

Spring 2021

控制系統
Control Systems

Unit 4C
PID Control

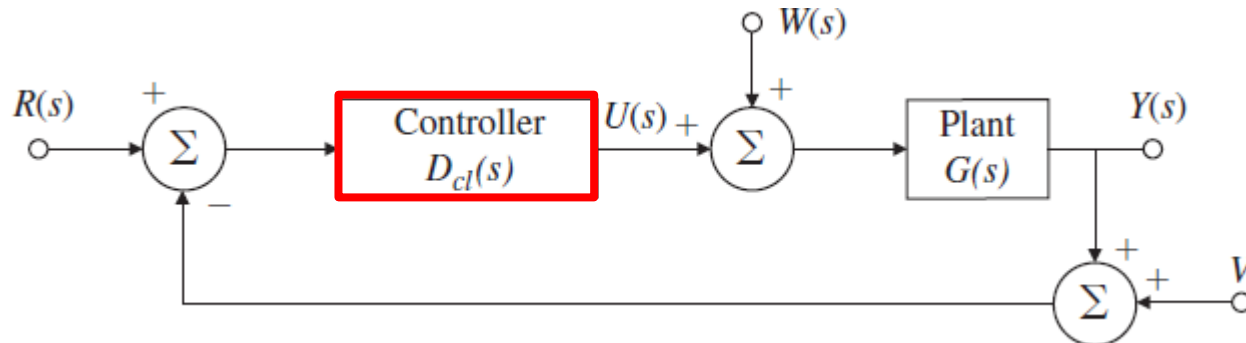
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NTU-EE

Feb – Jun, 2021

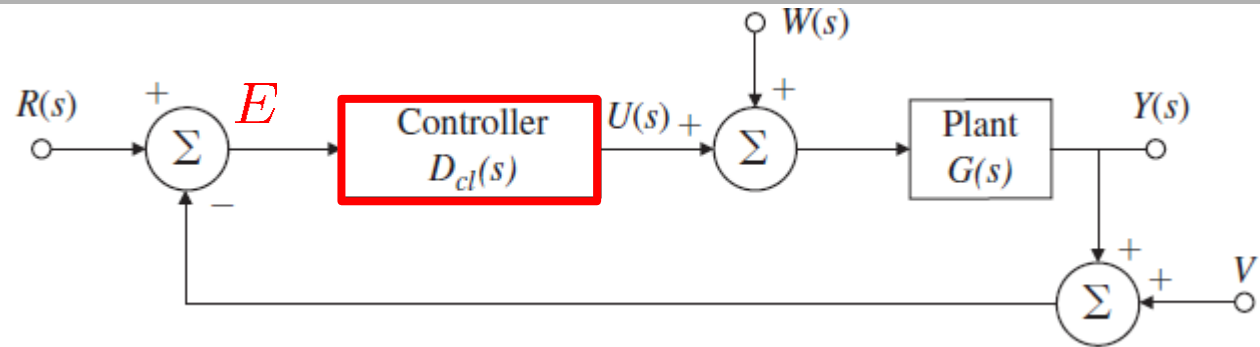
- PID Control:
 - Proportional: Simple Proportional Feedback
 - Integral: Eliminating Bias Offset
 - Derivative: Anticipatory

$$D_c(s) = k_P + \frac{k_I}{s} + k_D s$$



- Proportional Control

$$u(t) = k_P e(t)$$

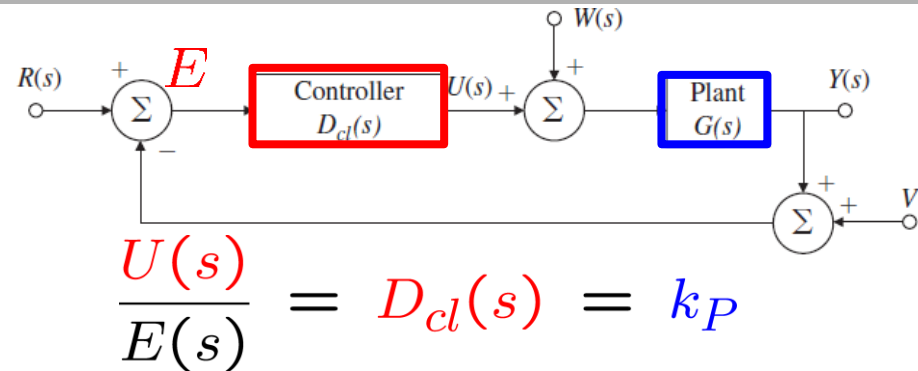


$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

- No dynamics, a static controller
- k_P : Proportional Gain
- An amplifier with a “knob” that can be adjusted up or down

- Consider a 2nd-order plant:

$$G(s) = \frac{A}{s^2 + a_1s + a_2}$$



- For the closed-loop system, the characteristic equation is:

$$\mathcal{T}(s) = \frac{Y(s)}{R(s)} = \frac{k_P G}{1 + k_P G}$$

$$1 + k_P G(s) = 0 \qquad 1 + k_P \frac{A}{s^2 + a_1s + a_2} = 0$$

$$s^2 + a_1s + a_2 + k_P A = 0$$

$$s^2 + a_1s + (a_2 + k_P A) = 0$$

$$\Rightarrow (a_2 + k_P A)$$

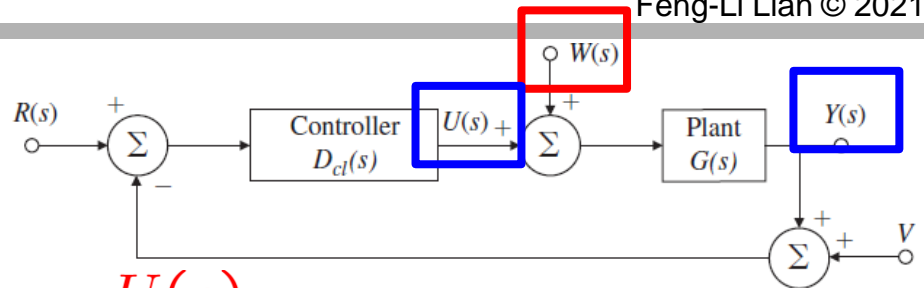
$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

\Rightarrow for natural frequency, not damping

- Consider a 2nd-order plant:

$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

$$r(t) = 1(t)$$



$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

```
zeta=0.707; wn=1; A=1;
```

```
k=1.5;  
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);  
[y,t] = step( sysy, t );
```

```
k=6;  
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);  
[y,t] = step( sysy, t );
```

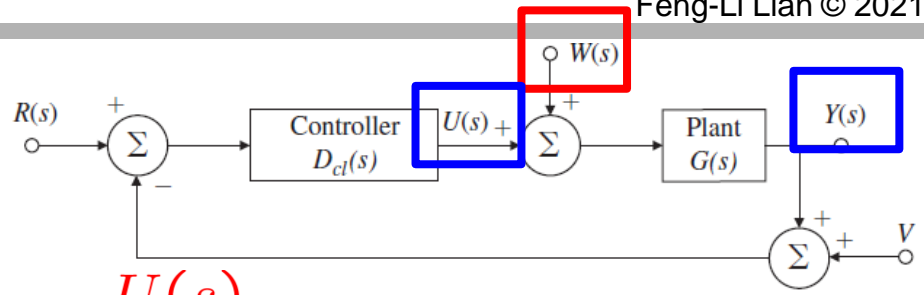
$$\frac{Y(s)}{W(s)} = \frac{G}{1 + k_P G}$$

$$\frac{U(s)}{W(s)} = \frac{-k_P G}{1 + k_P G}$$

- Consider a 2nd-order plant:

$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

$$r(t) = 1(t)$$



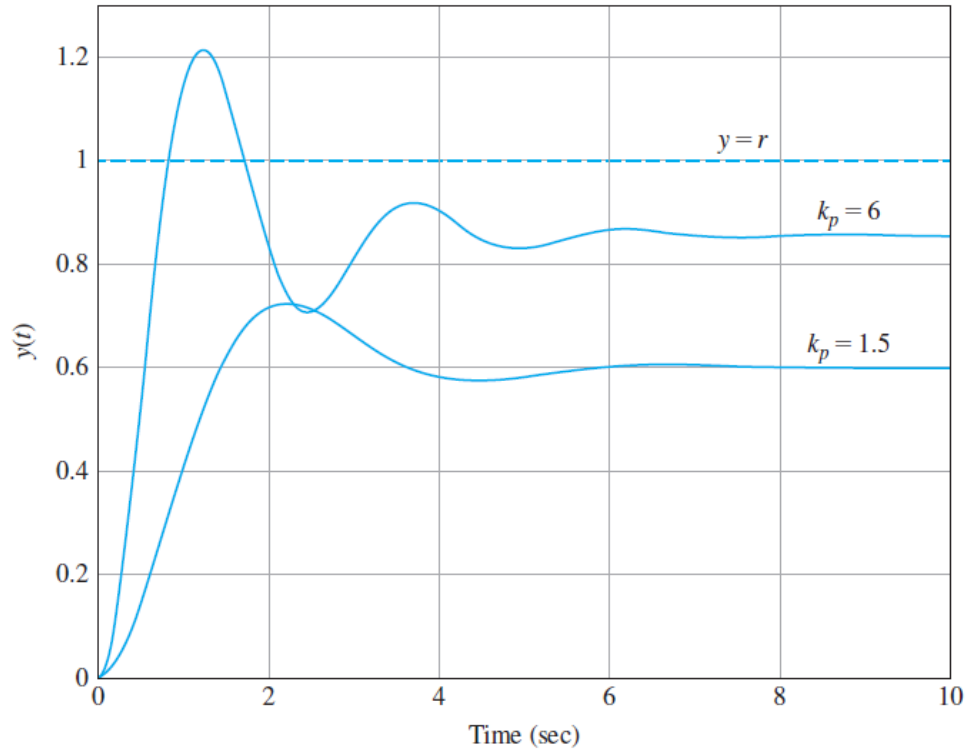
$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

```

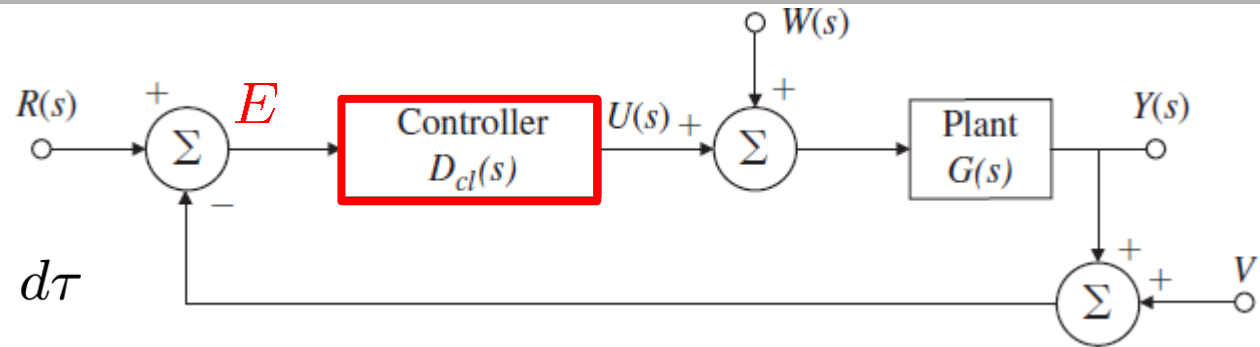
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k=1.5;
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);
[y,t] = step( sysy, t );

k=6;
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);
[y,t] = step( sysy, t );
    
```



● Integral Control

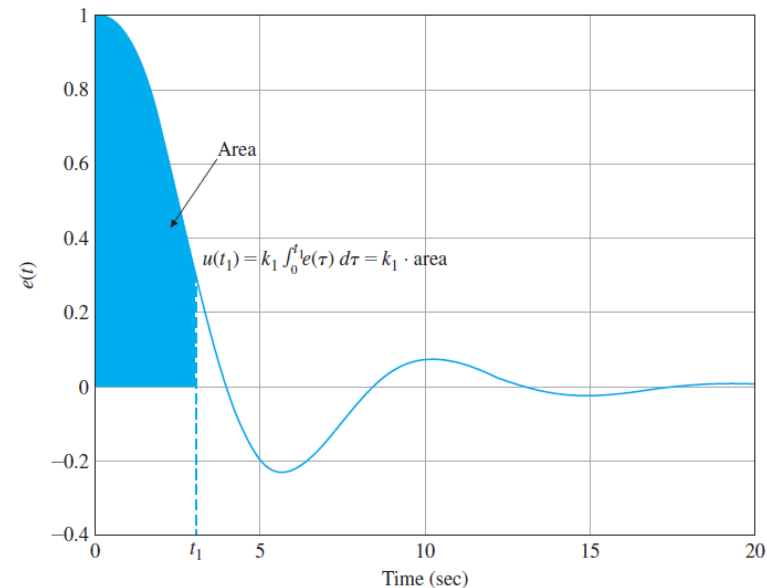


$$u(t) = k_I \int_{t_0}^t e(\tau) d\tau$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = \frac{k_I}{s}$$

● k_I : Integral Gain

- Sum of **all past values** of the tracking error
- Based on the **“history”** of the system error
- A **dynamic** controller



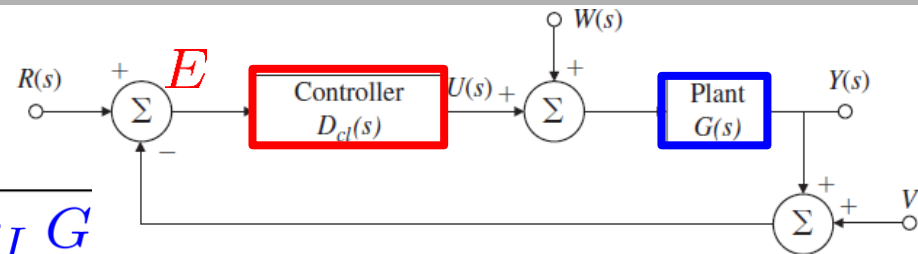
- The closed-loop system:

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \frac{k_I}{s} G} = \frac{s}{s + k_I G}$$

$$\frac{U(s)}{R(s)} = \frac{\frac{k_I}{s}}{1 + \frac{k_I}{s} G} = \frac{k_I}{s + k_I G}$$

$$\mathcal{T}(s) = \frac{Y(s)}{R(s)} = \frac{\frac{k_I}{s} G}{1 + \frac{k_I}{s} G} = \frac{k_I G}{s + k_I G}$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = \frac{k_I}{s}$$



- Unit-Step Reference Input: $r(t) = 1(t)$, $R(s) = 1/s$

$$y(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s} \frac{k_I G}{s + k_I G} = \frac{k_I G(0)}{0 + k_I G(0)} = 1$$

$$e(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s} \frac{s}{s + k_I G} = \frac{0}{0 + k_I G(0)} = 0$$

$$u(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s} \frac{k_I}{s + k_I G} = \frac{k_I}{0 + k_I G(0)} = G(0)^{-1} = 1$$

- The closed-loop system:

$$\frac{Y(s)}{W(s)} = \frac{s G}{s + k_I G}$$

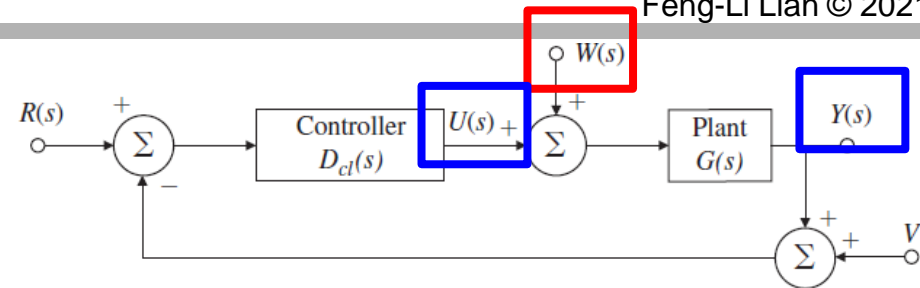
$$\frac{U(s)}{W(s)} = - \frac{k_I G}{s + k_I G}$$

- Unit-Step Disturbance Input:

$$w(t) = 1(t), W(s) = 1/s$$

$$\begin{aligned} y(\infty) &= \lim_{s \rightarrow 0} s \frac{1}{s} \frac{s G}{s + k_I G} \\ &= \frac{0 G(0)}{0 + k_I G(0)} = 0 \end{aligned}$$

$$\begin{aligned} u(\infty) &= \lim_{s \rightarrow 0} -s \frac{1}{s} \frac{k_I G}{s + k_I G} \\ &= - \frac{k_I G(0)}{0 + k_I G(0)} = -1 \end{aligned}$$



```

zeta=0.707; wn=1; A=1;

% Disturbance with integral control
ki = 0.5

sysy = s*A/(s^3+2*zeta*wn*s^2+wn^2*s+ki*A)
[y,t] = step(sysy,t);

sysu = -ki*A/(s^3+2*zeta*wn*s^2+wn^2*s+ki*A)
[u,t] = step(sysu,t);
    
```

- The closed-loop system:

$$\frac{Y(s)}{W(s)} = \frac{s G}{s + k_I G}$$

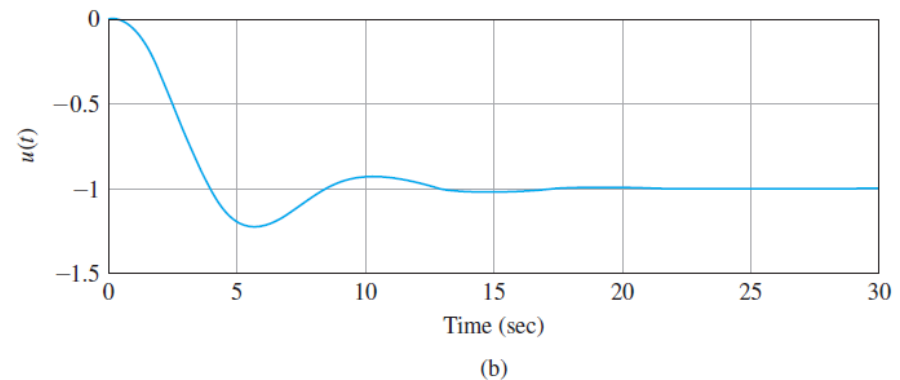
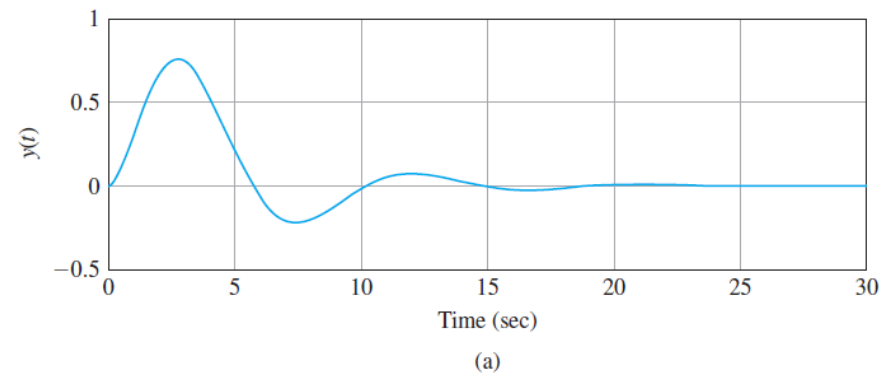
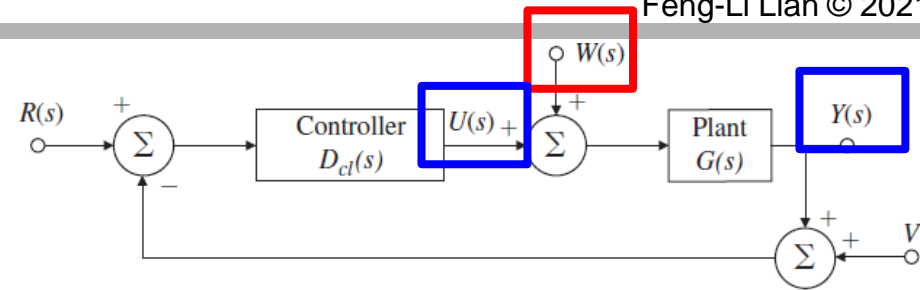
$$\frac{U(s)}{W(s)} = - \frac{k_I G}{s + k_I G}$$

- Unit-Step Disturbance Input:

$$w(t) = 1(t), W(s) = 1/s$$

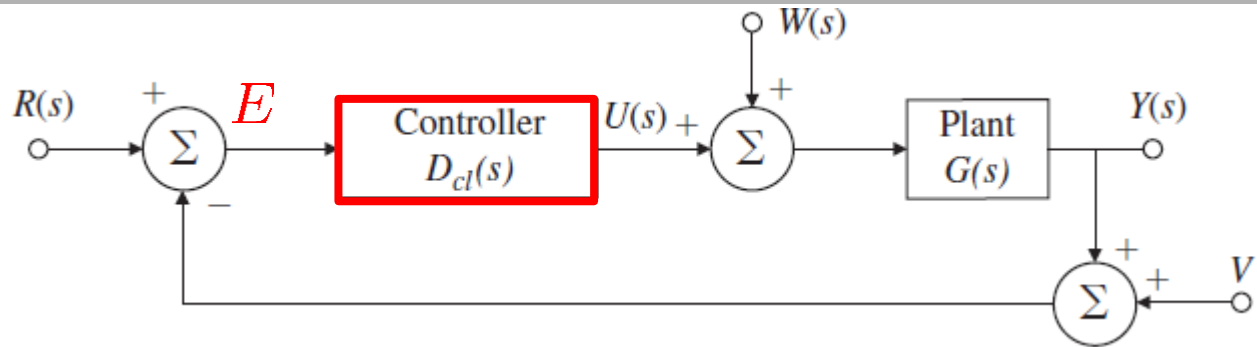
$$\begin{aligned} y(\infty) &= \lim_{s \rightarrow 0} s \frac{1}{s} \frac{s G}{s + k_I G} \\ &= \frac{0 G(0)}{0 + k_I G(0)} = 0 \end{aligned}$$

$$\begin{aligned} u(\infty) &= \lim_{s \rightarrow 0} -s \frac{1}{s} \frac{k_I G}{s + k_I G} \\ &= - \frac{k_I G(0)}{0 + k_I G(0)} = -1 \end{aligned}$$



● Derivative Control

$$u(t) = k_D \dot{e}(t)$$

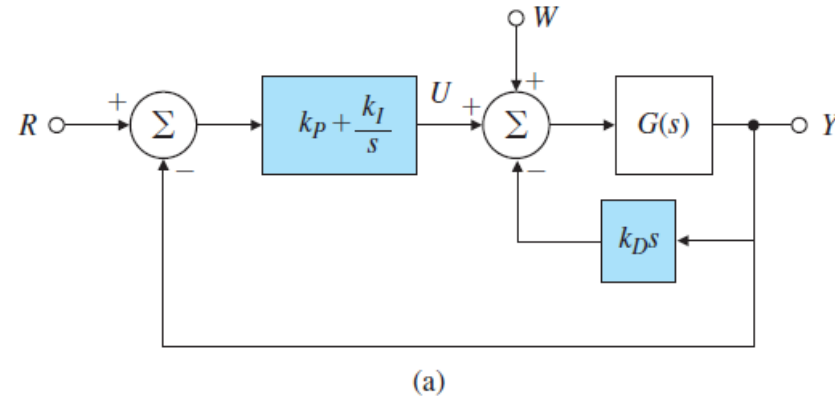


$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_D s$$

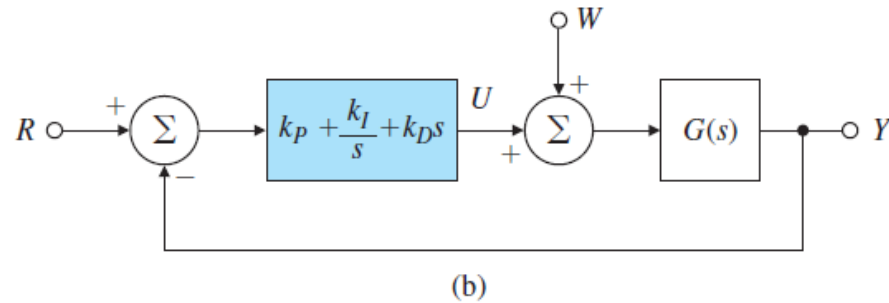
- k_D : Derivative Gain
- Rate feedback, know the **slope** of error signal
- **Anticipatory** behavior
- Improve the **stability** of closed-loop system
- Speed up the **transient** response
- Reduce **overshoot**
- Amplify **noise**

Derivative Control

- **Derivative Control**
 almost never used by itself.
- It is usually augmented by **proportional control**.
- **Derivative Control**
 does not supply desired end state.

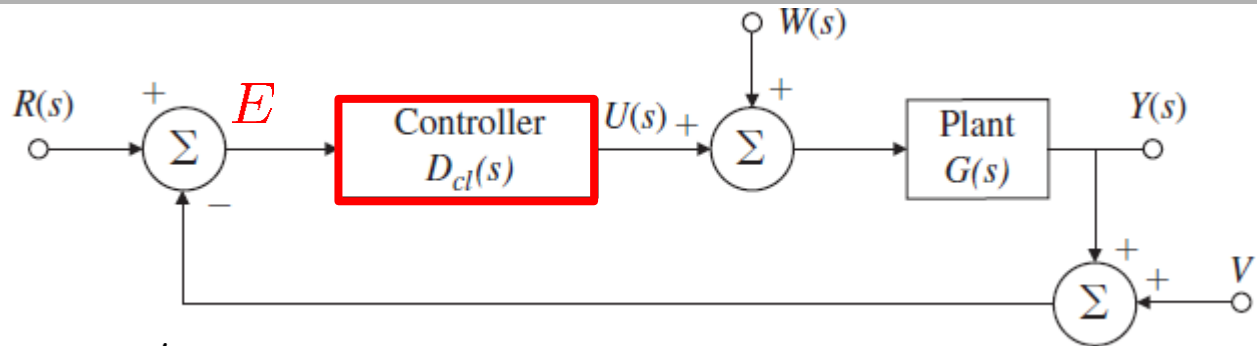


- $e(t) = \text{constant}$
 $\rightarrow d e(t) = 0$



- Provide **sharp** response to suddenly changing signals
- By using **velocity** sensor (e.g., tachometer)
- The closed-loop **characteristic equations** of (a) and (b) are **the same**.
 But, the **zeros** from reference to output are **different**.
 Because the **reference** is **not differentiated** in (a).

- PI Control



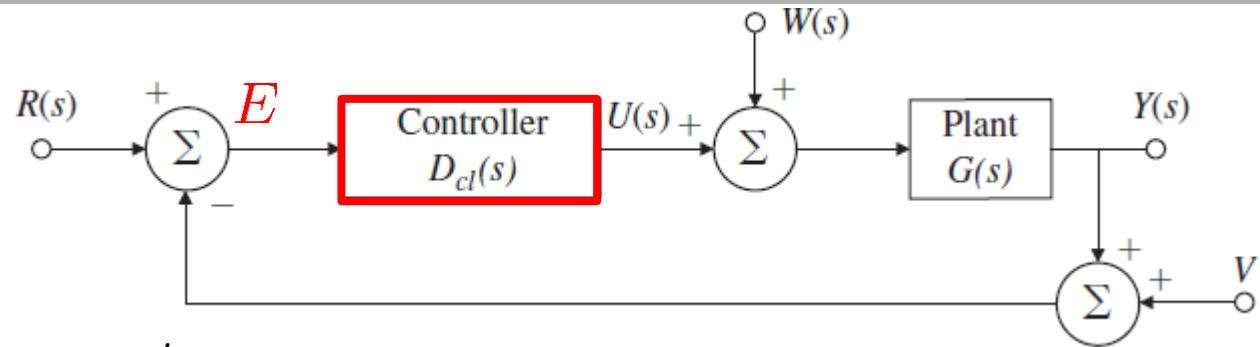
$$u(t) = k_P e(t) + k_I \int_{t_0}^t e(\tau) d\tau$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P + \frac{k_I}{s}$$

- Allow for a fast response

- I control raises the type to Type 1 and reject constant bias disturbance

● PID Control



$$u(t) = k_P e(t) + k_I \int_{t_0}^t e(\tau) d\tau + k_D \dot{e}(t)$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P + \frac{k_I}{s} + k_D s$$

● Characteristic Equation:

$$1 + \left(k_P + \frac{k_I}{s} + k_D s\right) \frac{A}{s^2 + a_1 s + a_2} = 0$$

$$1 + D_{cl}(s) G(s) = 0$$

$$s^2 + a_1 s + a_2 + A \left(k_P + \frac{k_I}{s} + k_D s \right) = 0$$

$$s^3 + a_1 s^2 + a_2 s + A (k_P s + k_I + k_D s^2) = 0$$

$$s^3 + (a_1 + A k_D) s^2 + (a_2 + A k_P) s + A k_I = 0$$

⇒ 3 roots by 3 parameters (gains)

Examples

- Example 4.5: PID Control of Motor Speed (from Ex 2.15)

- Parameters:

- J_m : 1.13×10^{-2} N m sec²/rad
- b : 0.028 N m sec/rad
- L_a : 10^{-1} H
- R_a : 0.45 Ω
- K_t : 0.67 N m/amp
- K_e : 0.67 V sec/rad

- Gain:

- K_P : 3
- K_I : 15
- K_D : 0.3

```

K=.0670; L1=0.1; J1=0.0113; R=0.45; b=0.0280;
kp=3; ki= 15; kd=0.3;
np=K;
dp=[L1*J1 R*J1+b*L1 R*b+K*K];
dclp=[L1*J1 R*J1+b*L1 R*b+K*K+K*kp];
nclp=K*kp;
nclpw=[L1 R];
dclpi=[L1*J1 R*J1+b*L1 R*b+K*K+K*kp K*ki];
nclpi=[K*kp K*ki];
nclpiw=[L1 R 0];
dclpid=[ L1*J1 R*J1+b*L1+K*kd R*b+K*K+K*kp K*ki];
nclpid=[K*kd K*kp K*ki];
nclpidw=[L1 R 0];
sysp      = tf( nclp, dclp );
syspw     = tf( nclpw, dclp );
syspi     = tf( nclpi, dclpi );
syspiw    = tf( nclpiw, dclpi );
syspid    = tf( nclpid, dclpid );
syspidw   = tf( nclpidw, dclpid );
    
```

Responses of P, PI, and PID control
 to: (a) step disturbance input
 to: (b) step reference input

Examples

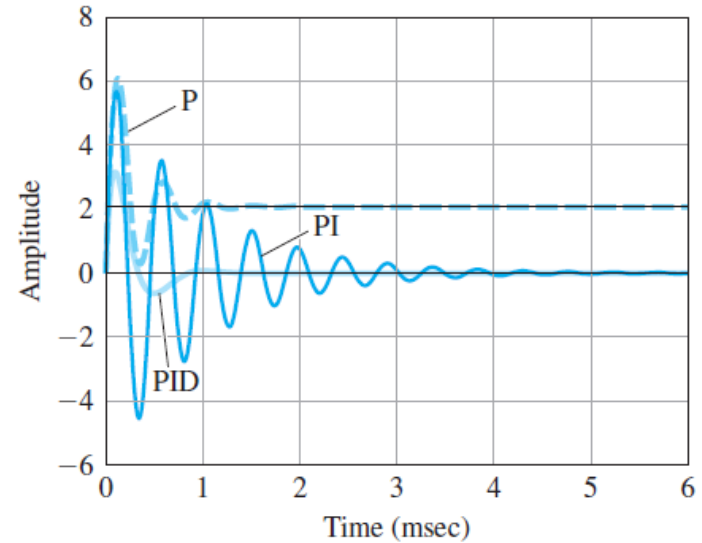
- Example 4.5: PID Control of Motor Speed (from Ex 2.15)

- Parameters:

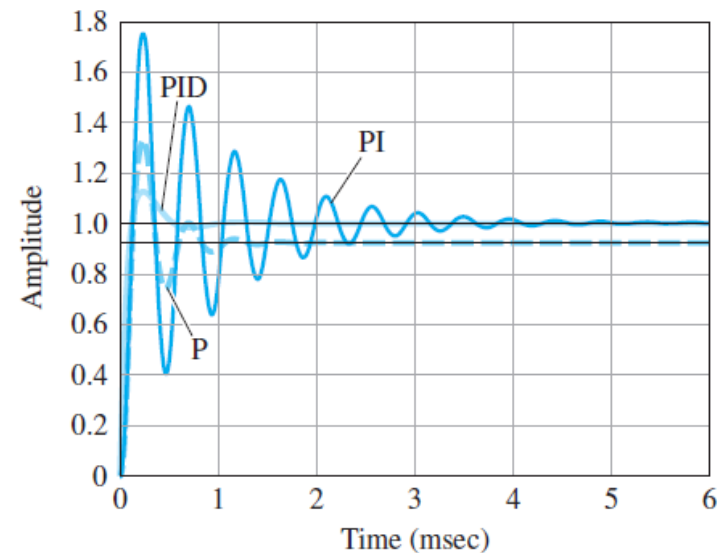
- J_m : 1.13×10^{-2} N m sec²/rad
- b : 0.028 N m sec/rad
- L_a : 10^{-1} H
- R_a : 0.45 Ω
- K_t : 0.67 N m/amp
- K_e : 0.67 V sec/rad

- Gain:

- K_P : 3
- K_I : 15
- K_D : 0.3



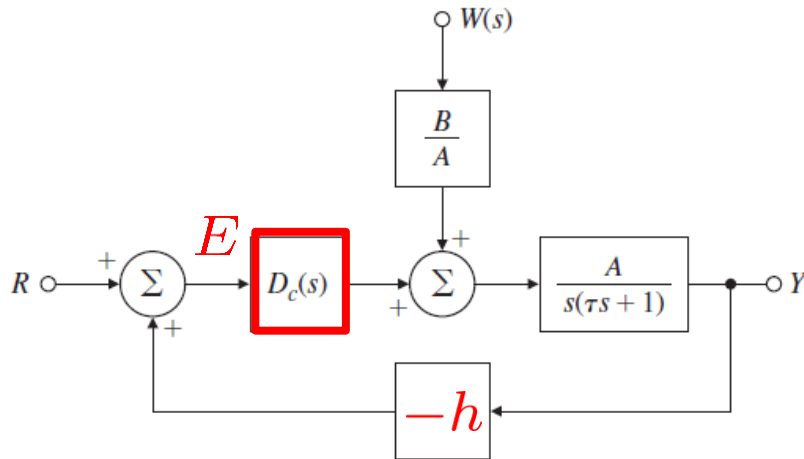
(a)



(b)

Responses of P, PI, and PID control
to: (a) step disturbance input
to: (b) step reference input

- Example 4.6: PD Control for DC Motor Position Control (From Ex 4.4)



$$(a) D_c(s) = k_P$$

$$(b) D_c(s) = k_P + \frac{k_I}{s}$$

- (a) The closed-loop transfer function from W to E (where $R = 0$) is:

$$\begin{aligned} T_w(s) &= \frac{-B}{s(\tau s + 1) + Ak_P h} \\ &= s^0 T_{w,0}(s) \end{aligned}$$

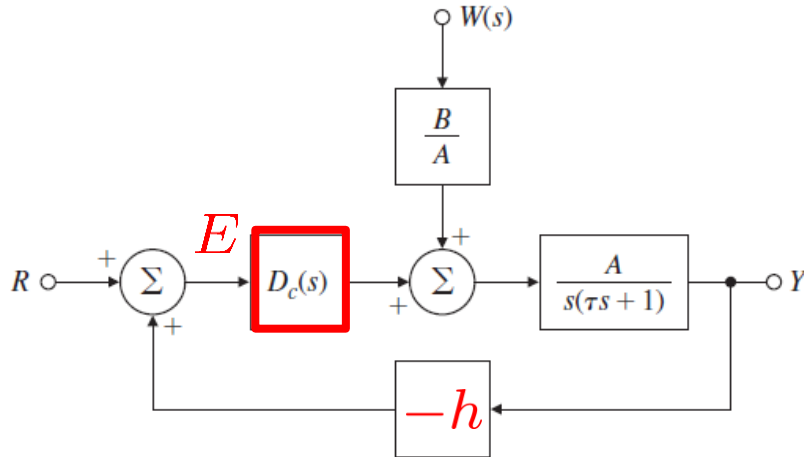
$$n = 0$$

$$K_{w,0} = -\frac{Ak_P h}{B}$$

- **Type 0:** Steady-state error to a **unit-step** disturbance input is:

$$e_{ss} = -\frac{B}{Ak_P h}$$

- Example 4.6: PD Control for DC Motor Position Control (From Ex 4.4)



$$(a) D_c(s) = k_P$$

$$(b) D_c(s) = k_P + \frac{k_I}{s}$$

- (b) The closed-loop transfer function from W to E (where $R = 0$) is:

$$\mathcal{T}_w(s) = \frac{-Bs}{s^2(\tau s + 1) + (k_P s + k_I)Ah}$$

$$n = 1$$

$$K_{w,n} = -\frac{Ak_I h}{B}$$

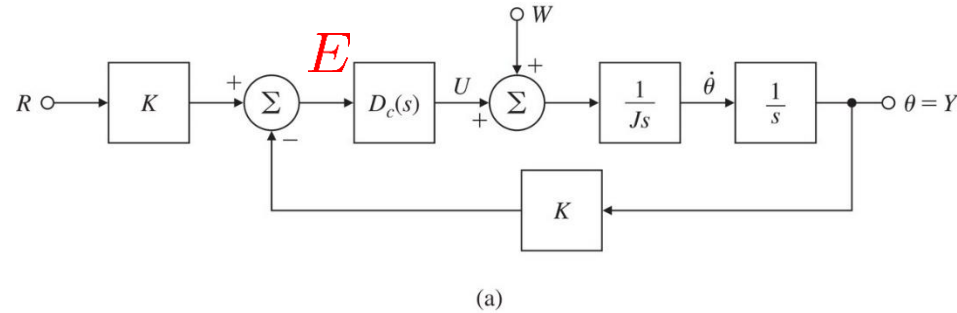
- **Type 1:** Steady-state error to a **unit-ramp** disturbance input is:

$$e_{ss} = -\frac{B}{Ak_I h}$$

Examples

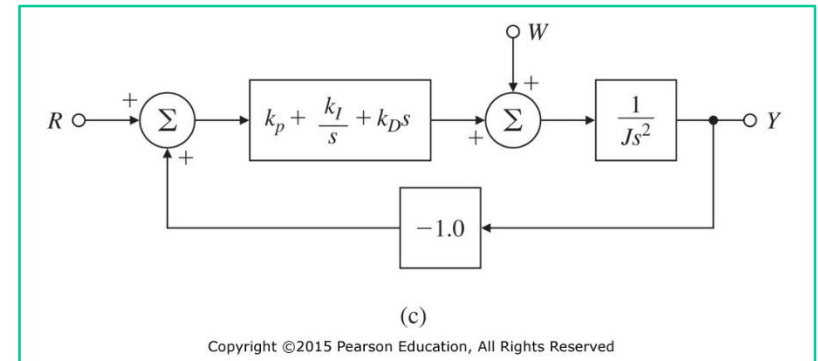
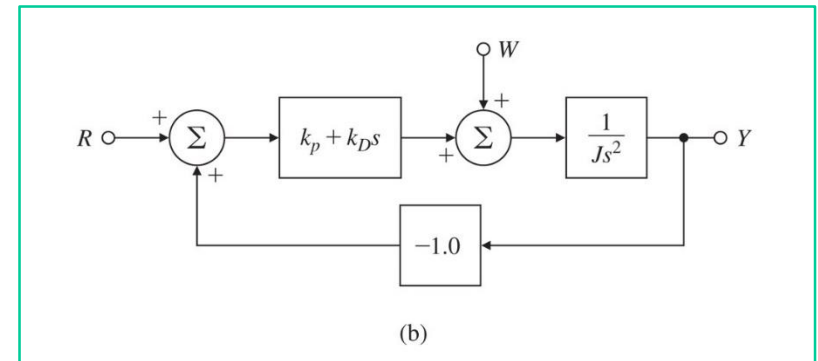
● Example 4.7: Satellite Attitude Control

- J : moment of inertia
- W : disturbance torque
- K : sensor and reference gain
- D_c : compensator



(b) $D_c(s) = k_P + k_D s$

(c) $D_c(s) = k_P + \frac{k_I}{s} + k_D s$



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Model of a satellite attitude control:

- (a) basic system
- (b) PD control
- (c) PID control

Examples

● Example 4.7: Satellite Attitude Control

(b) $D_c(s) = k_P + k_D s$

$$\mathcal{T}_w(s) = \frac{-1}{Js^2 + k_D s + k_P}$$

$$= \mathcal{T}_{w,0}(s)$$

$n = 0$ $K_{w,0} = -k_P$

■ Type 0: $e_{ss} = -\frac{1}{k_P}$

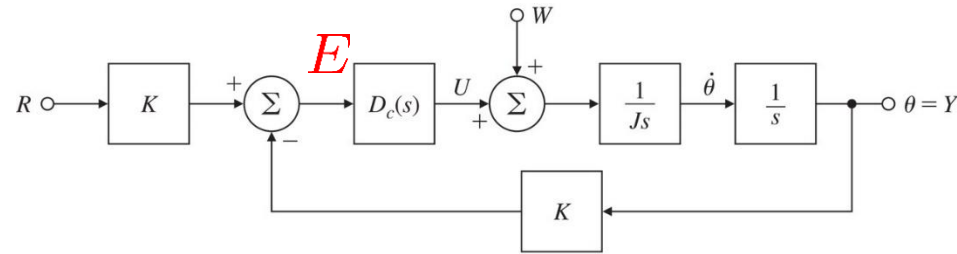
(c) $D_c(s) = k_P + \frac{k_I}{s} + k_D s$

$$\mathcal{T}_w(s) = \frac{-s}{Js^3 + k_D s^2 + k_P s + k_I}$$

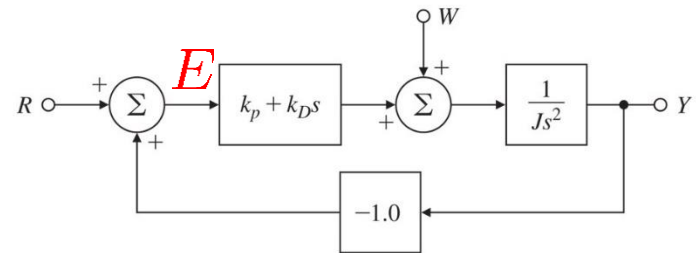
$$\mathcal{T}_{w,0}(s) = \frac{-1}{Js^3 + k_D s^2 + k_P s + k_I}$$

$n = 1$ $K_{w,n} = -k_I$

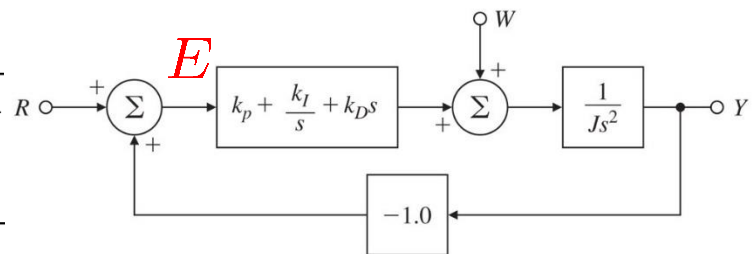
■ Type 1: $e_{ss} = -\frac{1}{k_I}$



(a)



(b)



(c)

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Model of a satellite attitude control:

(a) basic system

(b) PD control

(c) PID control