

Spring 2021

控制系統
Control Systems

Unit 6C

Non-Minimum Phase and Steady-State Errors

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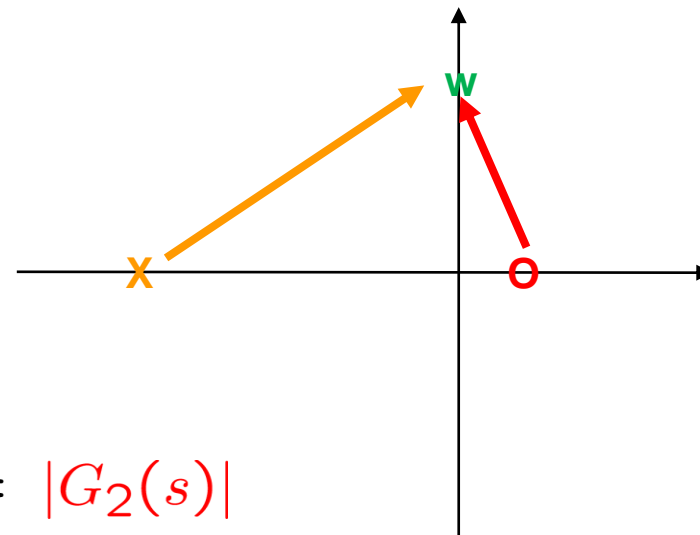
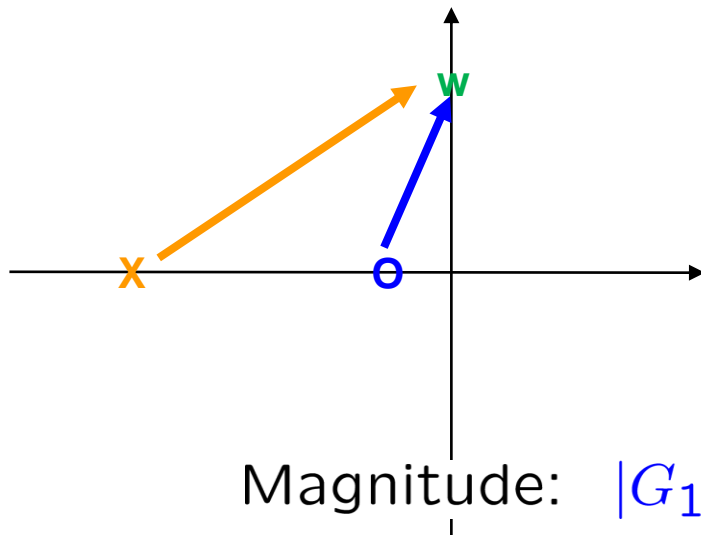
NTU-EE

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Non-Minimum-Phase Systems

$$G_1(s) = 10 \frac{s + 1}{s + 10}$$

$$G_2(s) = 10 \frac{s - 1}{s + 10}$$



Magnitude: $|G_1(s)| = |G_2(s)|$

Phase: $\angle G_1(s), \angle G_2(s)$: different

For this example,

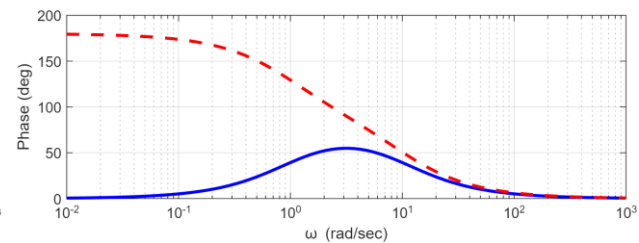
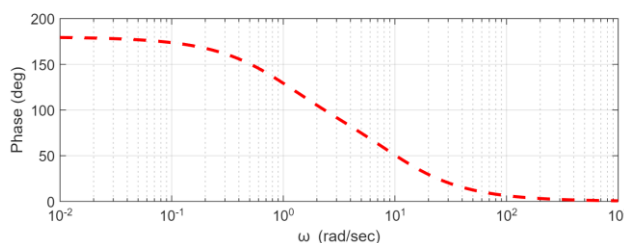
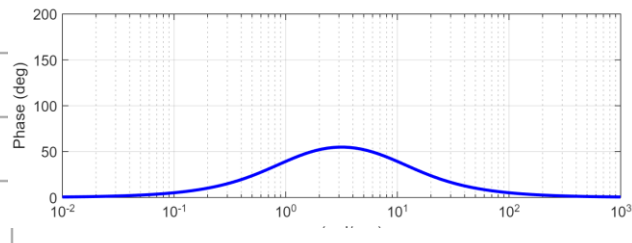
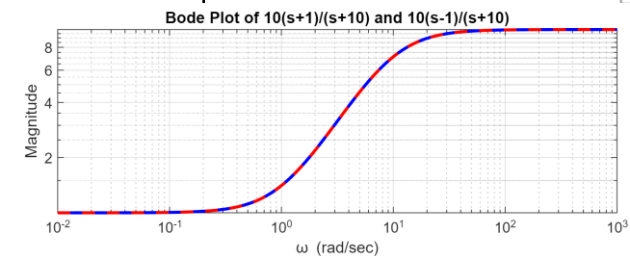
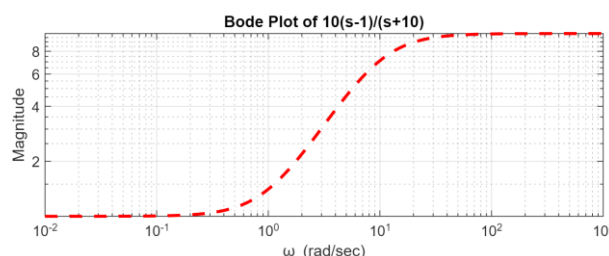
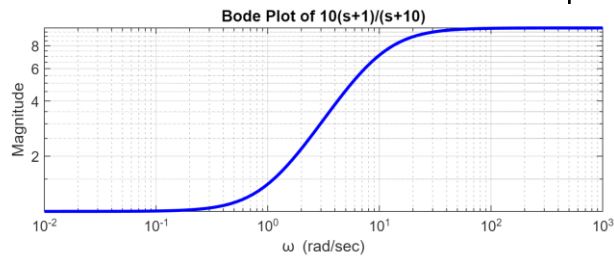
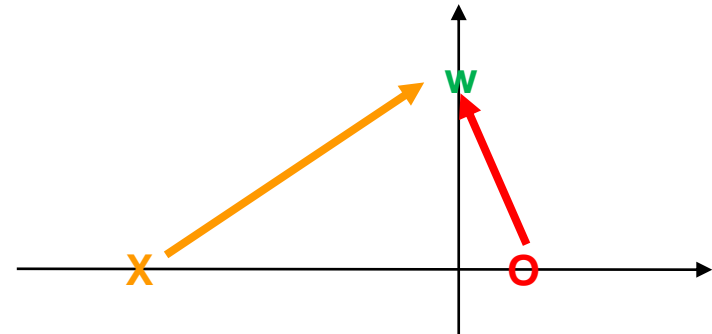
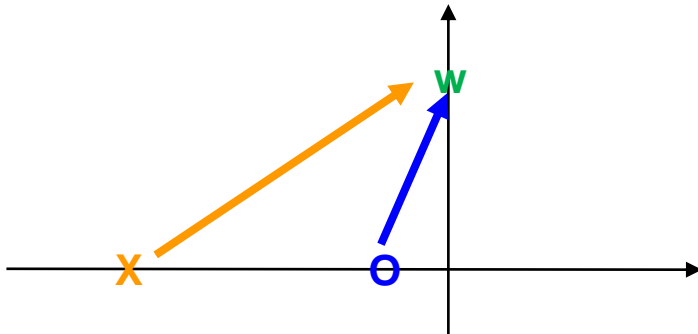
→ small w : $|\angle G_1(s) - \angle G_2(s)| = 180^\circ$

→ large w : $|\angle G_1(s) - \angle G_2(s)| = 0^\circ$

Non-Minimum-Phase Systems

$$G_1(s) = 10 \frac{s + 1}{s + 10}$$

$$G_2(s) = 10 \frac{s - 1}{s + 10}$$



→ small w : $|\angle G_1(s) - \angle G_2(s)| = 180^\circ$

→ large w : $|\angle G_1(s) - \angle G_2(s)| = 0^\circ$

- In Section 4.2,

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1 + K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

- As the **gain** of the open-loop transfer function **increases**,
- **Steady-State Error** of a feedback system **decreases**.

- In Section 6.1.1,

$$K G(j\omega) = K_0 (j\omega)^n \frac{(j\omega\tau_1 + 1)(j\omega\tau_2 + 1)\cdots}{(j\omega\tau_a + 1)(j\omega\tau_b + 1)\cdots}$$

- At very low frequencies, OL TF is **approximated by**

$$K G(j\omega) \approx K_0 (j\omega)^n$$

- **Larger** the **magnitude** on low-frequency asymptote,
- **Lower** steady-state errors

$$K G(j\omega) \approx K_0 (j\omega)^n$$

- For $n = 0$, a Type 0 system,

$$e_{ss} = \frac{1}{1 + K_p}$$

- For $n = -1$, a Type 1 system,

- The low-frequency asymptote has a slope of -1.
- The gain = K_0/ω and the velocity-error constant is:

$$K_v = K_0$$

- For a unity-feedback system with a unit-ramp input,
- The steady-state error is:

$$e_{ss} = \frac{1}{K_v}$$

Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1 + K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

- The easiest way of **determining K_v** in a **Type 1** system
- is to read the **magnitude** of the low-frequency **asymptote**
at **$\omega = 1$ rad/sec**,
- because this asymptote is **$A(\omega) = K_v / \omega$** .

- In some cases, the **lowest-frequency break point**
will be below **$\omega = 1$ rad/sec**;
- Therefore, the **asymptote** needs to **extend to $\omega = 1$ rad/sec**
in order to read **K_v** directly.

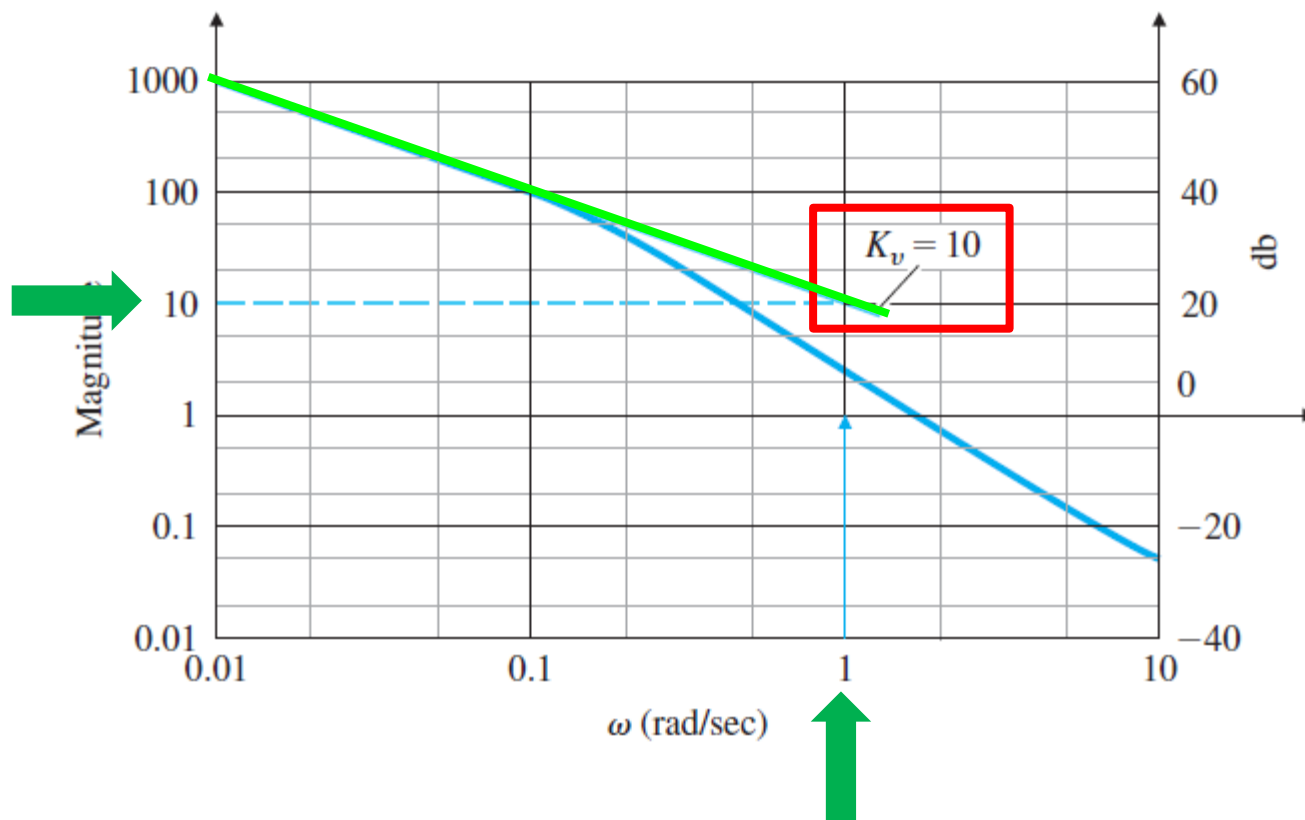
- Example 6.7: Computation of K_v

$$KG(s) = \frac{10}{s(s+1)}$$

- slope = -1

- Type 1 system

$$e_{ss} = \frac{1}{K_v} = 0.1$$



- Or, by $A(\omega) = K_v / \omega \rightarrow 1000 = K_v / 0.01 \rightarrow K_v = 10$