

Spring 2021

控制系統
Control Systems

Unit 6B
Bode Plot Techniques

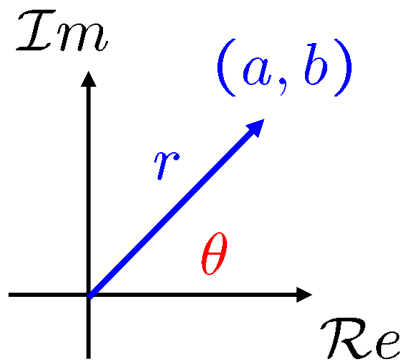
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Feb – Jun, 2021

- The hand plotting was developed by **H.W. Bode** at **Bell Laboratories** between **1932-1942**.
- Now, most control system designers **use computer programs** to illustrate the Bode plot.
- However, it is still important to **develop good intuition** so that you can **quickly identify erroneous** computer result and **perform sanity check** and determine **approximate result** by hand
- The idea in Bode's method is **to plot magnitude curves** using a **logarithmic scale** and **phase curves** using a **linear scale**

■ Magnitude & Phase Representation:



$$a + jb \Rightarrow \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow a + jb = r e^{j\theta}$$

$$|a + jb| = \sqrt{a^2 + b^2} \quad \left| \frac{1}{a + jb} \right| = \frac{1}{\sqrt{a^2 + b^2}}$$

$$\angle a + jb = \tan^{-1}\left(\frac{b}{a}\right) \quad \angle \frac{1}{a + jb} = -\tan^{-1}\left(\frac{b}{a}\right)$$

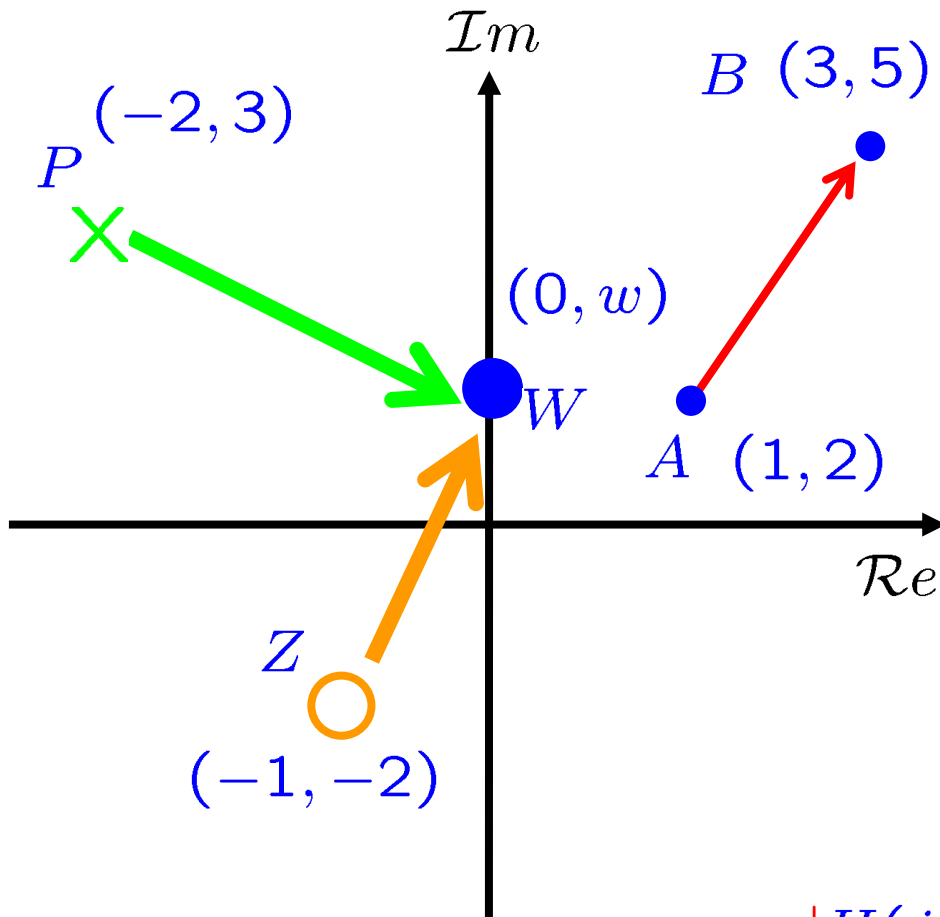
$$X(j\omega) = \text{Re}\{X(j\omega)\} + j \text{Im}\{X(j\omega)\} = |X(j\omega)| e^{j\angle X(j\omega)}$$

$$X(e^{j\omega}) = \text{Re}\{X(e^{j\omega})\} + j \text{Im}\{X(e^{j\omega})\} = |X(e^{j\omega})| e^{j\angle X(e^{j\omega})}$$

$|X(j\omega)|$ or $|X(e^{j\omega})|$: magnitude

$\angle X(j\omega)$ or $\angle X(e^{j\omega})$: phase angle

In s-plane:



$$\begin{aligned}\overrightarrow{AB} &= (3 + 5j) - (1 + 2j) \\ &= 2 + 3j\end{aligned}$$

$$\overrightarrow{AB} = (3, 5) - (1, 2) = (2, 3)$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2}$$

$$\angle \overrightarrow{AB} = \tan^{-1} \frac{(5 - 2)}{(3 - 1)}$$

$$H(s) = \frac{s - (-1 - 2j)}{s - (-2 + 3j)}$$

$$|H(jw)| = \frac{|jw - (-1 - 2j)|}{|jw - (-2 + 3j)|} = \frac{|\overrightarrow{ZW}|}{|\overrightarrow{PW}|}$$

$$\angle H(jw) = \angle \overrightarrow{ZW} - \angle \overrightarrow{PW}$$

- For example,

$$G(j\omega) = \frac{\overrightarrow{s_1} \overrightarrow{s_2}}{\overrightarrow{s_3} \overrightarrow{s_4} \overrightarrow{s_5}} = \frac{(r_1 e^{j\theta_1}) (r_2 e^{j\theta_2})}{(r_3 e^{j\theta_3}) (r_4 e^{j\theta_4}) (r_5 e^{j\theta_5})}$$

$$= \frac{r_1 r_2}{r_3 r_4 r_5} e^{j(\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)}$$

$$|G(j\omega)| = \frac{r_1 r_2}{r_3 r_4 r_5} \quad \angle G(j\omega) = (\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$\log_{10} |G(j\omega)| = \log_{10} r_1 + \log_{10} r_2 - \log_{10} r_3 - \log_{10} r_4 - \log_{10} r_5$$

- Power db: $|G(j\omega)|_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$

- Voltage db: $|G(j\omega)|_{\text{db}} = 20 \log_{10} \frac{V_2}{V_1}$

■ Advantages of working with Frequency Response

in terms of Bode Plots

1. Dynamic compensator design

can be based entirely on Bode plots.

2. Bode plots can be determined experimentally.

3. Bode plots of systems in series (or tandem) simply add, which is quite convenient.

4. The use of a log scale permits

a much wider range of frequencies

to be displayed on a single plot

than is possible with linear scales.

- The open-loop transfer function:

$$K G(s) = K \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - p_2) \cdots (s - p_n)}$$

$$K G(j\omega) = K_0 (j\omega)^n \frac{(j\omega\tau_1 + 1)(j\omega\tau_2 + 1) \cdots}{(j\omega\tau_a + 1)(j\omega\tau_b + 1) \cdots}$$

- For example,

$$K G(j\omega) = K_0 \frac{(j\omega\tau_1 + 1)}{(j\omega)^2 (j\omega\tau_a + 1)}$$

$$\angle K G(j\omega) = \angle K_0 + \angle (j\omega\tau_1 + 1) - \angle (j\omega)^2 - \angle (j\omega\tau_a + 1)$$

$$\begin{aligned} \log |K G(j\omega)| &= \log |K_0| + \log |(j\omega\tau_1 + 1)| \\ &\quad - \log |(j\omega)^2| - \log |(j\omega\tau_a + 1)| \\ |K G(j\omega)|_{\text{db}} &= 20 \log |K_0| + 20 \log |(j\omega\tau_1 + 1)| \\ &\quad - 20 \log |(j\omega)^2| - 20 \log |(j\omega\tau_a + 1)| \end{aligned}$$

▪ Class 1: Singularities at the origin

$$K_0 (j\omega)^n$$

▪ Class 2: First-order term

$$(j\omega\tau + 1)^{\pm 1}$$

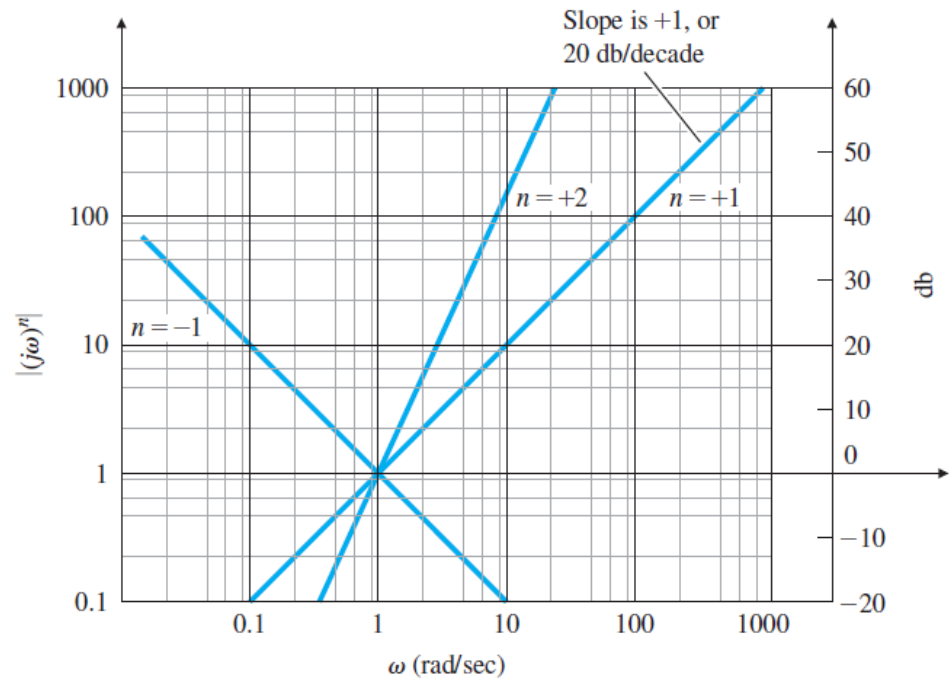
▪ Class 3: Second-order term

$$\left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\zeta \frac{j\omega}{\omega_n} + 1 \right]^{\pm 1}$$

▪ Class 1: $K_0 (j\omega)^n$

$$\begin{aligned} \log K_0 |(j\omega)^n| \\ = \log K_0 + n \log |j\omega| \end{aligned}$$

$$\angle K_0 (j\omega)^n = n \times 90^\circ$$



Class 2: Magnitude

$$(j\omega\tau + 1)^{\pm 1}$$

a) For $\omega\tau \ll 1$, $j\omega\tau + 1 \cong 1$

b) For $\omega\tau \gg 1$, $j\omega\tau + 1 \cong j\omega\tau$

▪ $\omega = 1/\tau$: Break Point

$$? G(s) = \frac{1}{10s + 1}$$

▪ For example,

$$G(s) = 10s + 1$$

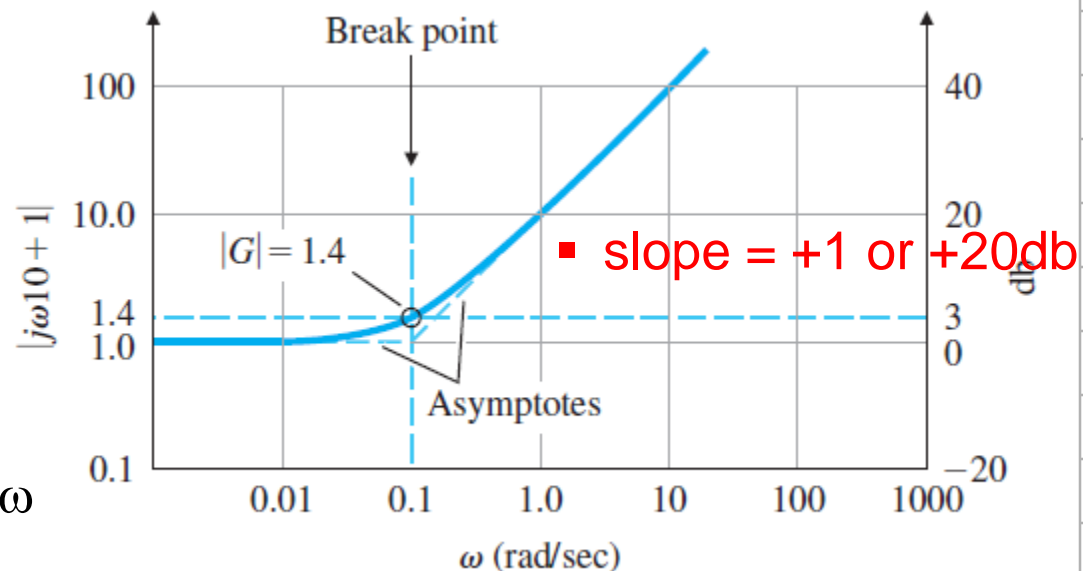
$$G(j\omega) = j(10\omega) + 1$$

a) For $10\omega \ll 1$, $j10\omega + 1 \cong 1$

b) For $10\omega \gg 1$, $j10\omega + 1 \cong j10\omega$

▪ $\omega = 1/10$: Break Point

$$|G(j0.1)| = |j(1) + 1| = 1.414 = +3 \text{ db}$$



Class 2: Phase

$$(j\omega\tau + 1)^{\pm 1}$$

- a) For $\omega\tau \ll 1$, $\angle 1 = 0^\circ$
- b) For $\omega\tau \gg 1$, $\angle j\omega\tau = 90^\circ$
- c) For $\omega\tau \cong 1$, $\angle (j\omega\tau + 1) \cong 45^\circ$

$$? G(s) = \frac{1}{10s + 1}$$

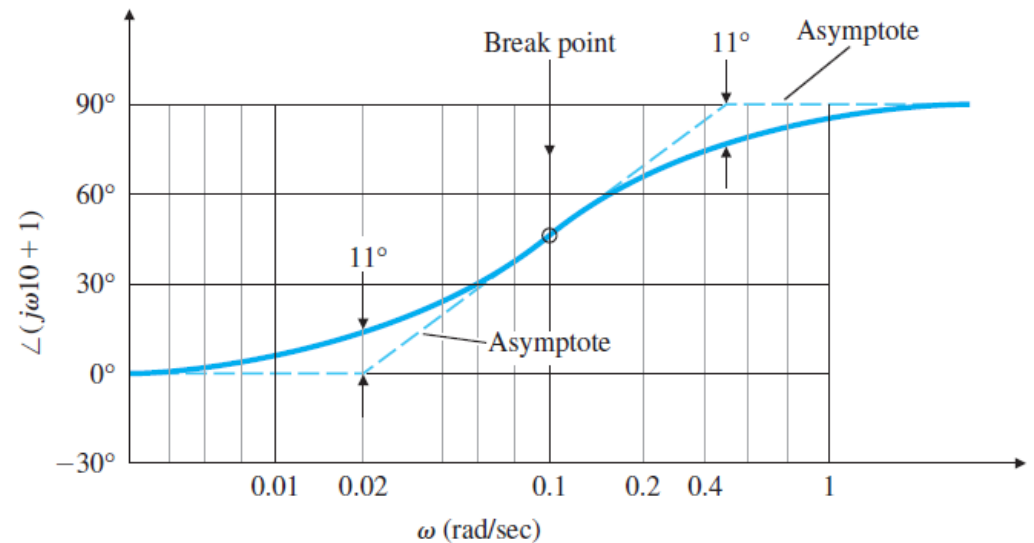
$\omega = 1/\tau$: Break Point

For example,

$$G(s) = 10s + 1$$

$$G(j\omega) = j(10\omega) + 1$$

$\omega = 1/10$: Break Point

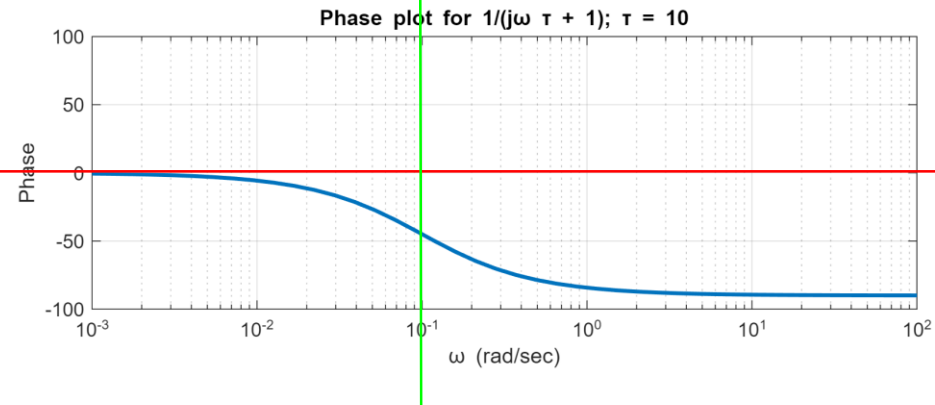
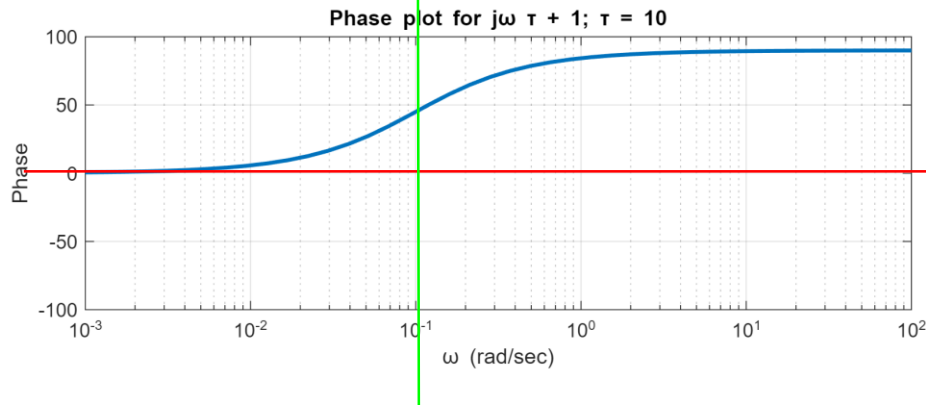
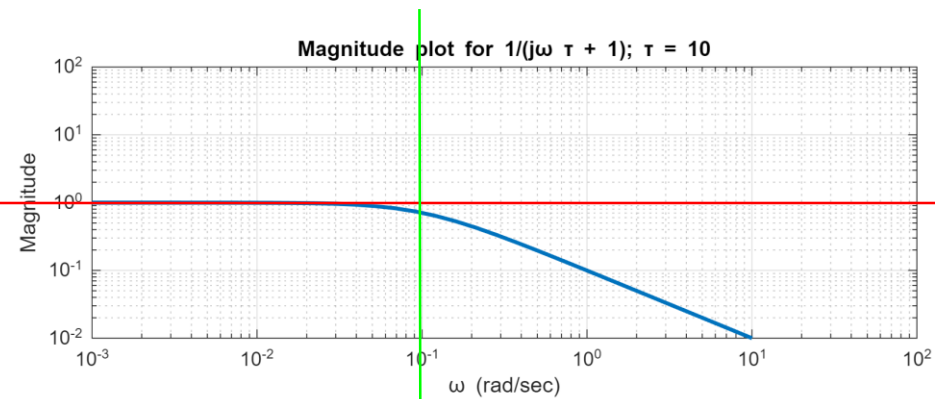
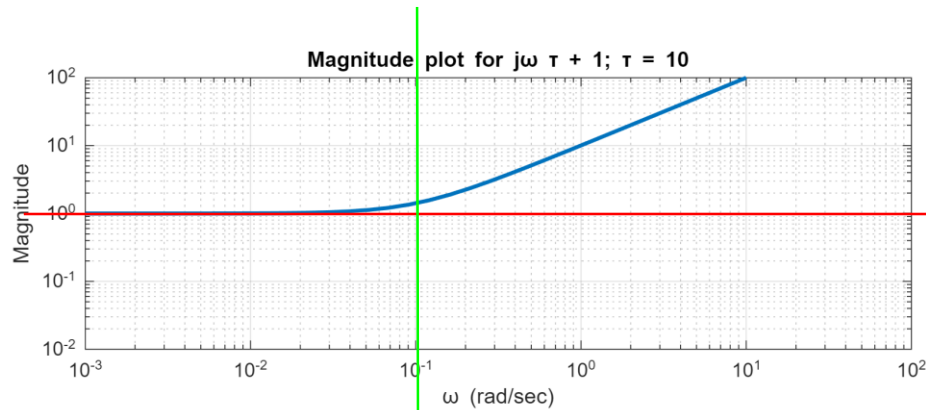


Class 2:

$$G(s) = 10s + 1$$

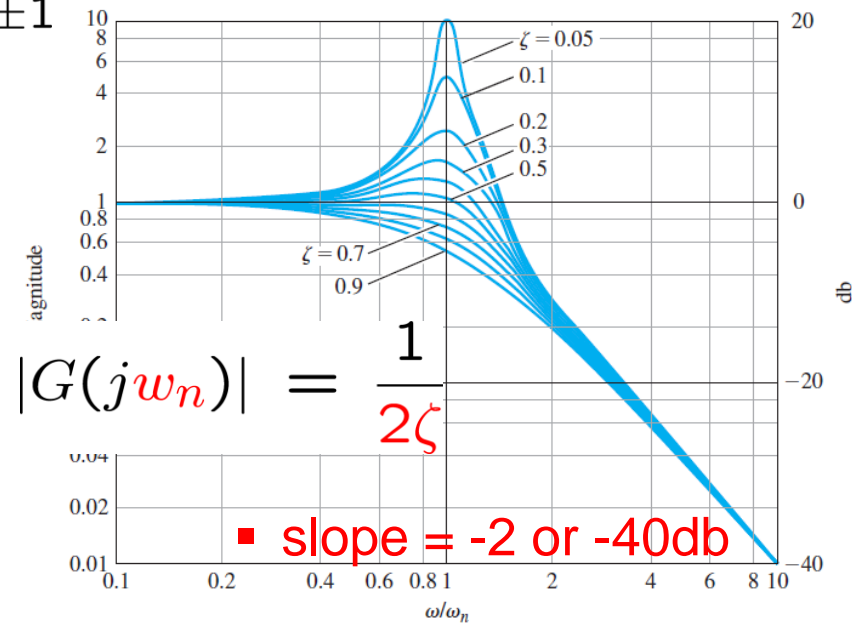
$$(j\omega\tau + 1)^{\pm 1}$$

$$G(s) = \frac{1}{10s + 1}$$



■ **Class 3:** $\left[\left(\frac{jw}{w_n} \right)^2 + 2\zeta \frac{jw}{w_n} + 1 \right]^{\pm 1}$

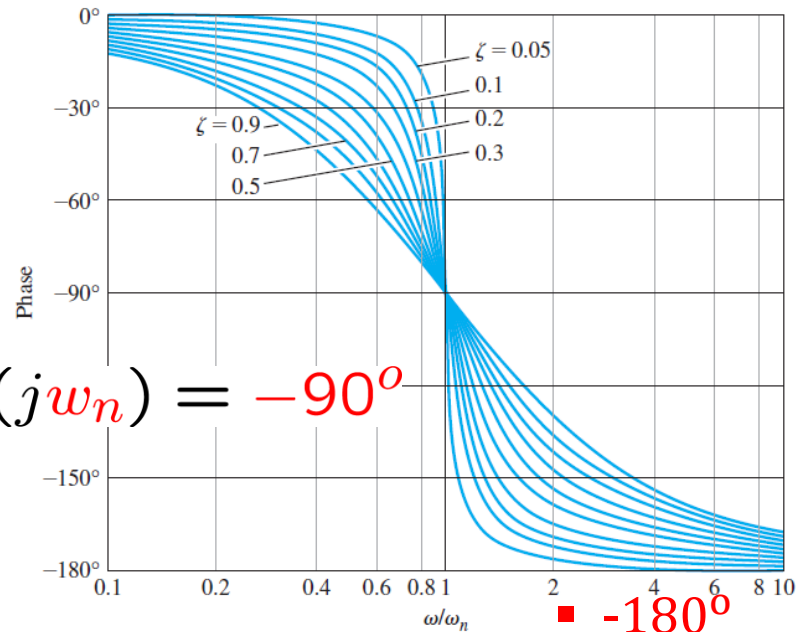
$$|G(s = jw)|$$



$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$

$$\angle G(jw_n) = -90^\circ$$



Examples

- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50

- (2) Asymptotes

- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

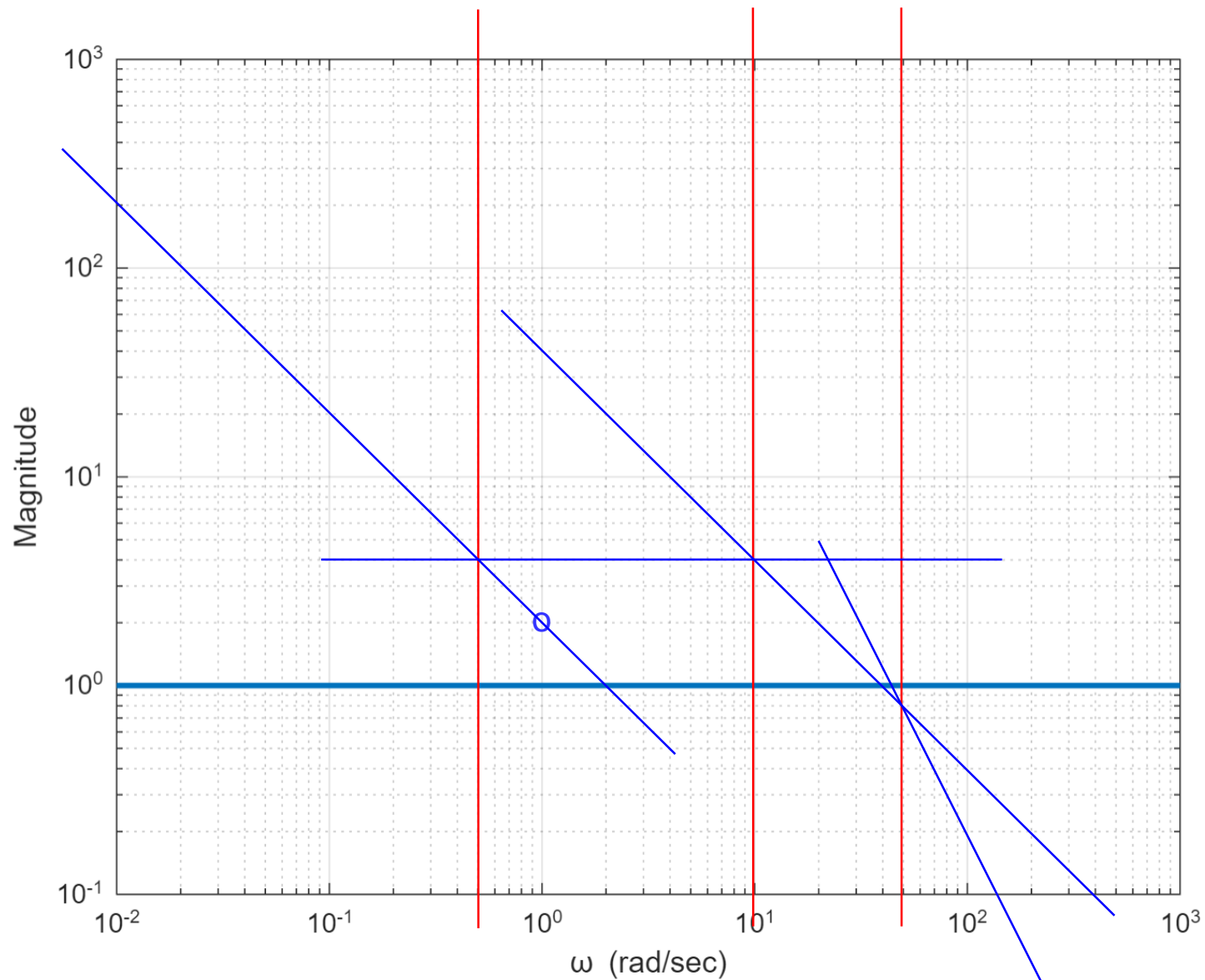
- $\omega \ll 0.5$: slope = -1 (or -20 db/decade)

- $0.5 < \omega < 10$: slope = 0 (or 0 db/decade)

- $10 < \omega < 50$: slope = -1 (or -20 db/decade)

- $50 < \omega$: slope = -2 (or -40 db/decade)

- Example 6.3: Bode Plot for Real Poles and Zeros

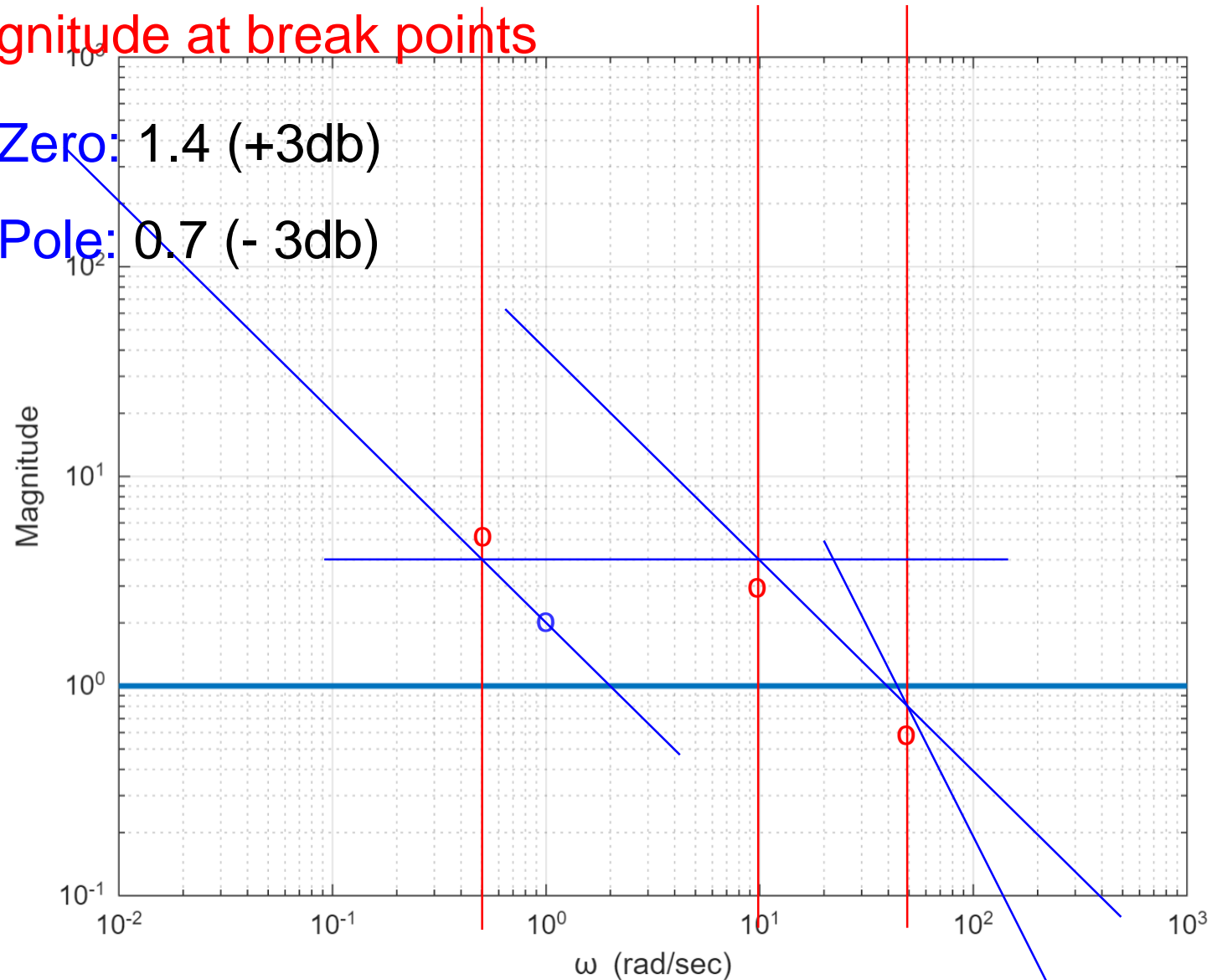


- Example 6.3: Bode Plot for Real Poles and Zeros

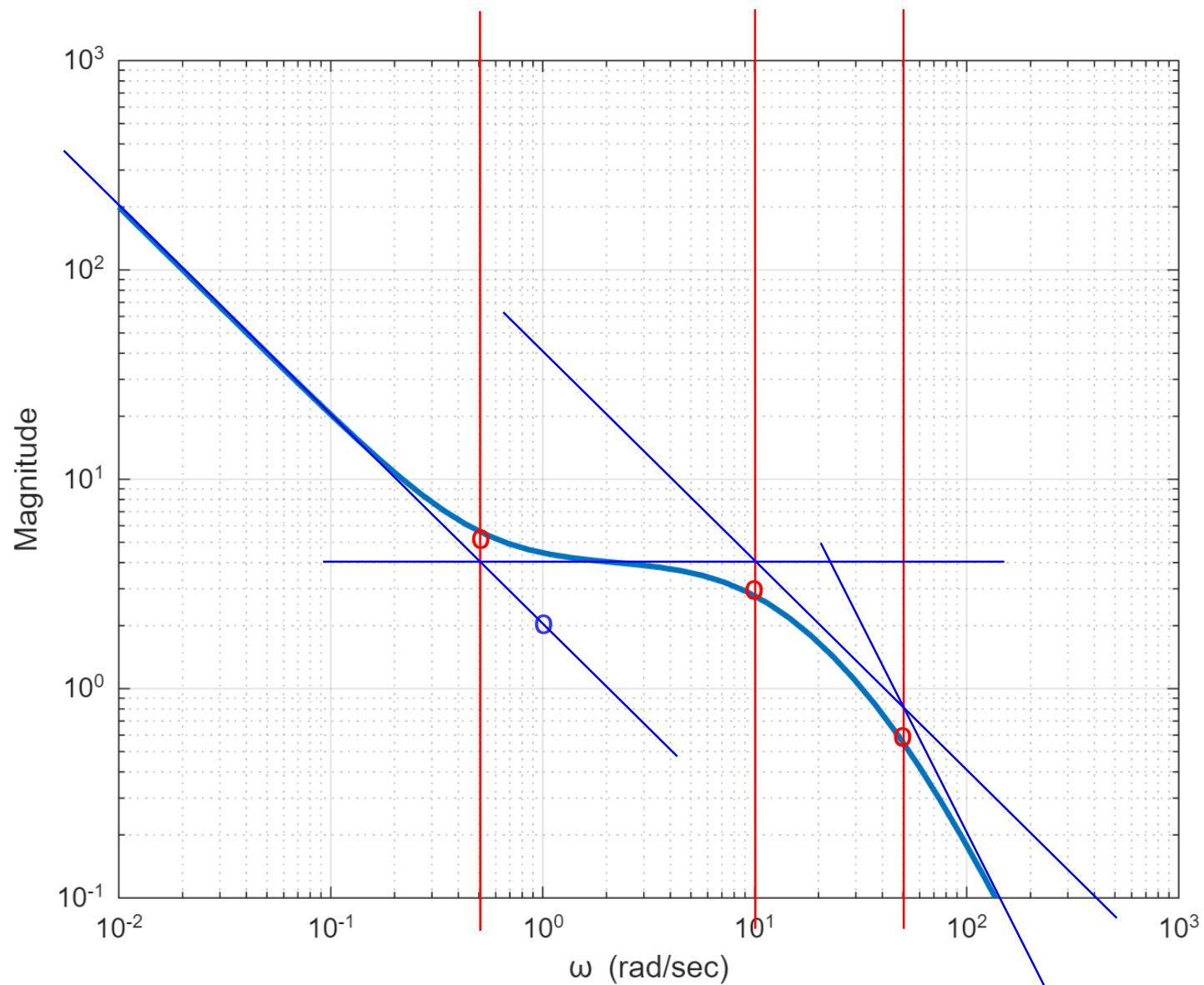
- (3) Magnitude at break points

- By Zero: 1.4 (+3db)

- By Pole: 0.7 (- 3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K G(s) = \frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

- (1) Break points

$$K G(j\omega) = \frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

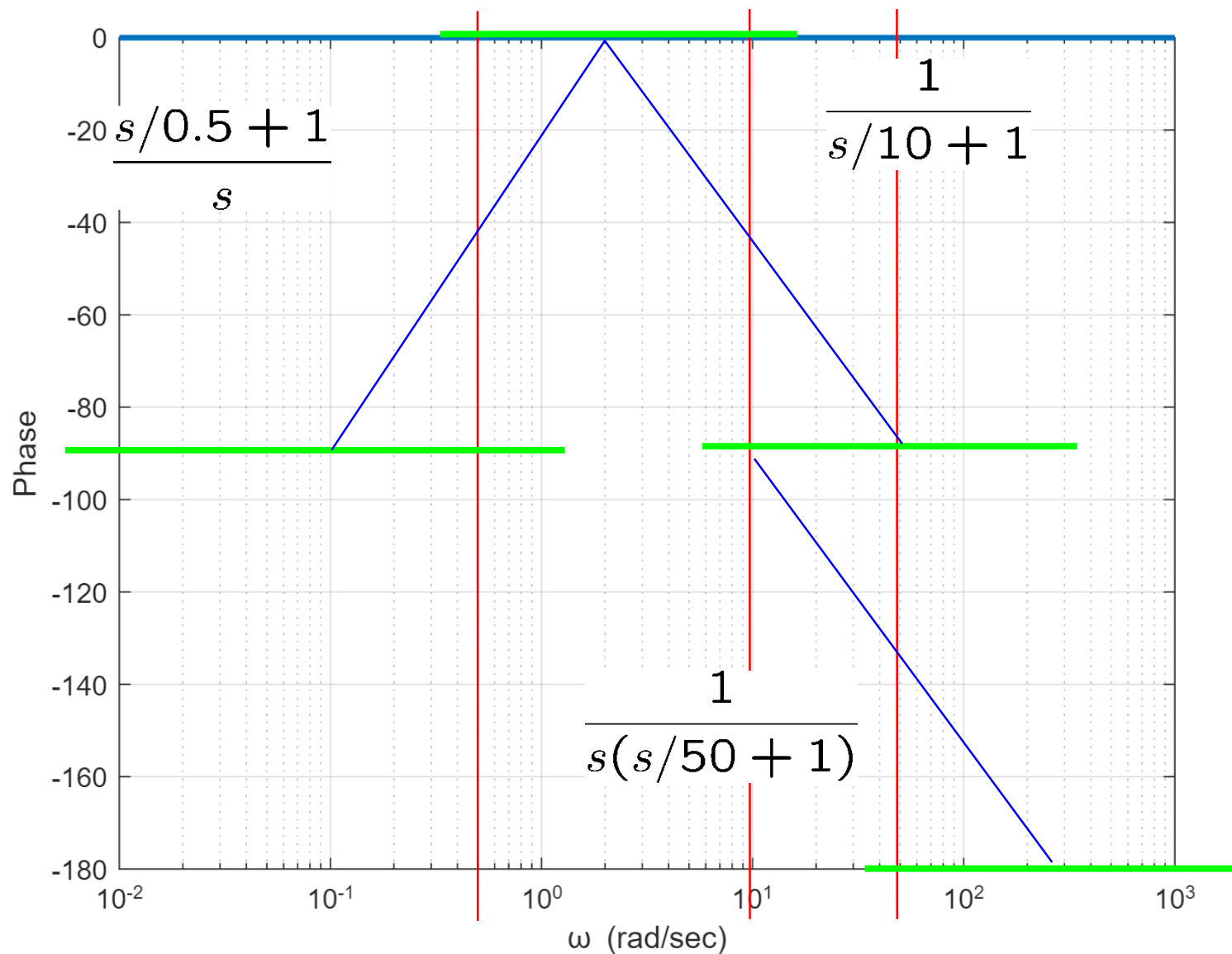
- Break points: 0.5, 10, 50

- (4) Phase

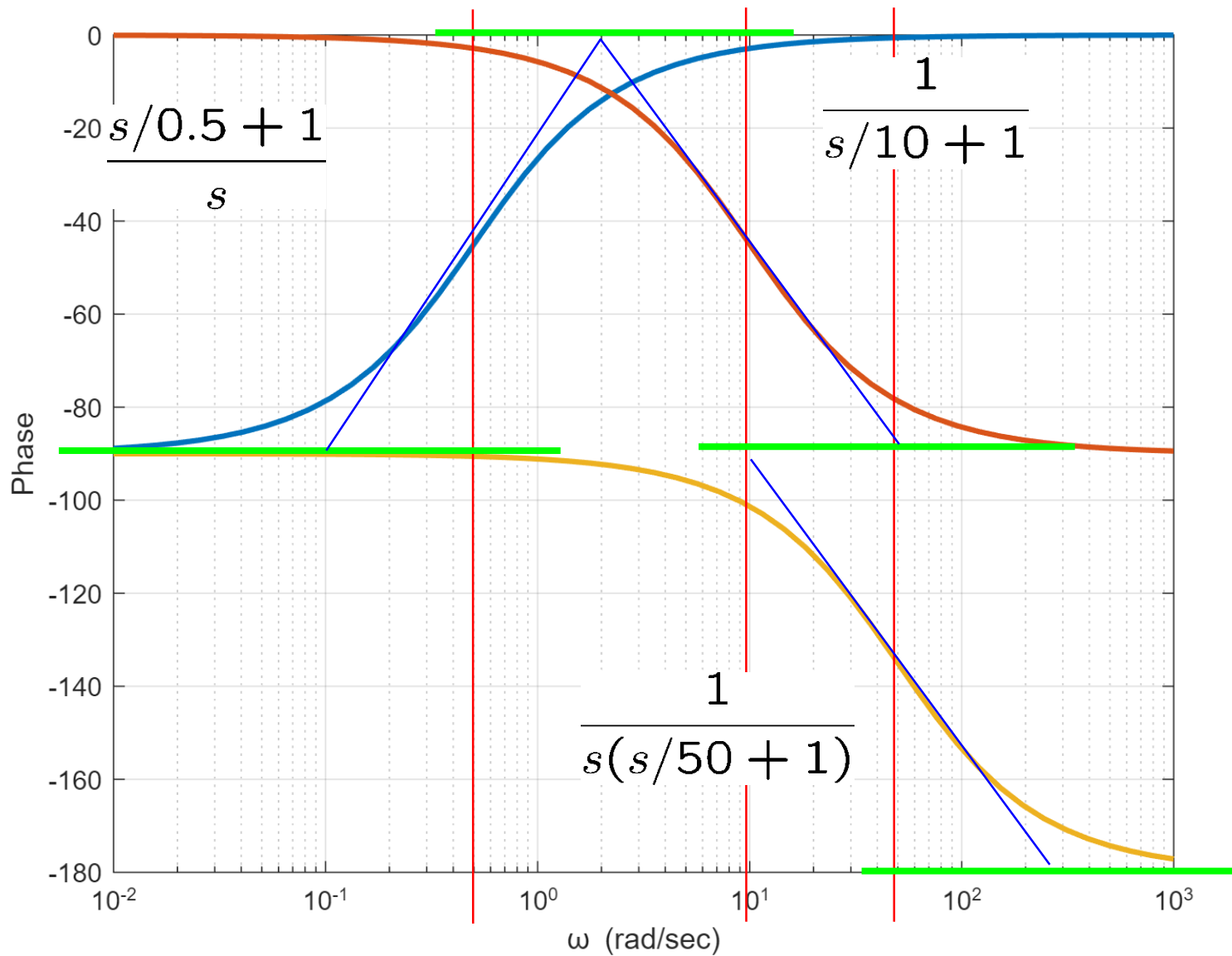
- Low-Frequency Asymptote: $K G(j\omega) = \frac{2}{(j\omega)}$ for $\omega < 0.1$

- $\omega \ll 0.5$: phase = -90°
- $0.5 < \omega < 10$: phase = 0°
- $10 < \omega < 50$: phase = -90°
- $50 < \omega$: phase = -180°

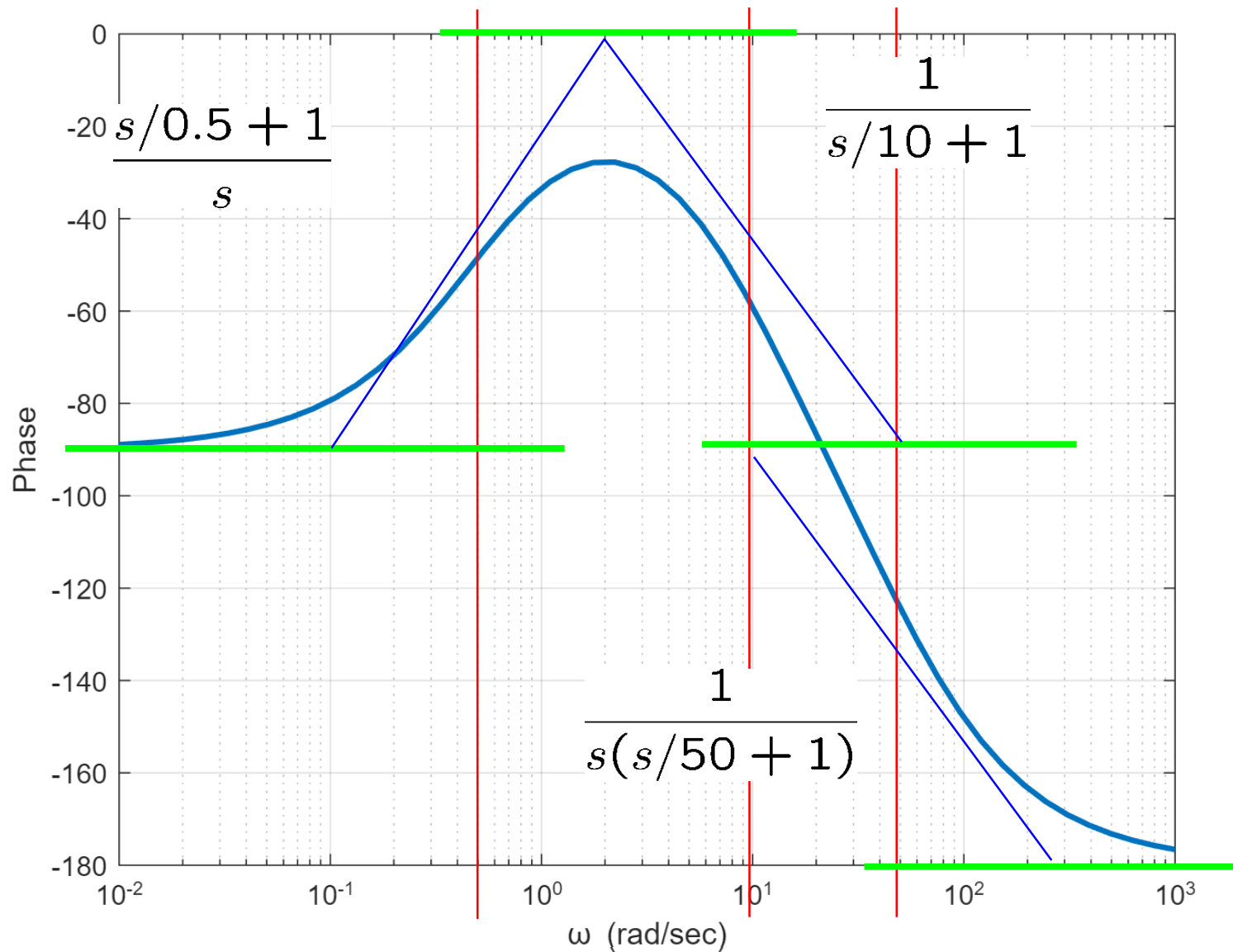
- Example 6.3: Bode Plot for Real Poles and Zeros



Example 6.3: Bode Plot for Real Poles and Zeros



■ Example 6.3: Bode Plot for Real Poles and Zeros

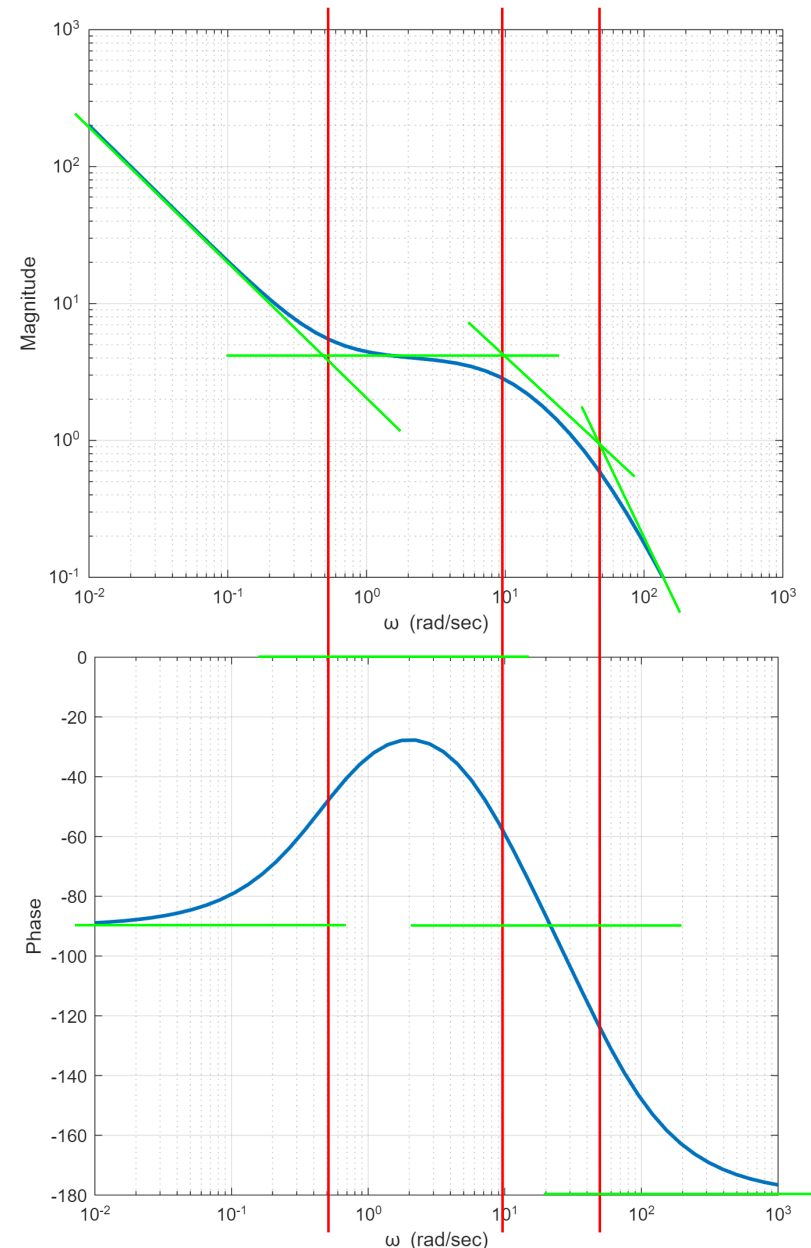


Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000 (s + 0.5)}{s (s + 10) (s + 50)}$$

$$\frac{2 \left[\frac{j\omega}{0.5} + 1 \right]}{(j\omega) \left[\frac{j\omega}{10} + 1 \right] \left[\frac{j\omega}{50} + 1 \right]}$$

- Break points: 0.5, 10, 50

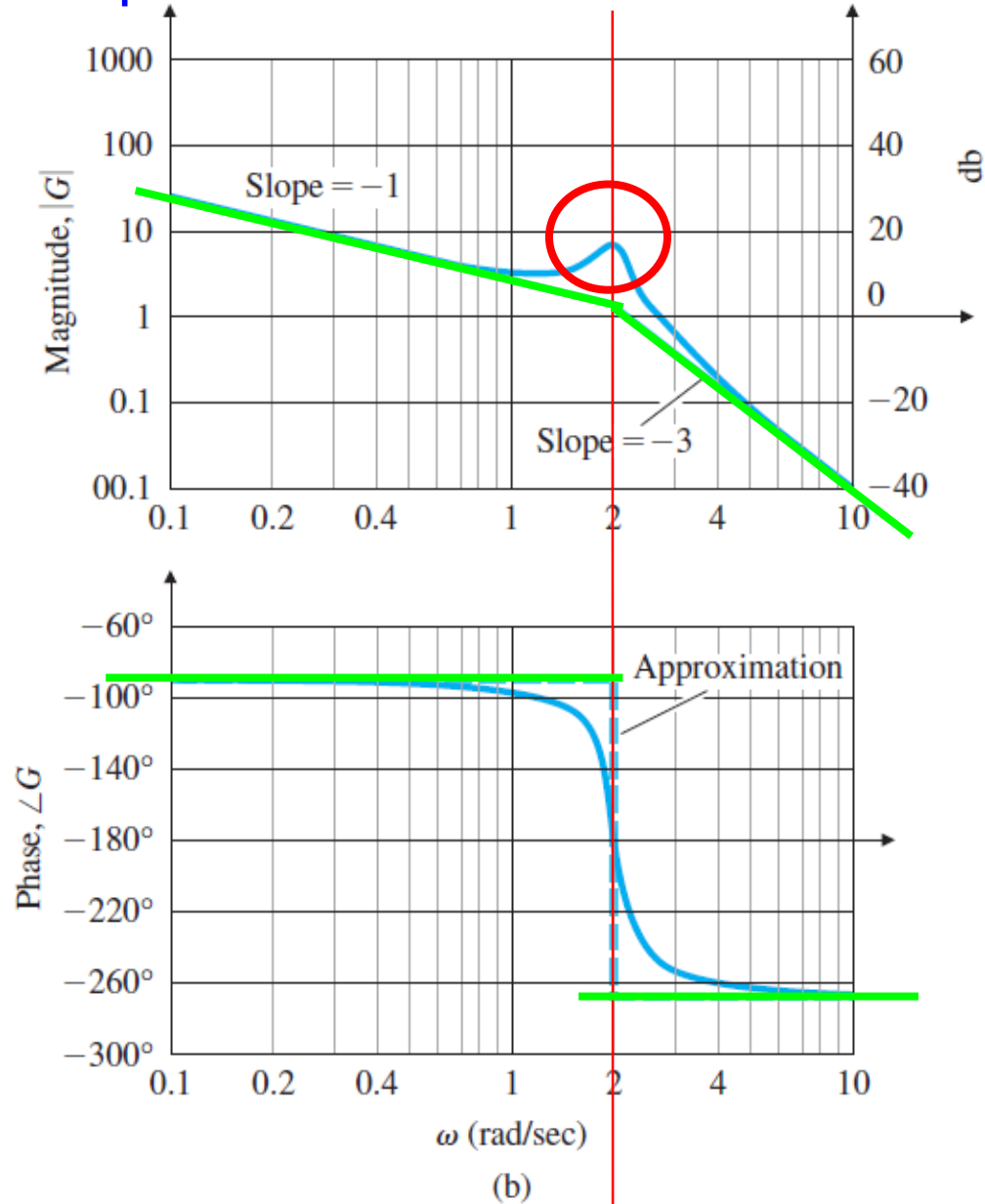


Example 6.4: Bode Plot for Complex Poles

$$K G(s) = \frac{10}{s [s^2 + 0.4s + 4]}$$

$$= \frac{10}{4} \frac{1}{s \left[\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1 \right]}$$

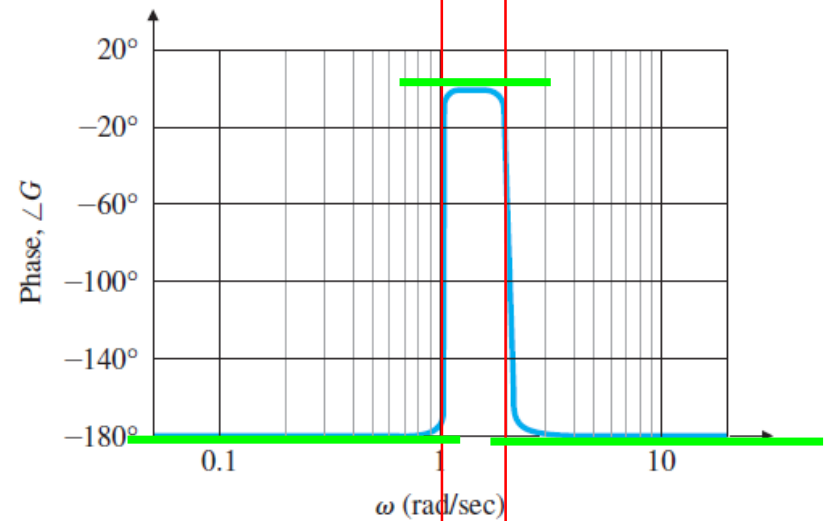
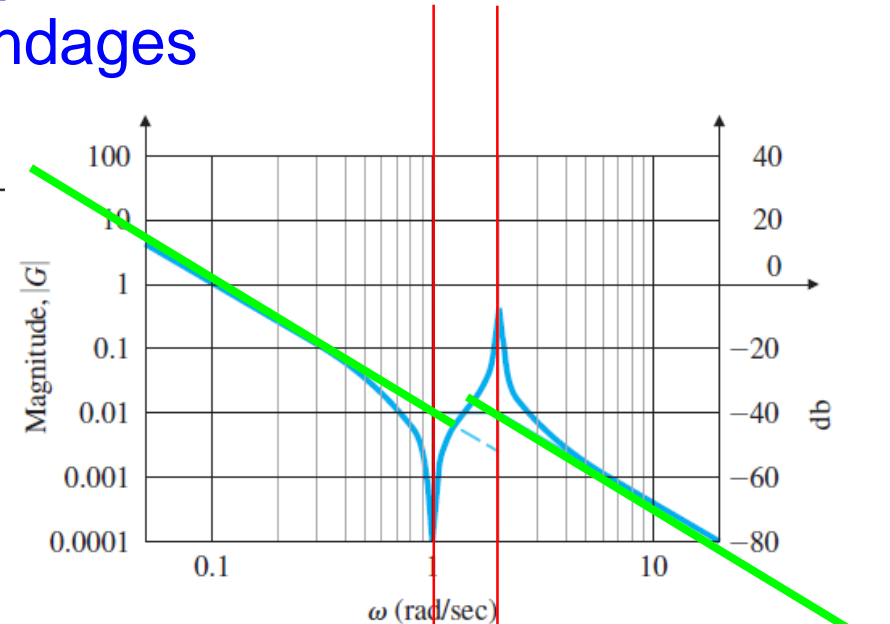
- Break points: 2



- Example 6.5: Bode Plot for Complex Poles and Zeros:
Satellite with Flexible Appendages

$$K G(s) = \frac{0.01 [s^2 + 0.01s + 1]}{s^2 [(s^2/4) + 0.02(s/2) + 1]}$$

- Break points: 1, 2



(b)

Example 6.6: Computer-Aided Bode Plot for Complex Poles and Zeros

```
num = 0.01*[1 0.01 1];  
den = conv([1 0 0],[.25 0.01 1]);
```

```
w = logspace(-2,2,1000);
```

```
[m,p] = bode(num, den, w);
```

```
subplot(2,1,1)  
loglog(w, m, 'LineWidth', 2);
```

```
subplot(2,1,2)  
semilogx(w, p, 'LineWidth', 2);
```

