Spring 2021

控制系統
Control Systems

Unit 5C
Selected Illustrative Root Loci

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Example 0: Double Integrator

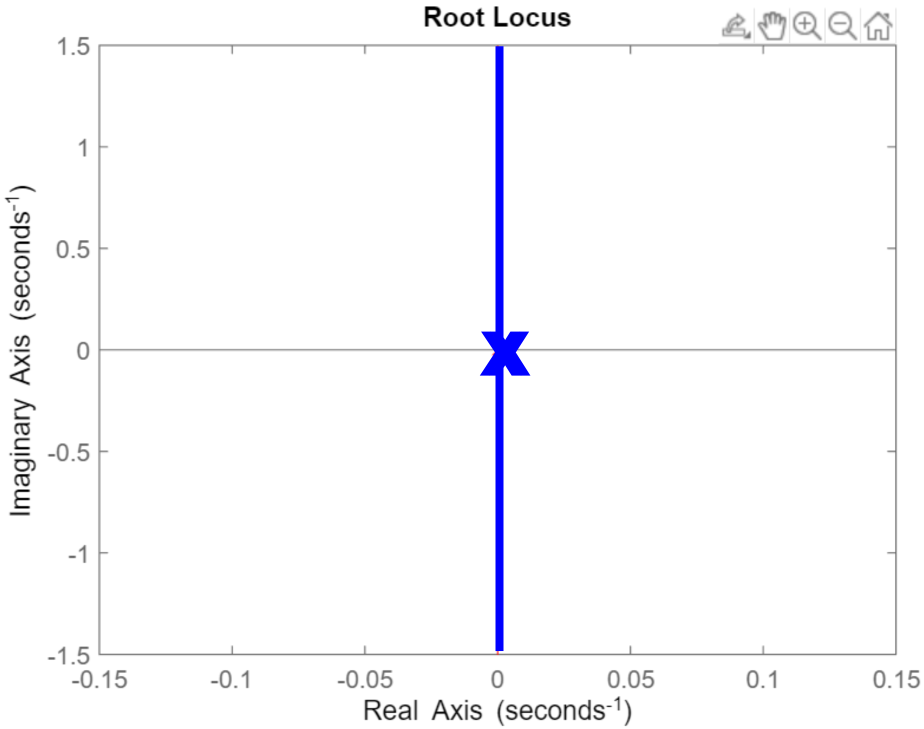
$$G(s) = \frac{1}{s^2}$$

- Satellite attitude, hard-disk drive, motor, etc.

- With P controller

$$\Rightarrow 1 + K_P \frac{1}{s^2} = 0$$

- Rule 1:
  - 2 branches start at s = 0
- Rule 2:
  - No locus on real axis
- Rule 3:
  - Asymptotes: +- 90∘
- Rule 4:
  - Depart at +- 90∘



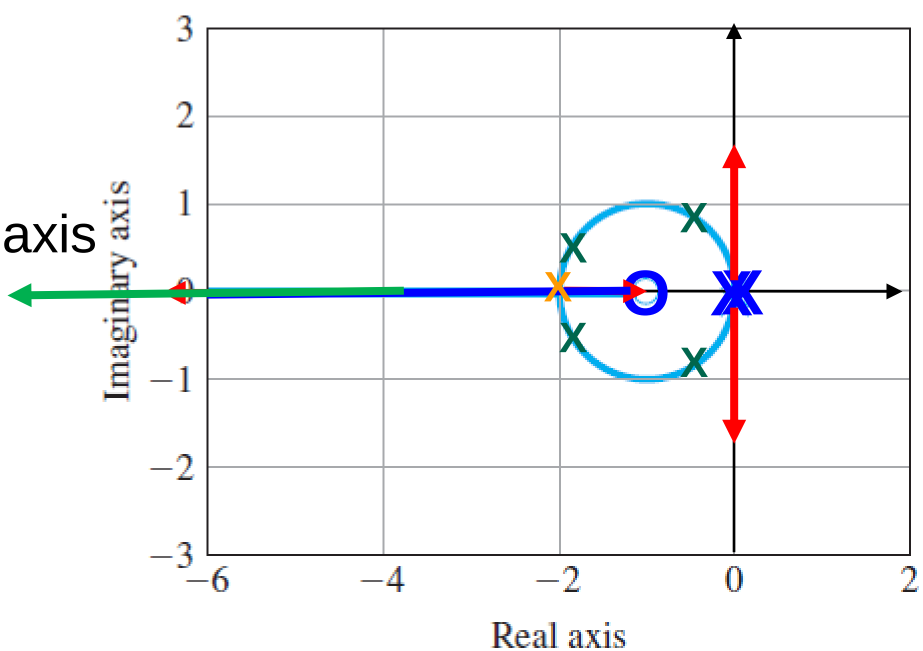
Example 5.3: Satellite Attitude Control w/ PD Control

$$\Rightarrow 1 + [\, K_P + K_D\, s \,]\, \frac{1}{s^2} = 0$$

$$\Rightarrow K = K_D \quad \Rightarrow \frac{K_P}{K_D} = 1 \quad \Rightarrow 1 + K\, \frac{s+1}{s^2} = 0$$

- Additional zero: pull the locus into the LHP

- Rule 1:
  - 2 branches start at s = 0, one to s = -1, one to ∞
- Rule 2:
  - Real axis: to the left of s = -1
- Rule 3:
  - Asymptotes: along negative real axis
- Rule 4:
  - Depart at +- 90◦
- Rule 5:
  - Rejoin and break at s = -2



Example 5.3: Satellite Attitude Control w/ PD Control

- Practically, differentiation is not good
- Use the following approximation:

$$\Rightarrow\ D_c(s) = K_P + \frac{K_D\, s}{\frac{s}{p} + 1} = K_P + \frac{K_D\, p\, s}{s + p}$$

$$= (K_P + pK_D)\,\frac{s + \frac{pK_P}{K_P + pK_D}}{s + p}$$

$$= K\,\frac{s + z}{s + p} \qquad K = K_P + pK_D \qquad z = \frac{pK_P}{K_P + pK_D}$$

$$\Rightarrow\ 1 + D_c(s)\, G(s) = 0$$

$$\Rightarrow\ 1 + K\, L(s) = 0$$

$$\Rightarrow\ 1 + K\,\frac{(s+z)}{(s+p)}\,\frac{1}{s^2} = 0$$

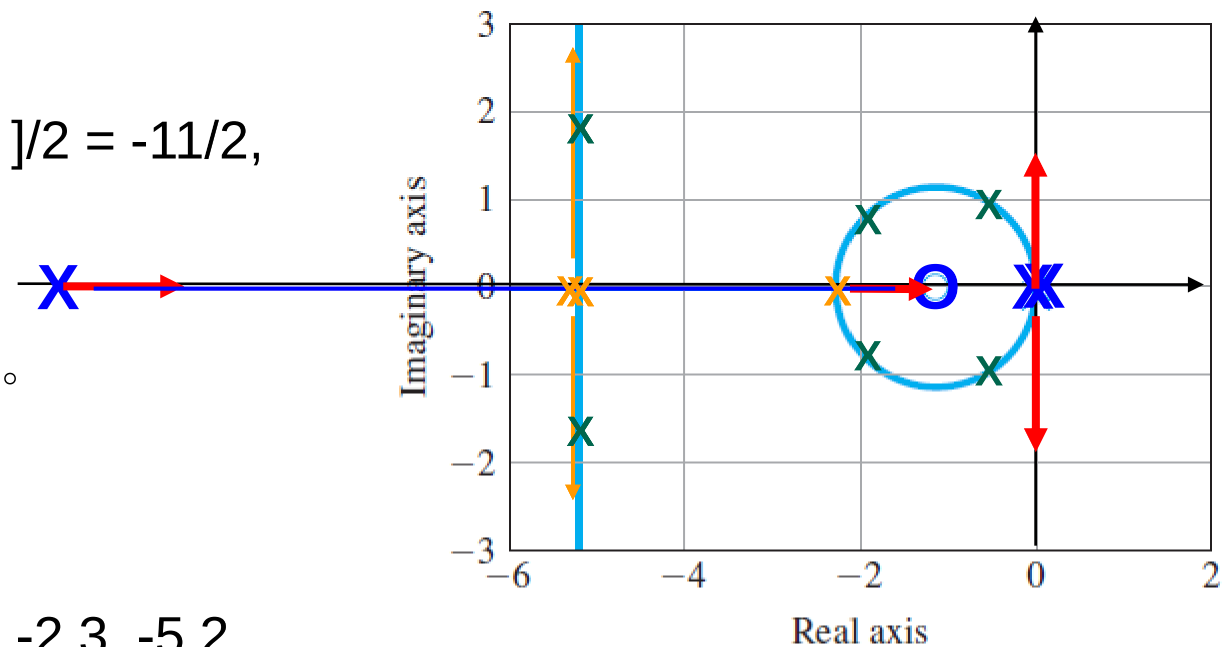
- Lead compensator: $z < p$

Example 5.4: Satellite Control w/ Modified PD or Lead Compensation

$z = 1, \quad p = 12$

$$\Rightarrow 1 + K \frac{(s+1)}{s^2 (s+12)} = 0$$

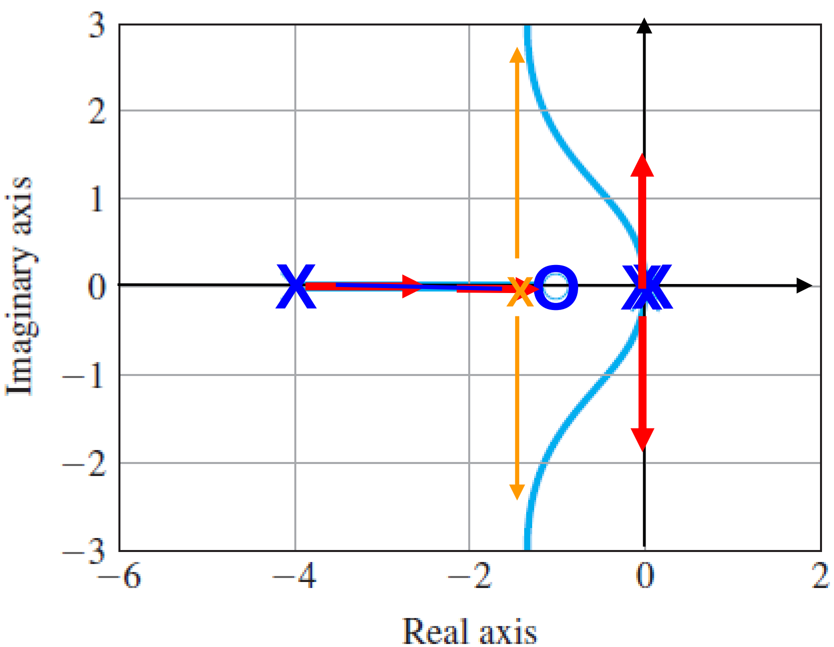
- **Rule 1:**
  - 3 branches, two start at s = 0, one at s = -12
- **Rule 2:**
  - Real axis: -12 <= s <= -1
- **Rule 3:**
  - Asymptotes: 3-1=2, centered at [-12-(-1)]/2 = -11/2, at +- 90◦
- **Rule 4:**
  - Depart at s=0: +- 90◦
  - Depart at s=-12: 0◦
- **Rule 5:**
  - Multiple roots: at s = -2.3, -5.2



Example 5.5: Satellite Control w/ Lead Having a Relatively Small Value for the Pole

$z = 1, \quad p = 4 \quad \Rightarrow 1 + K \frac{(s+1)}{s^2(s+4)} = 0$

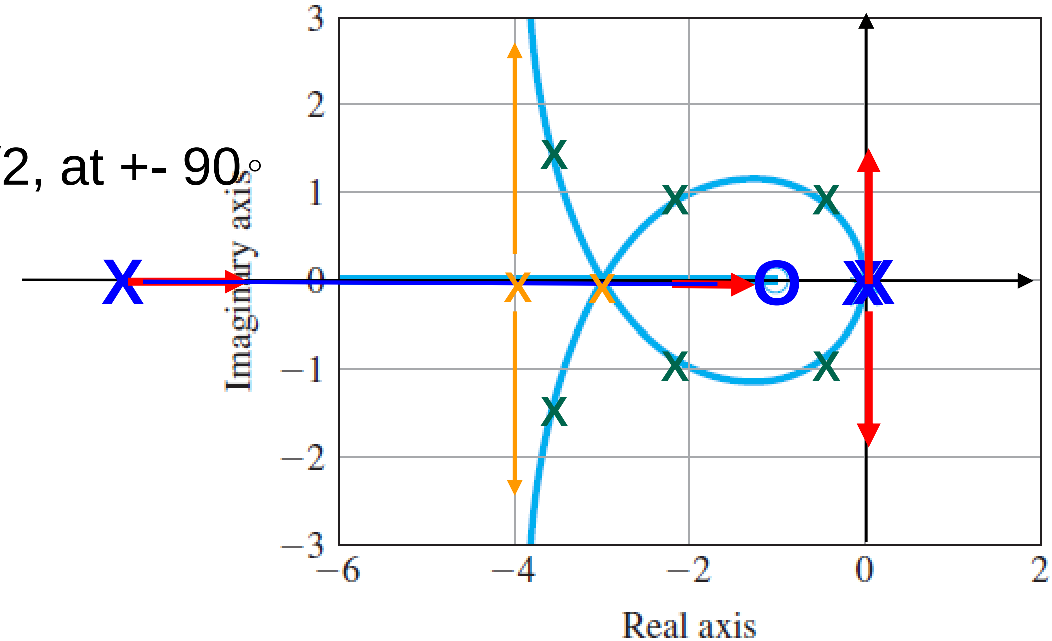
- Rule 1:
  - 3 branches, two start at s = 0, one at s = -4
- Rule 2:
  - Real axis: -4 <= s <= -1
- Rule 3:
  - Asymptotes: 3-1=2, centered at [-4-(-1)]/2 = -3/2, at +- 90◦
- Rule 4:
  - Depart at s=0: +- 90◦



Example 5.6: Satellite w/ Transition Value for the Pole

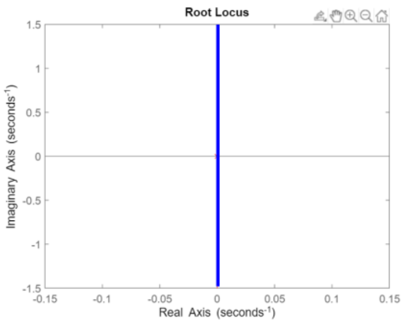
$$z = 1, \quad p = 9 \qquad \Rightarrow 1 + K \frac{(s+1)}{s^2(s+9)} = 0$$

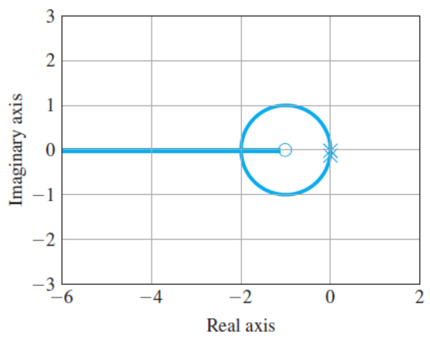
- Rule 1:
  - 3 branches, two start at s = 0, one at s = -9
- Rule 2:
  - Real axis: -9 <= s <= -1
- Rule 3:
  - Asymptotes: 3-1=2, centered at [-9-(-1)]/2 = -8/2, at +- 90◦
- Rule 4:
  - Depart at s=0: +- 90◦
- At s = -3,
  - arrival: +60◦, -60◦, 0◦
  - depart: +120◦, -120◦, 0◦



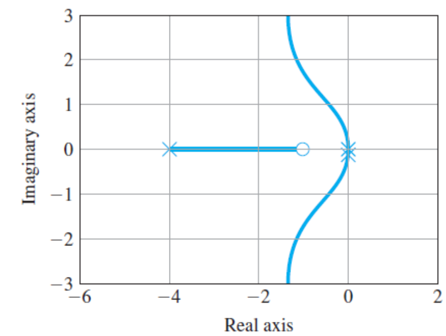
Summary of Examples 5.3 5.4, 5.5, 5.6: Satellite Control

$$\Rightarrow 1 + K_P \frac{1}{s^2} = 0$$

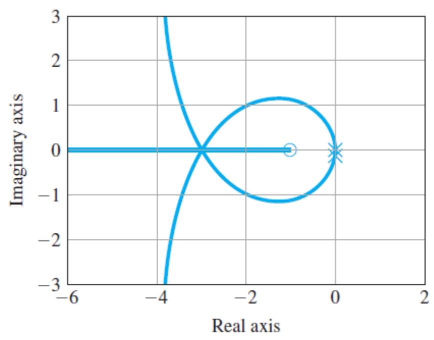




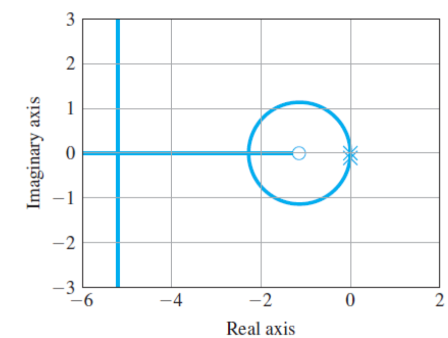
$$\Rightarrow 1 + K \frac{s+1}{s^2} = 0$$



$$\Rightarrow 1 + K \frac{(s+1)}{s^2(s+4)} = 0$$



$$\Rightarrow 1 + K \frac{(s+1)}{s^2(s+9)} = 0$$



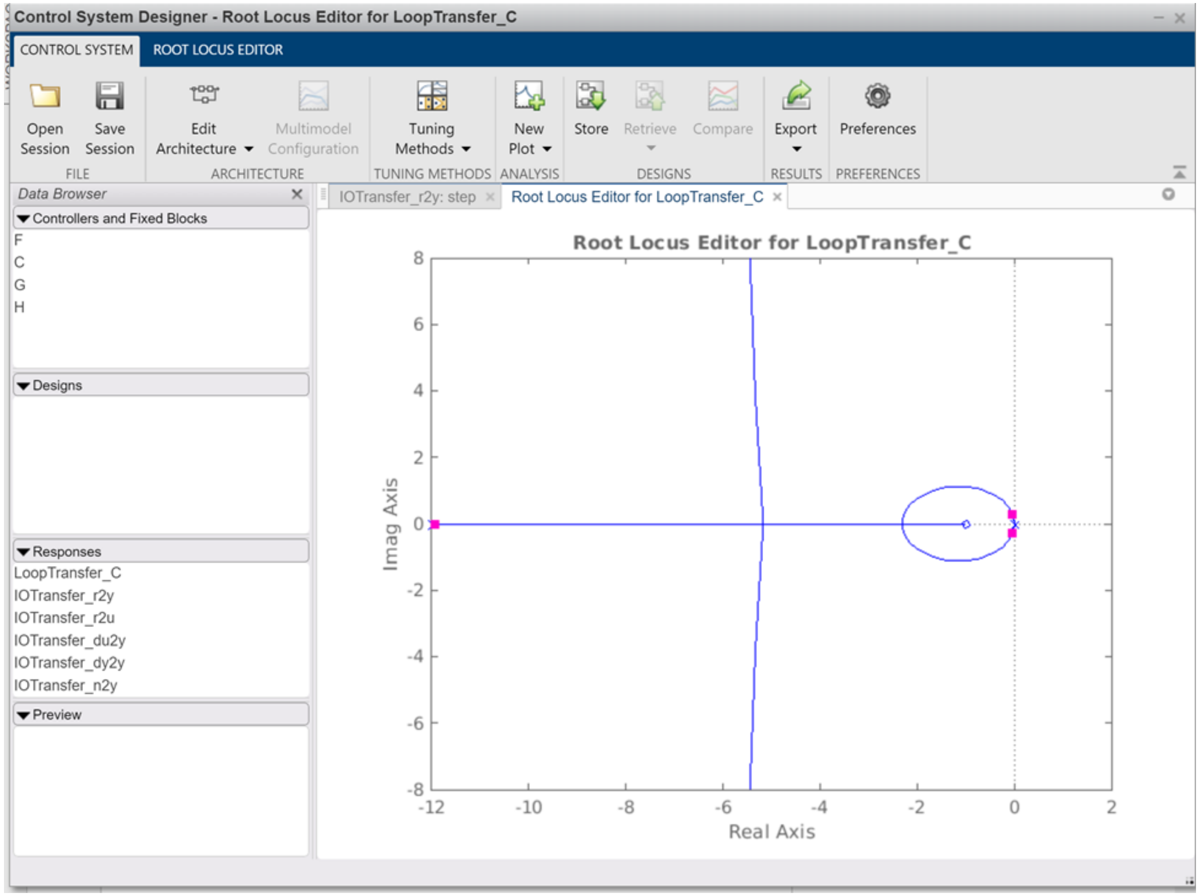
$$\Rightarrow 1 + K \frac{(s+1)}{s^2(s+12)} = 0$$

Example 5.7: Exercise to use rltool (Matlab)

```
s = tf( 's' )

sysL = ( s+1 )/(s^2);
% sysL = (s+1)/(s^2*(s+12));

rltool( sysL );
% sisotool( 'rlocus', sysL );
```



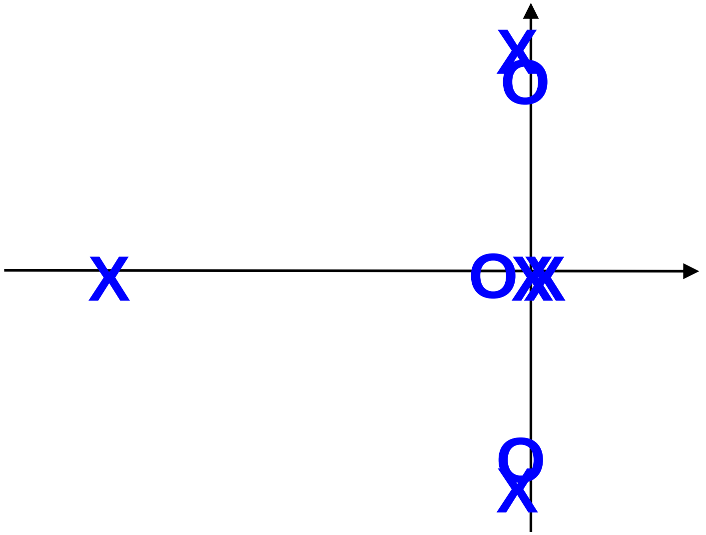
Example 5.8: Satellite Control w/ Collocated Flexibility

$$G(s) = \frac{(s+0.1)^2+6^2}{s^2[(s+0.1)^2+6.6^2]}$$

$$\Rightarrow 1 + D_c(s)\,G(s) = 0 \qquad \Rightarrow D_c(s) = K\frac{(s+1)}{(s+12)}$$

$$\Rightarrow 1 + K\,L(s) = 0 \qquad \Rightarrow L(s) = \frac{(s+1)}{(s+12)}\,\frac{(s+0.1)^2+6^2}{s^2[(s+0.1)^2+6.6^2]}$$

- Rule 1:
  - 5 branches,
  - 3 -> finite zeros
  - 2 -> asymptotes



- Rule 2:
  - Real axis: -12 <= s <= -1

Example 5.8: Satellite Control w/ Collocated Flexibility

- Rule 3:
  - Asymptotes: 5-3 = 2
  - Centered at [-12 -0.1 -0.1 - (-0.1 -9.1 -1)] / 2 = -11/2, at +- 90◦
- Rule 4:
  - Depart at s = -0.1+ j6.6:

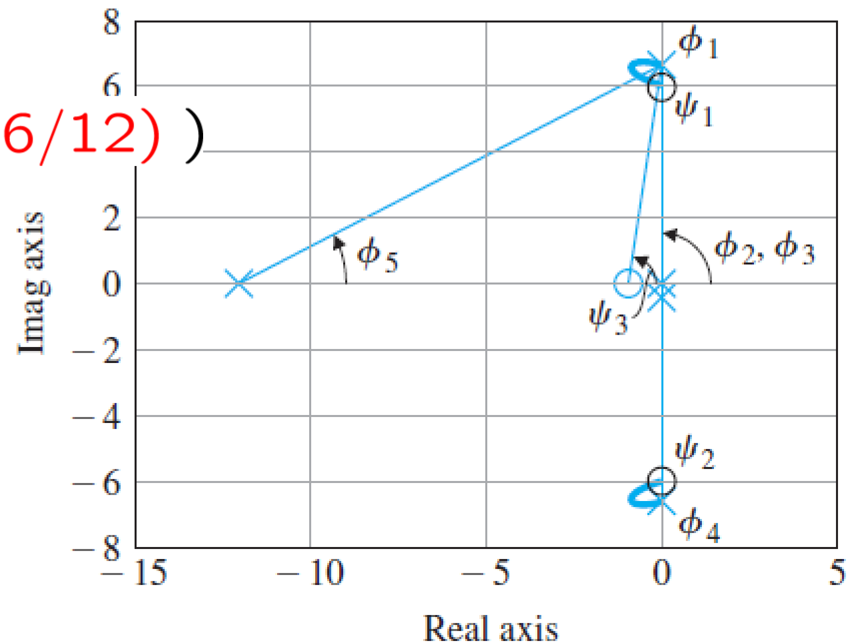
$$\phi_1 = \psi_1 + \psi_2 + \psi_3 - (\phi_2 + \phi_3 + \phi_4 + \phi_5) - 180$$

$$= 90 + 90 + \tan^{-1}(6.6)$$

$$- (90 + 90 + 90 + \tan^{-1}(6.6/12))$$

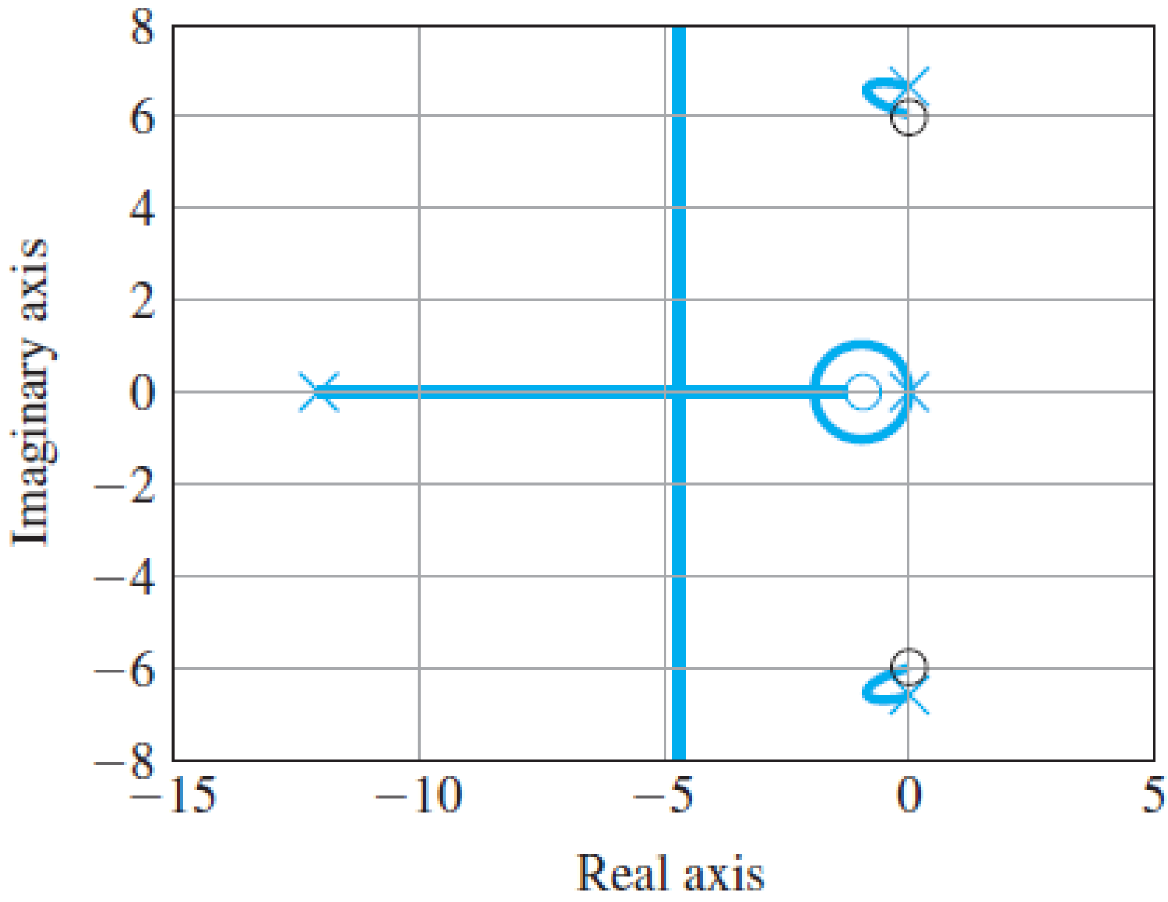
$$- 180$$

$$= 142.6$$



- Example 5.8: Satellite Control w/ Collocated Flexibility

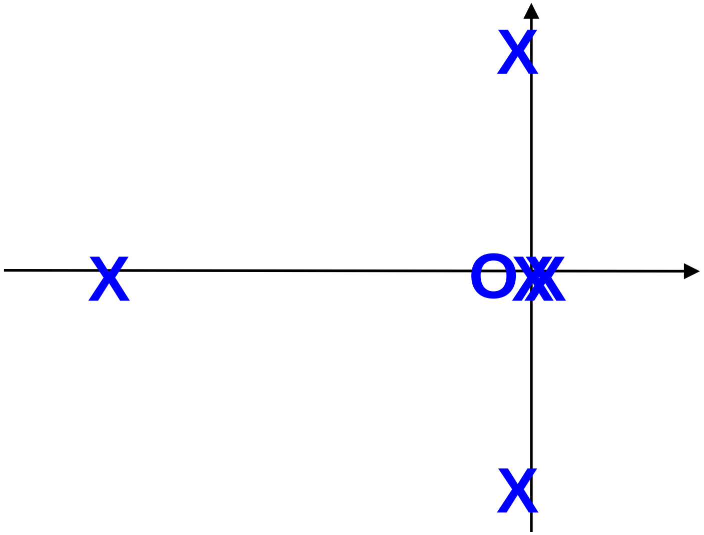
$$\Rightarrow \; L(s) = \frac{(s+1)}{(s+12)} \frac{(s+0.1)^2 + 6^2}{s^2[(s+0.1)^2 + 6.6^2]}$$



Example 5.9: For Non-Collocated Case

$$\Rightarrow K\ L(s) = D_c(s)\ G(s) \qquad = K \frac{(s+1)}{(s+12)} \frac{1}{s^2[(s+0.1)^2+6.6^2]}$$

- Rule 1:
  - 5 branches,
  - 1 -> finite zero
  - 4 -> asymptotes
- Rule 2:
  - Real axis: -12 <= s <= -1
- Rule 3:
  - Asymptotes: 5-1 = 4,
  - Centered at [-12-0.2-(-1)] / 4 = -11.2/4,
    at +- 45°, +- 135°



Example 5.9: For Non-Collocated Case

$$\Rightarrow K\,L(s) = D_c(s)\,G(s) = K\,\frac{(s+1)}{(s+12)}\,\frac{1}{s^2[(s+0.1)^2+6.6^2]}$$

- Rule 4:
  - Depart at s= -0.1+ j6.6:

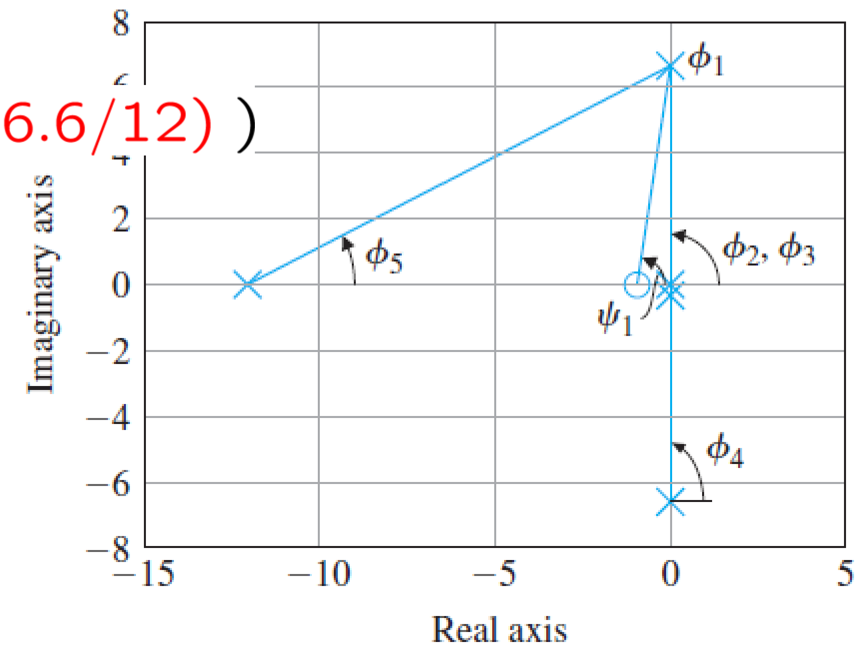
$$\phi_1 = \psi_1 - (\phi_2 + \phi_3 + \phi_4 + \phi_5) - 180$$

$$= \tan^{-1}(6.6)$$

$$-(90 + 90 + 90 + \tan^{-1}(6.6/12))$$

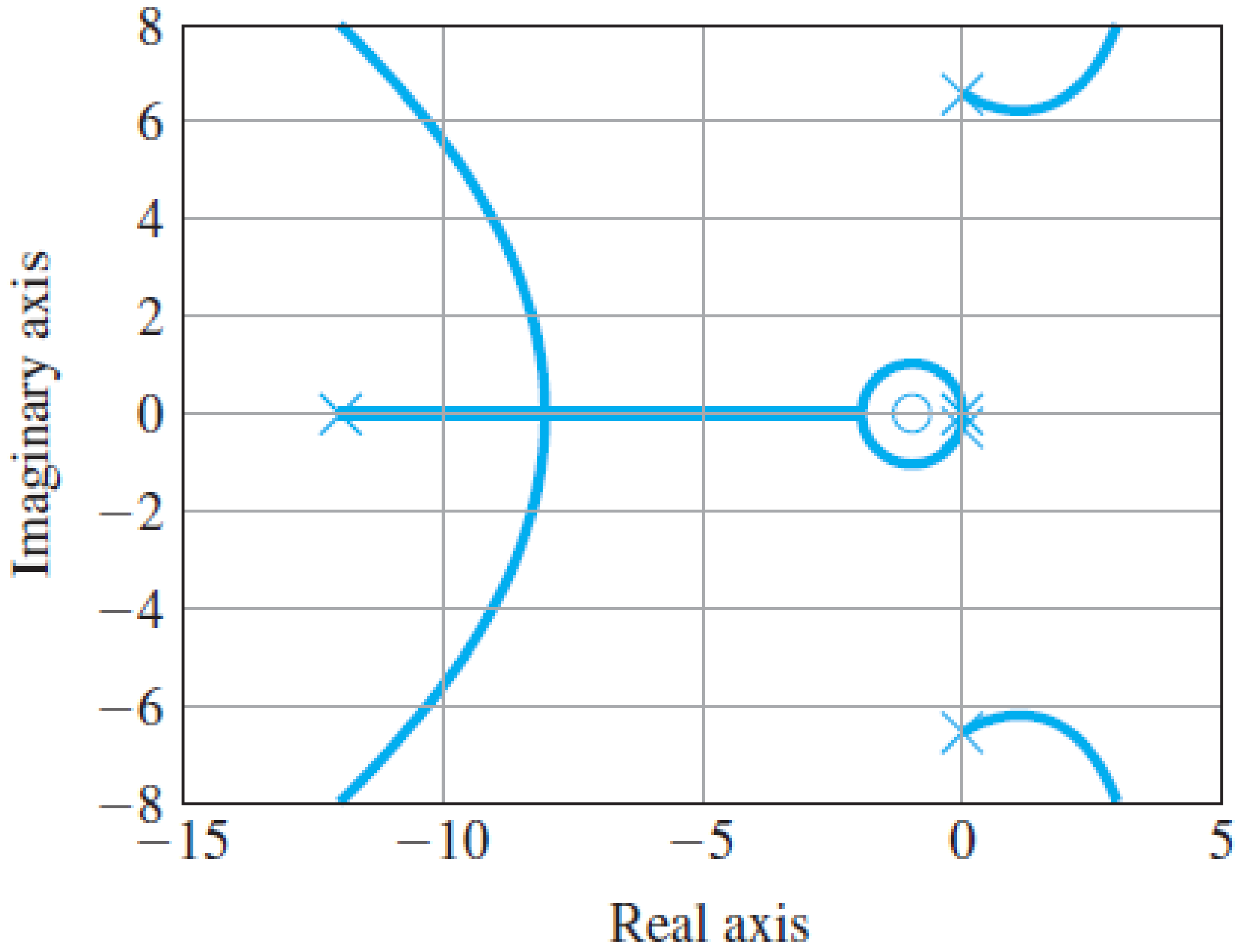
$$-180$$

$$= -37.4$$



Example 5.9: For Non-Collocated Case

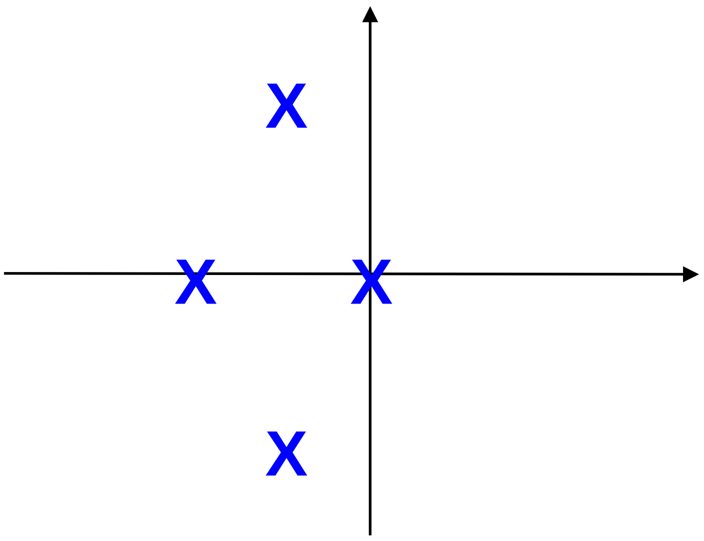
$$\Rightarrow K\, L(s) = D_c(s)\, G(s) = K \frac{(s+1)}{(s+12)} \frac{1}{s^2[(s+0.1)^2+6.6^2]}$$



Example 5.10: Having Complex Multiple Roots

$$\Rightarrow 1 + K\,L(s) = 0 \quad \Rightarrow L(s) = \frac{1}{s\,(s+2)\,[(s+1)^2+4]}$$

- Rule 1:
  - 4 branches,
  - 4 -> asymptotes
- Rule 2:
  - Real axis: -2 <= s <= 0
- Rule 3:
  - Asymptotes: 4-0 = 4,
  - Centered at [-2-1-1-0 + (0)] / 4 = -1,
    at +- 45°, +- 135°



Example 5.10: Having Complex Multiple Roots

$$\Rightarrow 1 + K\, L(s) = 0 \quad \Rightarrow L(s) = \frac{1}{s\,(s+2)\,[(s+1)^2+4]}$$

- **Rule 4:**
  - Depart at s= -1+ j2:

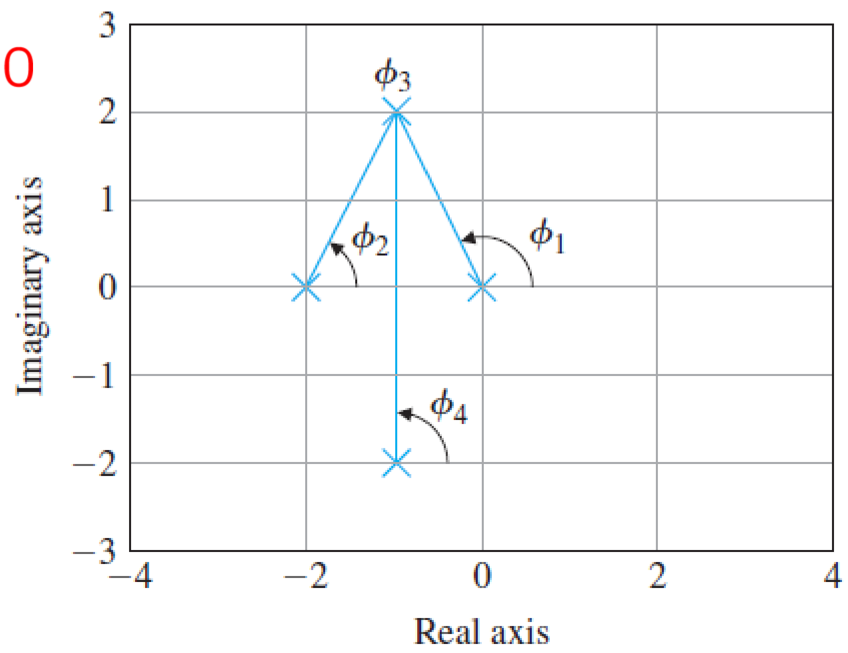
$$\phi_{dep} =$$

$$\phi_3 = -(\phi_1 + \phi_2 + \phi_4) + 180$$

$$= -\tan^{-1}\left(\frac{2}{-1}\right) - \tan^{-1}\left(\frac{2}{1}\right) - 90$$

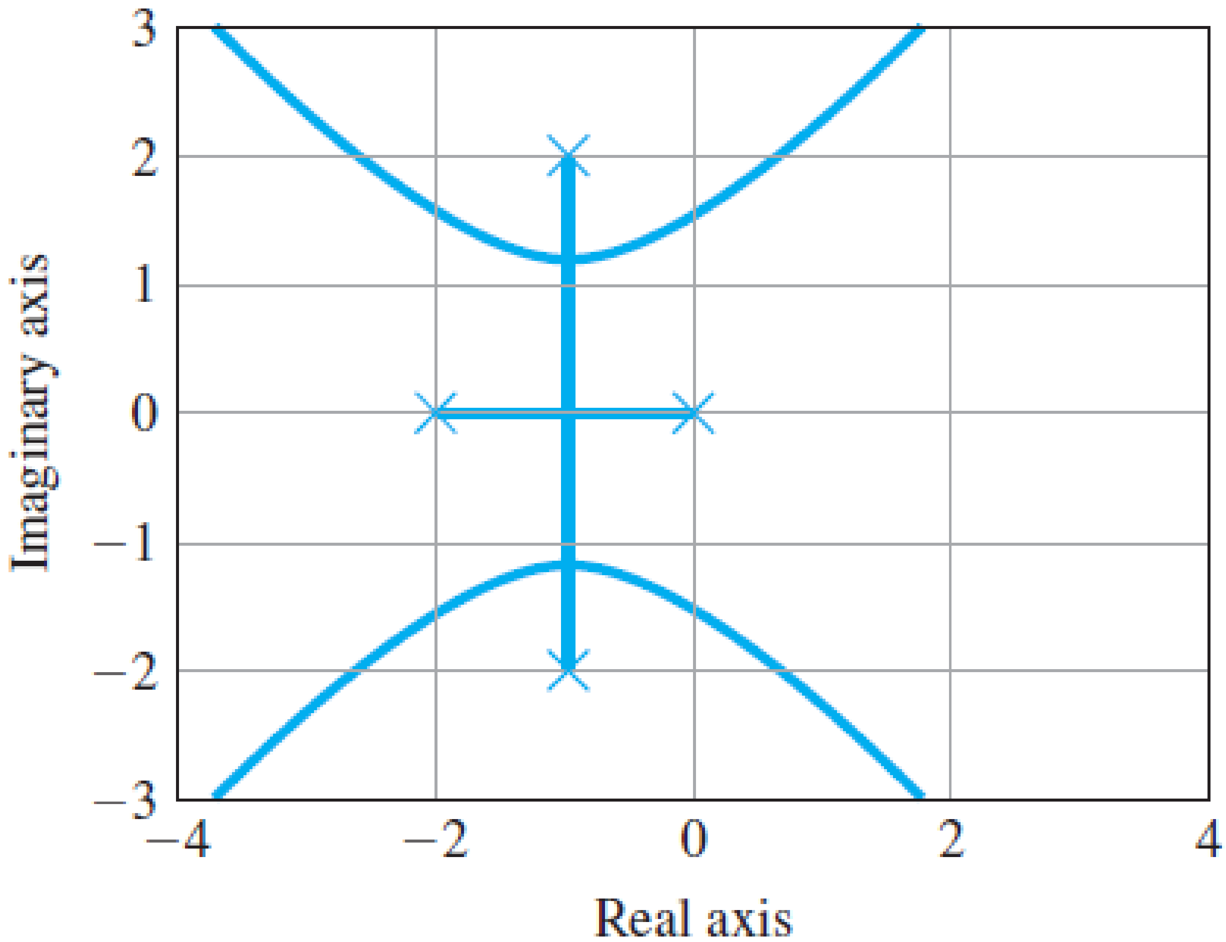
$$+ 180$$

$$= -90$$



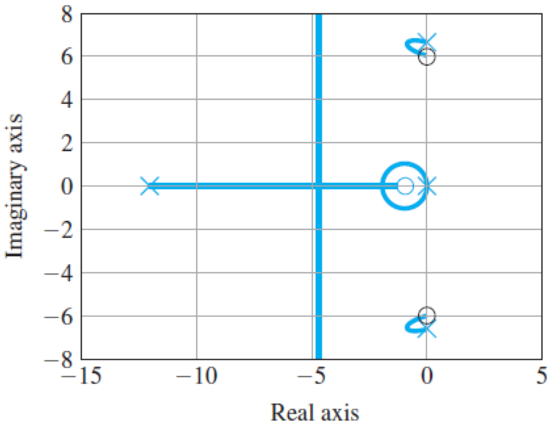
- Example 5.10: Having Complex Multiple Roots

$$\Rightarrow \; 1 + K\,L(s) = 0 \quad \Rightarrow \; L(s) = \frac{1}{s\,(s+2)\,[(s+1)^2+4]}$$

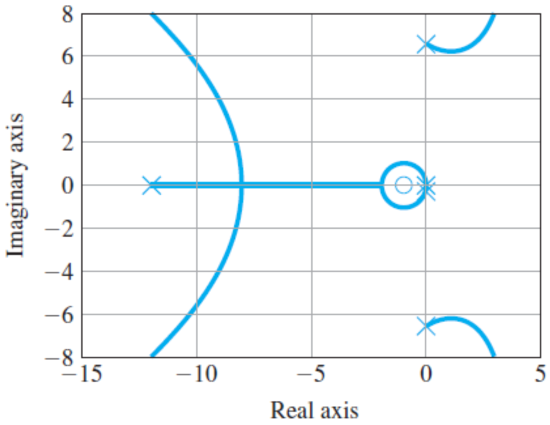


- Examples 5.8, 5.9, 5.10

$$\frac{(s+1)}{(s+12)}\;\frac{(s+0.1)^2+6^2}{s^2[(s+0.1)^2+6.6^2]}$$



$$\frac{(s+1)}{(s+12)}\;\frac{1}{s^2[(s+0.1)^2+6.6^2]}$$



$$\frac{1}{s\,(s+2)\,[(s+1)^2+4]}$$