Spring 2021

控制系統
Control Systems

Unit 5D
Design Using Dynamic Compensation

Feng-Li Lian

NTU-EE

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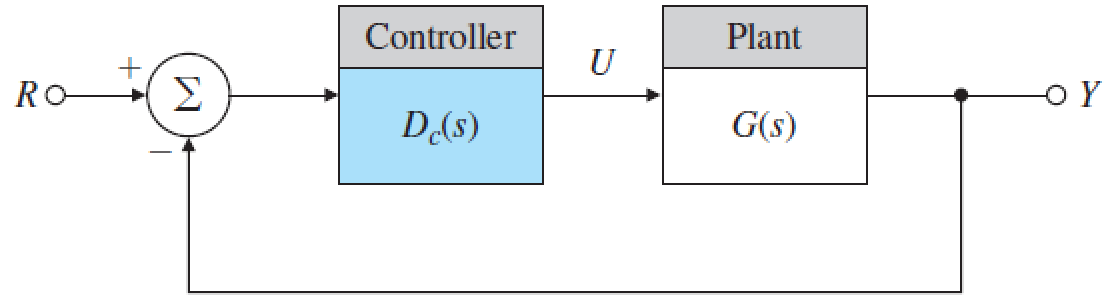
- Three categories of Particularly Simple and Effective Designs:
- Lead Compensation:
  - Approximates the function of PD control and
  - Acts mainly to speed up a response

    by lowering rise time and decreasing the transient overshoot
- Lag Compensation:
  - Approximates the function of PI control and
  - Is usually used to improve the steady-state accuracy
- Notch Compensation:
  - To achieve stability for systems with lightly damped flexible modes

Design Using Dynamic Compensation

- **Model:**



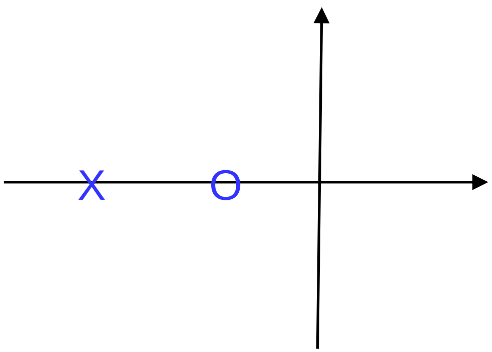
$$D_c(s) = K\frac{s+z}{s+p}$$

$$\Rightarrow 1 + D_c(s)\,G(s) = 0$$

$$\Rightarrow 1 + K\,L(s) = 0$$

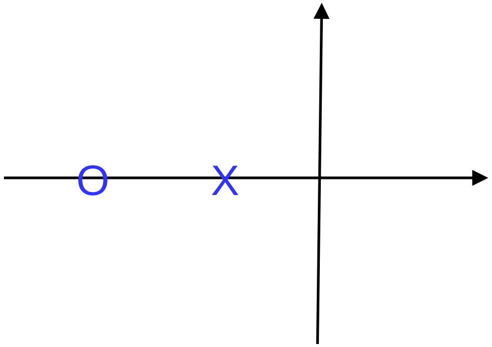
- **Lead Compensation:** if z < p
  - Provides a positive phase shift

- Lead



- **Lag Compensation:** if z > p
  - Provides a negative phase shift

- Lag



Design Using Lead Compensation

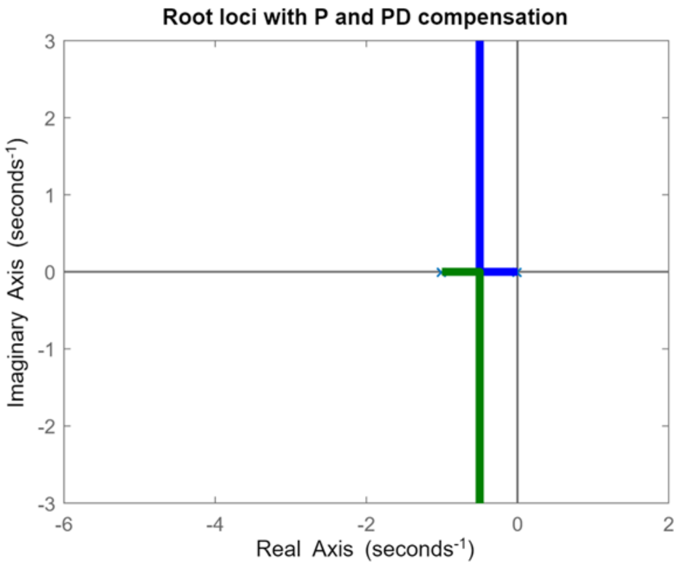
▪ 2nd-Order Position Control System

$$D_c(s) = K$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K \frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K = 0$$

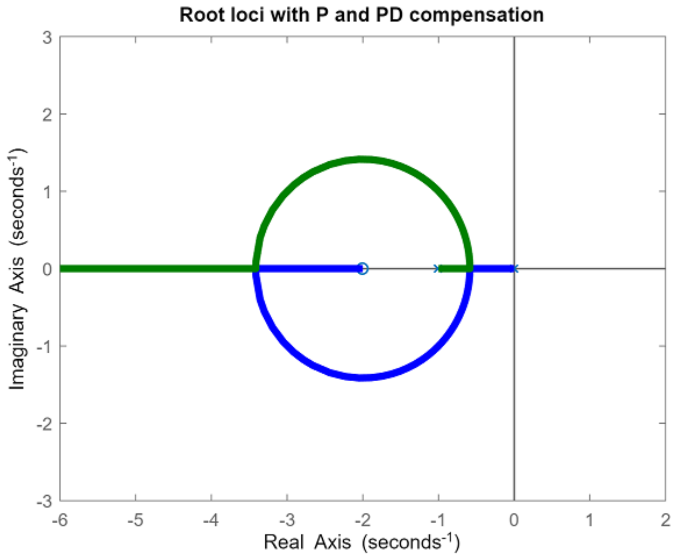


$$D_c(s) = K\,(s+2)$$

$$G(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow 1 + K\,(s+2)\frac{1}{s(s+1)} = 0$$

$$\Rightarrow s(s+1) + K(s+2) = 0$$



Design Using Lead Compensation

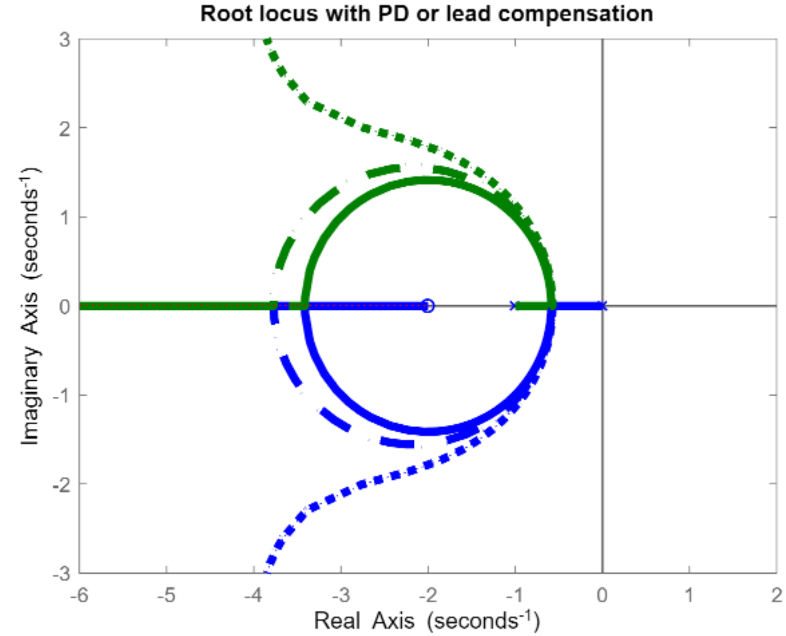
▪ 2nd-Order Position Control System

$$D_c(s) = K\frac{s+2}{s+p}$$

$$G(s) = \frac{1}{s(s+1)}$$

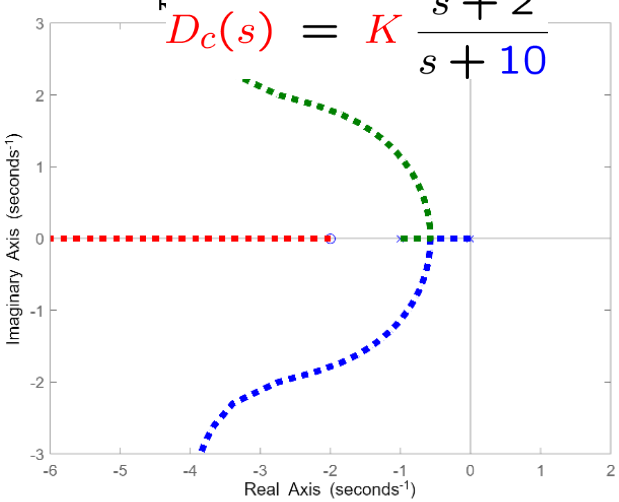
$$\Rightarrow 1 + K\frac{s+2}{s+p}\frac{1}{s(s+1)} = 0$$

$$\Rightarrow 1 + K\frac{(s+2)}{s(s+1)(s+p)} = 0$$

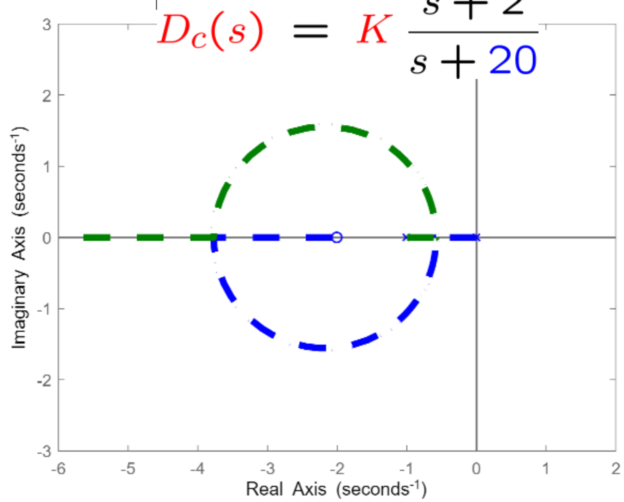


▪ p = 10, 20

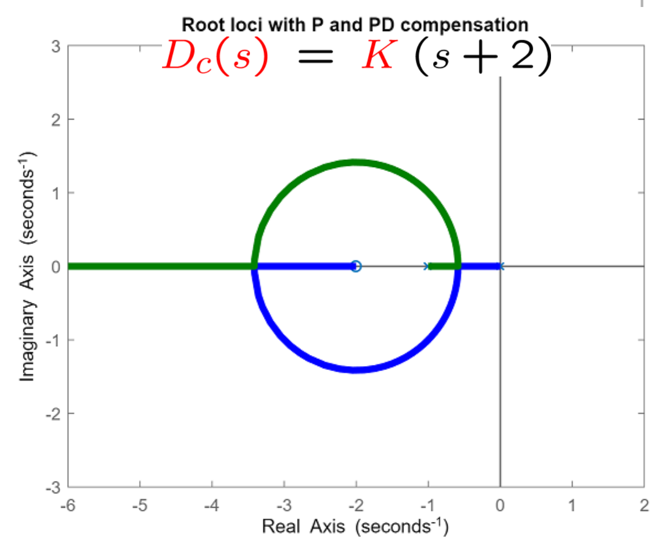
$$D_c(s) = K\frac{s+2}{s+10}$$



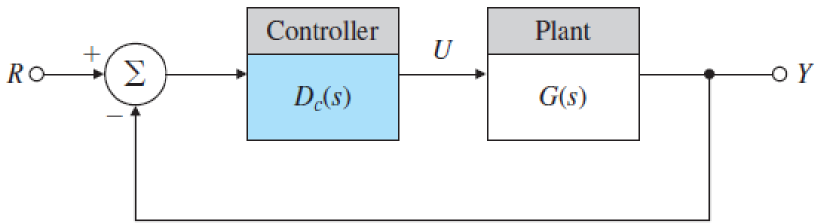
$$D_c(s) = K\frac{s+2}{s+20}$$



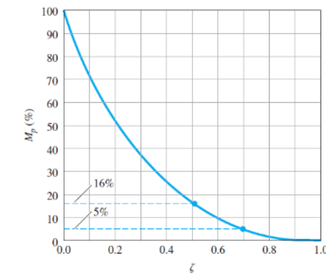
$$D_c(s) = K(s+2)$$



Example 5.11: Design Using Lead Compensation



- Overshoot <= 20%
- Rise Time <= 0.3 sec



$$\Rightarrow \quad \zeta \geq 0.5$$

$$\Rightarrow \quad w_n \approx \frac{1.8}{0.3} \approx 6 \quad \Rightarrow \quad w_n \geq 7$$

$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = K \frac{s+2}{s+10}$$

$$1 + K \frac{s+2}{s+10} \frac{1}{s(s+1)} = 0$$

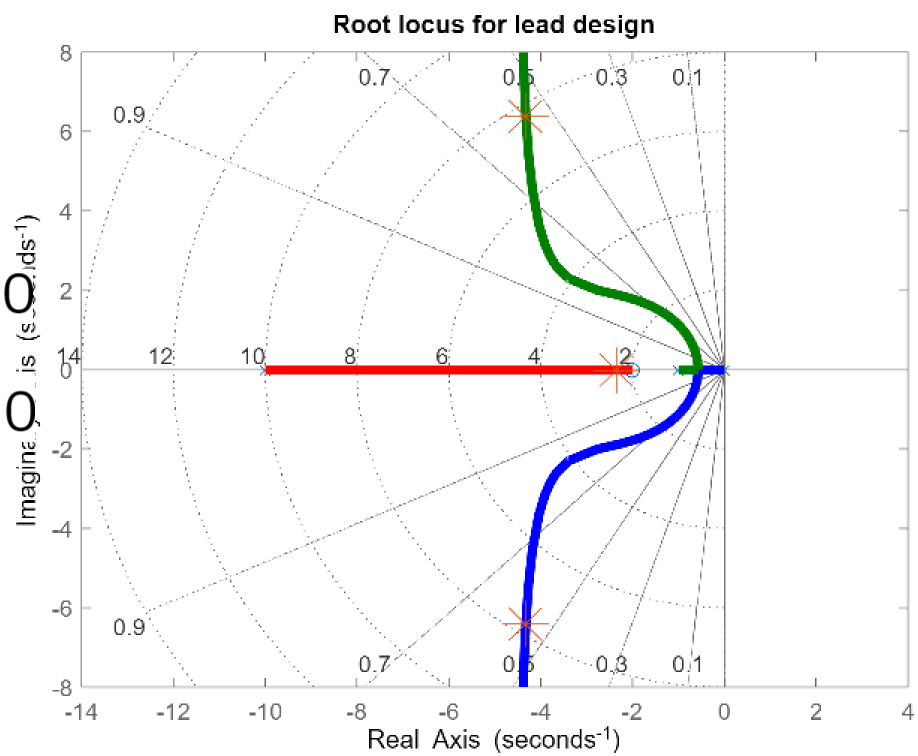
$$(s+10)s(s+1) + K(s+2) = 0$$

$$s^3 + 11s^2 + 10s + K(s+2) = 0$$

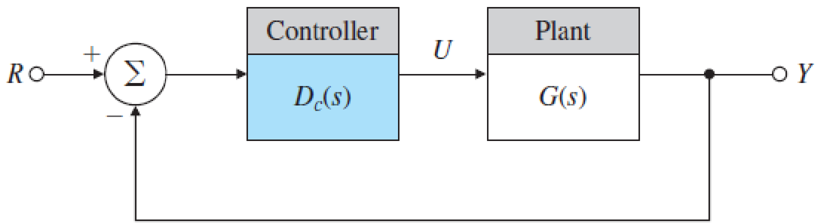
$$\Rightarrow \quad K = 70$$

$$\Rightarrow \quad s^3 + 11s^2 + 80s + 140 = 0$$

$$\Rightarrow \quad s = -2.34, -4.33 \pm 6.40i$$



▪ Example 5.11: Design Using Lead Compensation



- Overshoot <= 20%
- Rise Time <= 0.3 sec

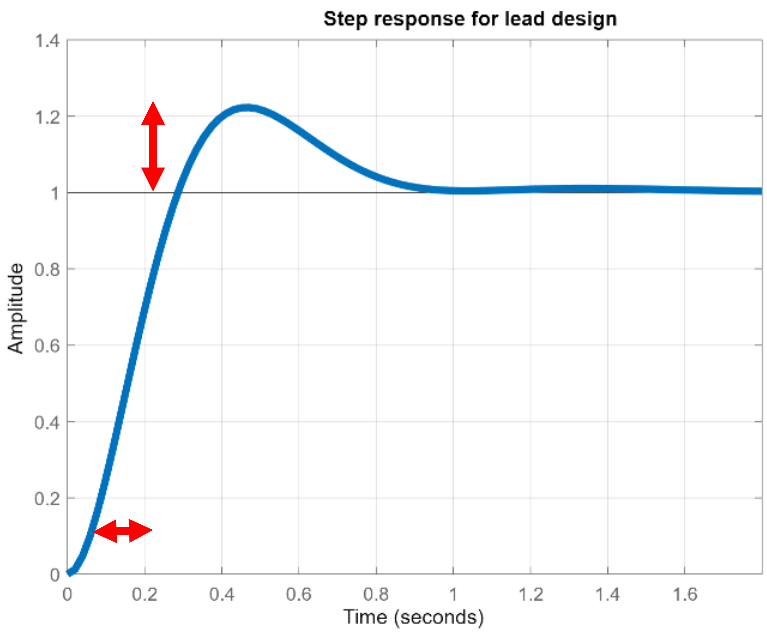
$$\Rightarrow \zeta \geq 0.5$$

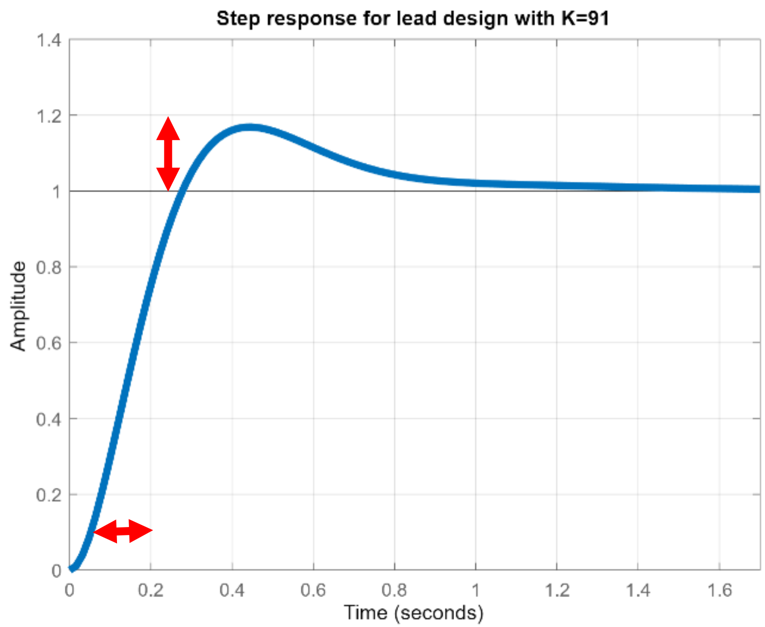
$$\Rightarrow w_n \approx \frac{1.8}{0.3} \approx 6 \quad \Rightarrow w_n \geq 7$$

$$G(s) = \frac{1}{s(s+1)}$$

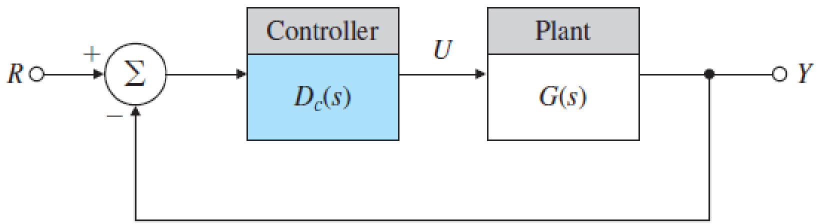
$$D_c(s) = 70\,\frac{s+2}{s+10}$$

$$D_c(s) = 91\,\frac{s+2}{s+13}$$





Example 5.11: Design Using Lead Compensation



- Overshoot <= 20%
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$$\Rightarrow \zeta \geq 0.5$$

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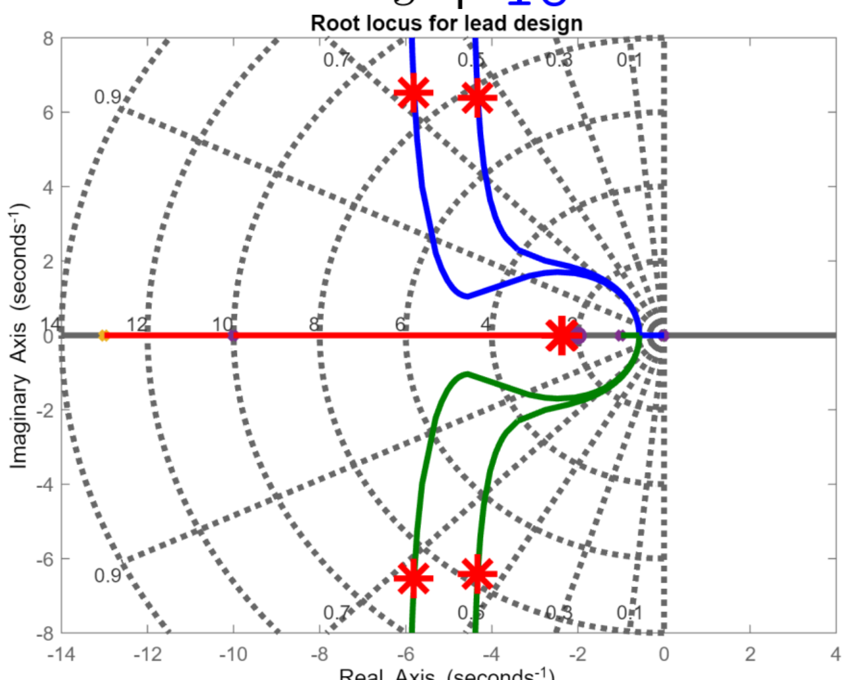
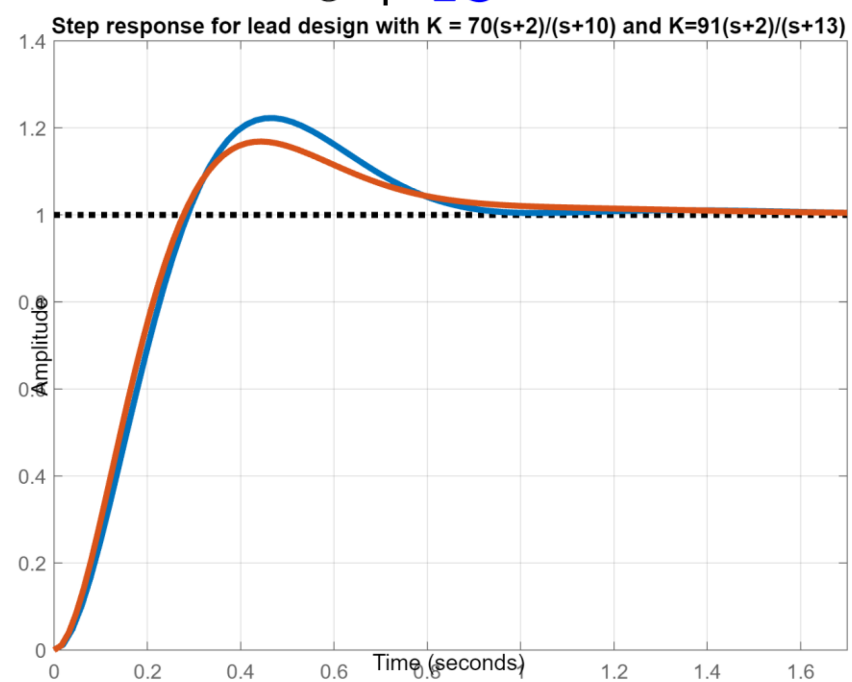
$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = 70\,\frac{s+2}{s+10}$$

$$D_c(s) = 91\,\frac{s+2}{s+13}$$





- 2nd-Order Position Control System

$$G(s) = \frac{1}{s(s+1)}$$

$$D_c(s) = \frac{s+z}{s+p}, \quad z > p$$

$$D_{c2}(s) = \frac{s+0.05}{s+0.01}$$

$$D_c(0) = \frac{z}{p} = 3 \text{ to } 10$$

- Lead Compensation:

IF $K_v = 70\,\text{sec}^{-1}$

$$K D_{c1}(s) = 91\frac{s+2}{s+13}$$

$$K_v = \lim_{s\to 0} s\, K\, D_{c1}(s)\, G(s) = \lim_{s\to 0} s\,(91)\frac{s+2}{s+13}\frac{1}{s(s+1)} = 14$$

- Reduce the velocity error by a factor of 5

$$\Rightarrow \frac{z}{p} = 5$$

$$\Rightarrow z = 0.05$$

$$\Rightarrow p = 0.01$$

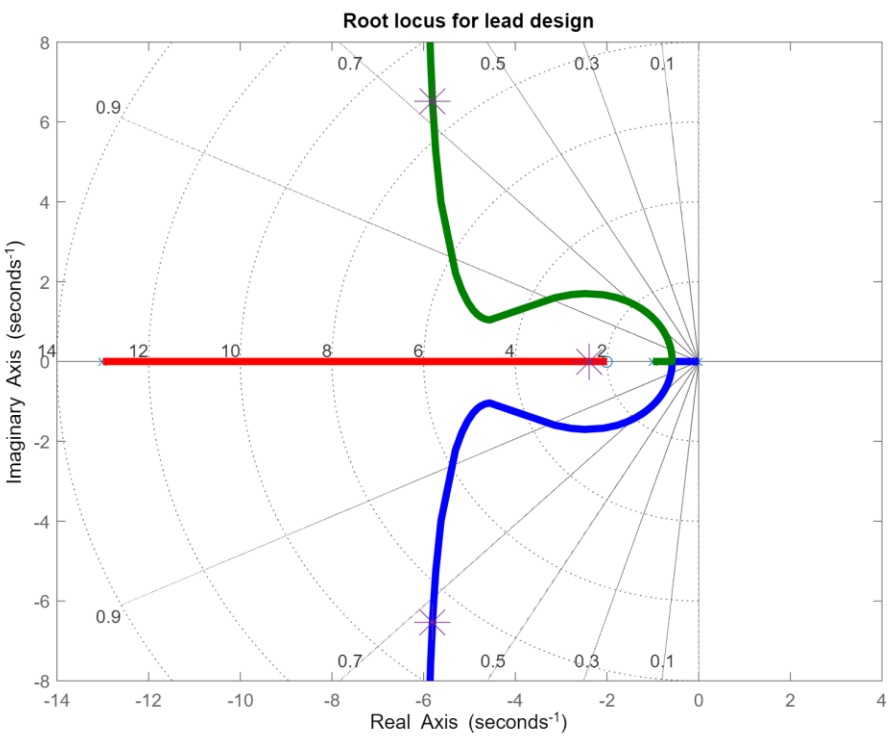
Design Using Lag Compensation

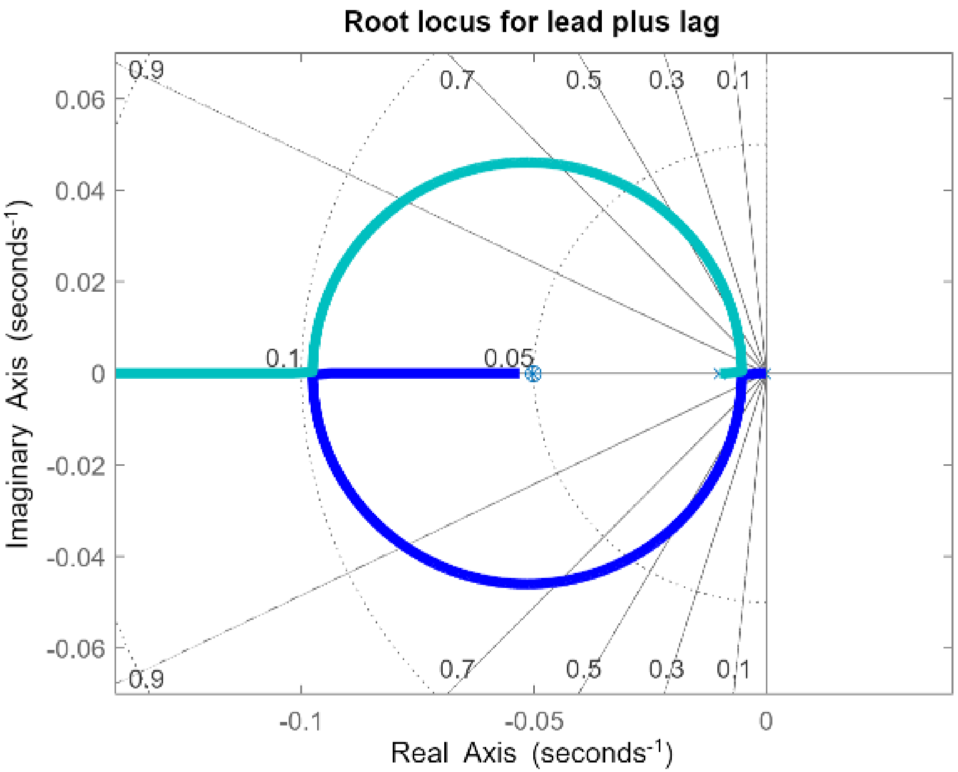
- 2nd-Order Position Control System

$$G(s) = \frac{1}{s(s+1)}$$

$$KD_{c1}(s) = 91\frac{s+2}{s+13}$$

$$KD_{c2}(s) = 91\frac{s+0.05}{s+0.01}$$





Design Using Notch Compensation

- Suppose the design with lead and lag compensation

$$KD_c(s) = 91 \frac{s+2}{s+13} \frac{s+0.05}{s+0.01}$$

- Has a substantial oscillation at about 50 rad/sec

$$G(s) = \frac{2500}{s(s+1)(s^2+s+2500)}$$

- Notch Compensation (Phase Stabilization)

$$D_{notch}(s) = \frac{s^2 + 2\zeta w_0 s + w_0^2}{(s+w_0)^2}$$

$$= \frac{s^2 + 0.8s + 3600}{(s+60)^2}$$

Design Using Notch Compensation

- Suppose the design with lead and lag compensation

$$G(s) = \frac{2500}{s(s+1)(s^2+s+2500)} \quad D_{c1}(s) = \frac{s+2}{s+13} \quad K = 91 \quad D_{c2}(s) = \frac{s+0.05}{s+0.01} \quad D_{notch}(s) = \frac{s^2+0.8s+3600}{(s+60)^2}$$