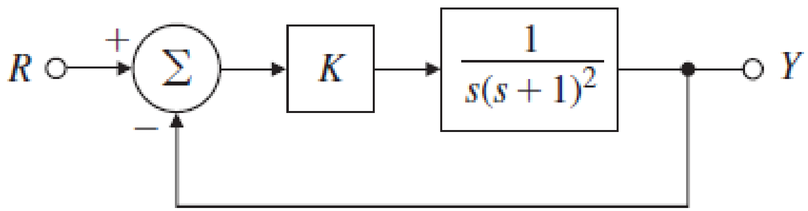
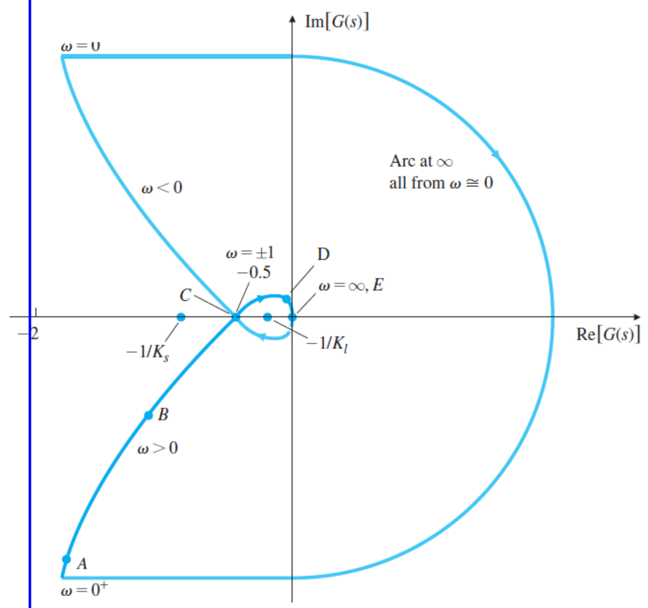




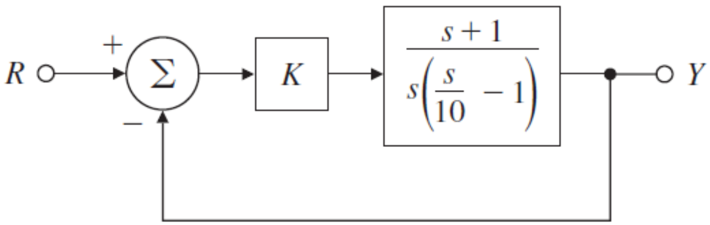
▪ Example 6.9: Nyquist Plot for a Third-Order System

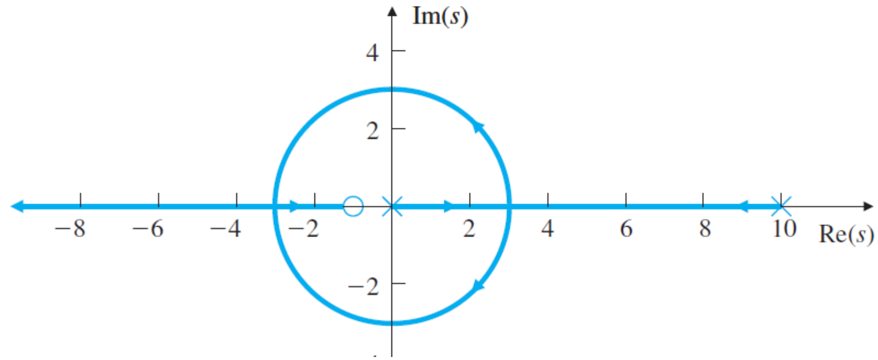


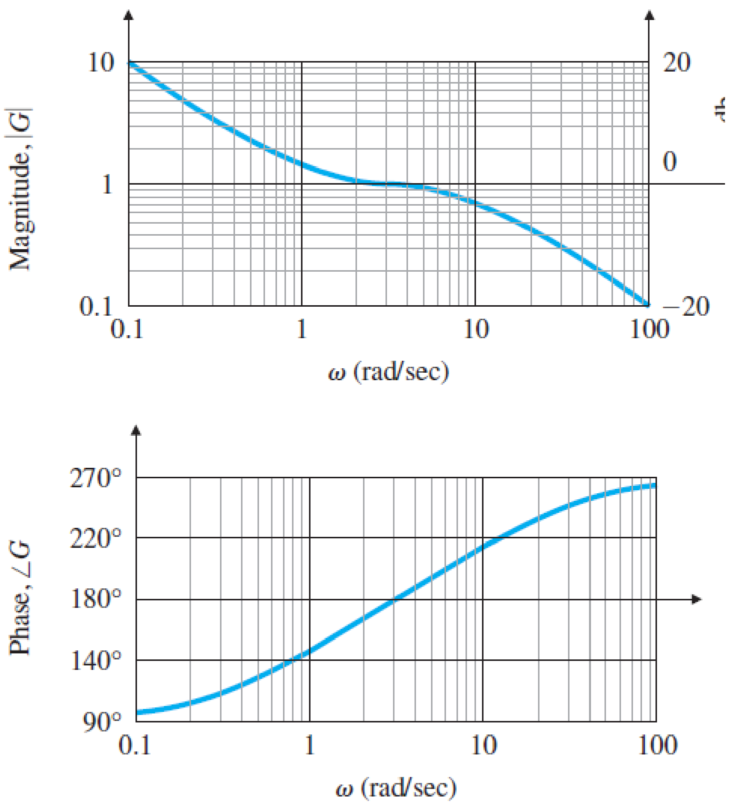

```
w=logspace(-2,3,100);

num = 1;
den = conv([1 0],[1 2 1]);

[re,im] = nyquist(num,den,w);

plot(re,im,re,-im,'LineWidth',2);

ii=[11 21 47 61];

plot(re(ii),im(ii),'*');
plot(-.5,0,'*')
```



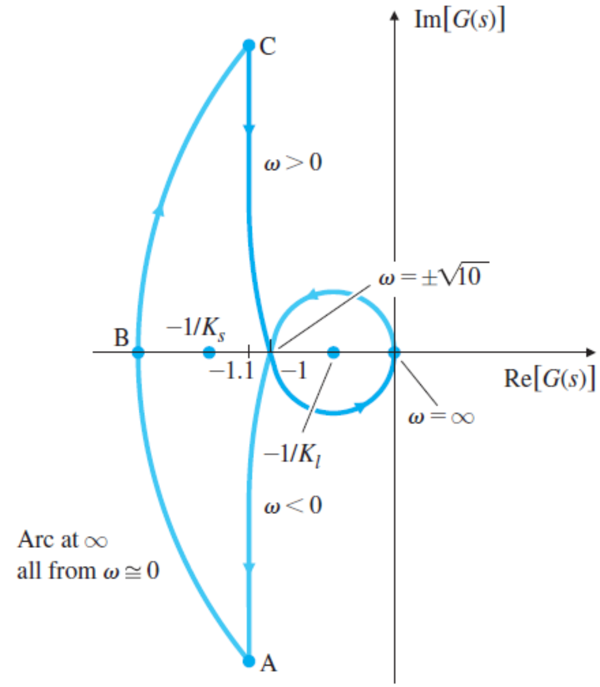
Example 6.10: Nyquist Plot for an Open-Loop Unstable System



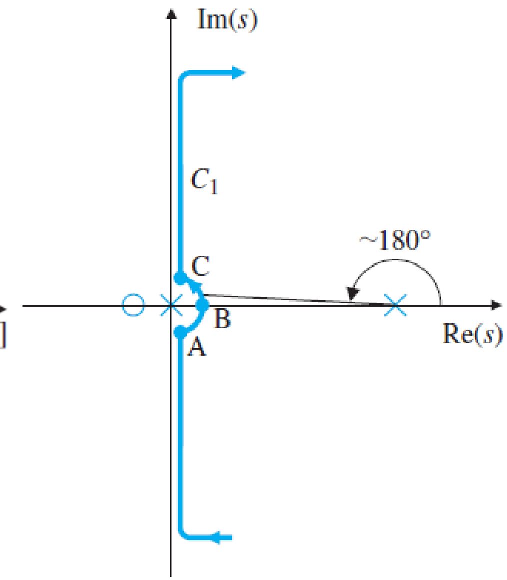




(b)

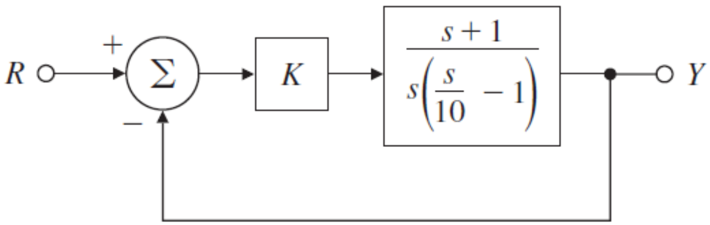


(a)

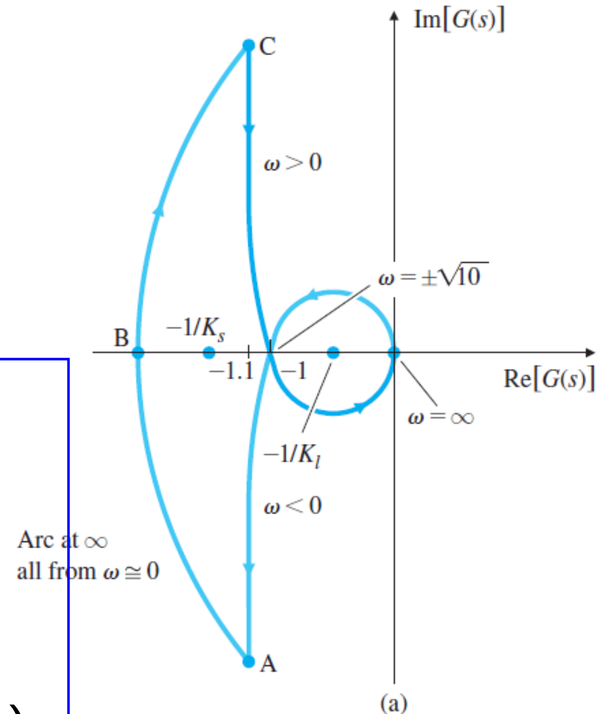


(b)

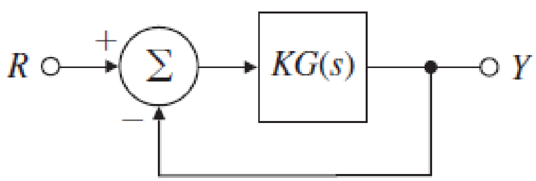
▪ Example 6.10: Nyquist Plot for an Open-Loop Unstable System



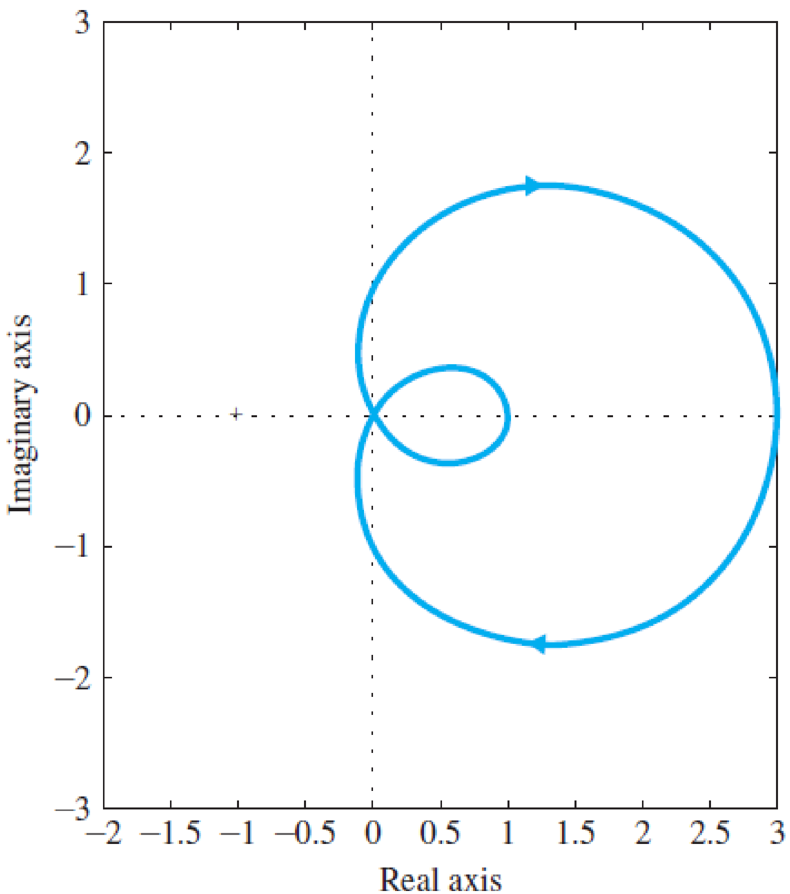

```
w = logspace(-3,3,100);

num = [1 1];
den = conv([1 0],[0.1 -1]);

[re,im] = nyquist( num, den, w );
plot( re, im, re, -im );
```



- Example 6.11: Nyquist Plot Characteristics



$$G(s) = \frac{s^2+3}{(s+1)^2}$$



- Never cross negative-real axis
- Stable for K > 0

```
w = logspace(-3,3,100);

num = [1 0 3];
den = conv([1 1],[1 1]);

[re,im] = nyquist( num, den, w );
plot( re, im, re, -im );

sysG = (s^2 + 3)/(s+1)^2;
nyquist( sysG );
```