Spring 2021

控制系統
Control Systems

Unit 4A
The Basic Equations of Control

Feng-Li Lian
NTU-EE

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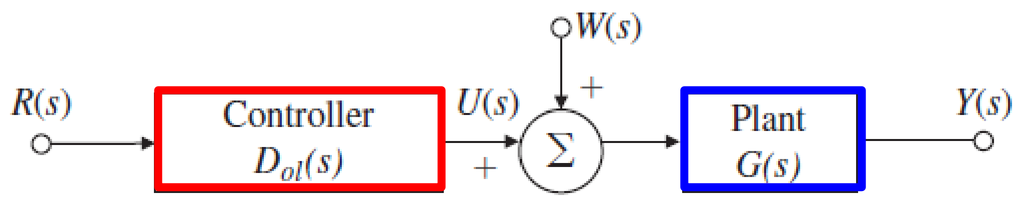
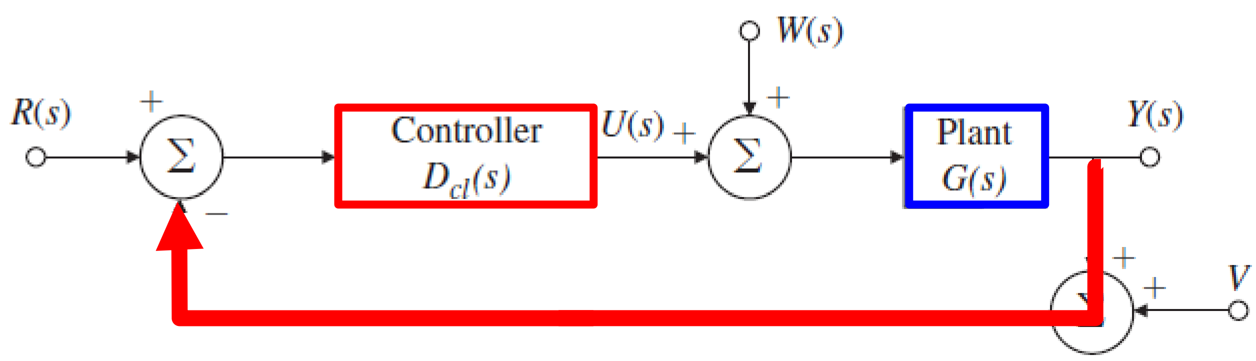
Open-Loop and Closed-Loop Systems

- Open-loop system showing
  reference, R, control, U, disturbance, W, and output Y

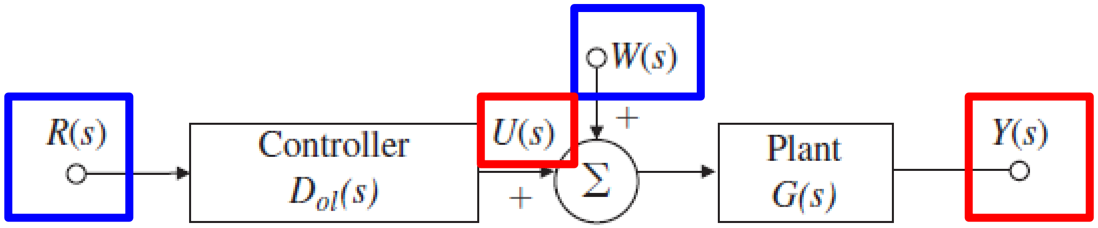




- Closed-loop system showing
  reference, R, control, U, disturbance, W, output, Y, and sensor noise, V

Open-Loop and Closed-Loop Systems

- Open-loop system showing reference, R, control, U, disturbance, W, and output Y



$$U = D_{ol} R$$

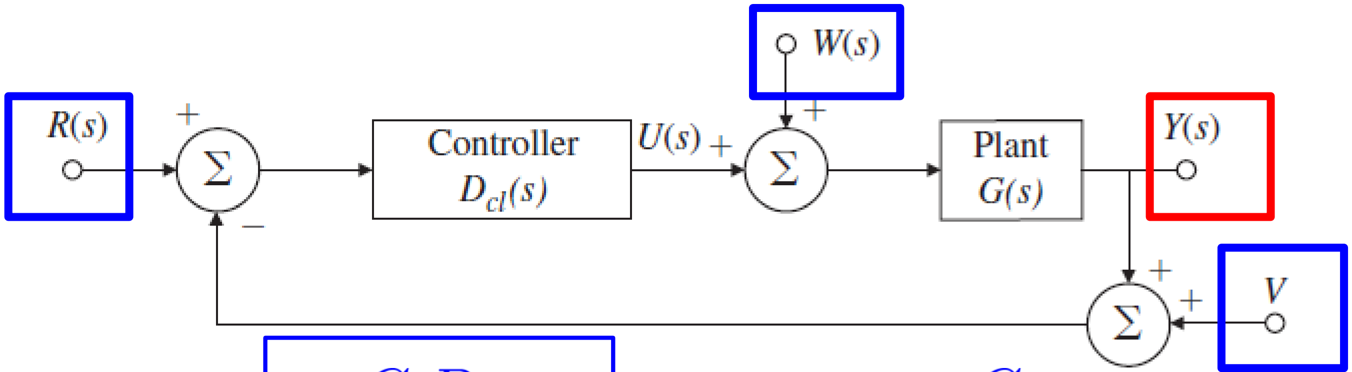
$$Y_{ol} = G (U + W) = \boxed{G D_{ol}} R + G W$$

$$T_{ol} = \frac{Y_{ol}}{R} = G D_{ol}$$

$$E_{ol} = R - Y_{ol} = R - (G D_{ol} R + G W) = (1 - G D_{ol}) R - G W$$

Open-Loop and Closed-Loop Systems

- Closed-loop system showing
  reference, R, control, U, disturbance, W, output, Y, and sensor noise, V



$$Y_{cl} = \boxed{\frac{G\,D_{cl}}{1 + G\,D_{cl}}} R + \frac{G}{\boxed{1 + G\,D_{cl}}} W + \boxed{\frac{-\,G\,D_{cl}}{1 + G\,D_{cl}}} V$$

$$\frac{G\,D_{cl}}{1 + G\,D_{cl}} = \mathcal{T} \qquad \frac{1}{1 + G\,D_{cl}} = \mathcal{S} \qquad \mathcal{T} + \mathcal{S} = 1$$

$$Y_{cl} = \mathcal{T}\,R + G\,\mathcal{S}\,W - \mathcal{T}\,V$$

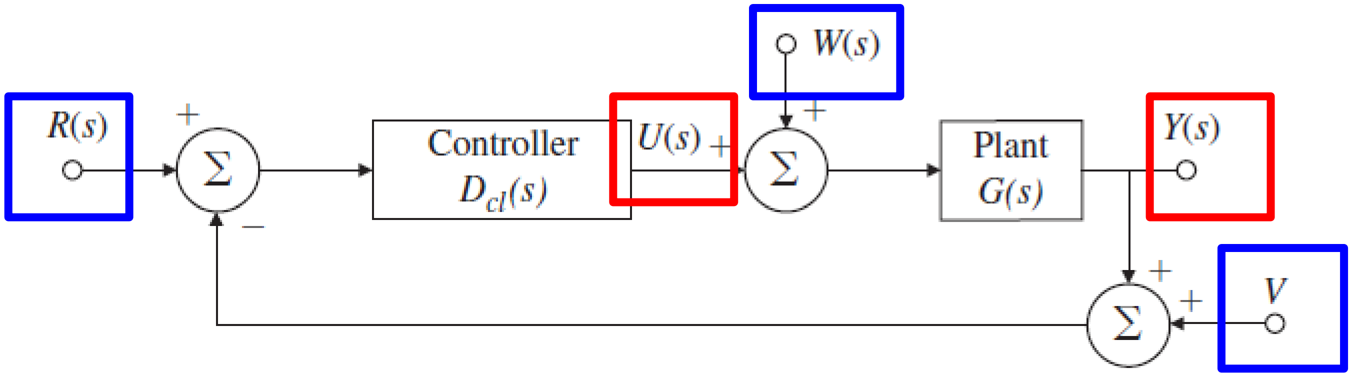
$$E_{cl} = R - Y_{cl} = (1 - \mathcal{T})\,R - G\,\mathcal{S}\,W + \mathcal{T}\,V$$

$$= \frac{1}{1 + G\,D_{cl}} R - \frac{G}{1 + G\,D_{cl}} W + \frac{G\,D_{cl}}{1 + G\,D_{cl}} V$$

$$= \mathcal{S}\,R - G\,\mathcal{S}\,W + \mathcal{T}\,V$$

Open-Loop and Closed-Loop Systems

- Closed-loop system showing
  reference, R, control, U, disturbance, W, output, Y, and sensor noise, V



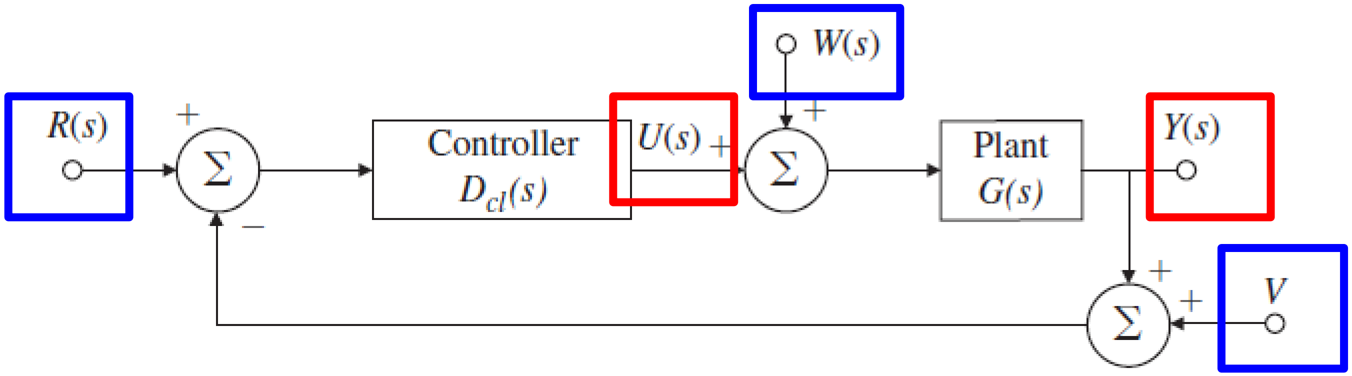
$$U = \boxed{\frac{D_{cl}}{1 + G\,D_{cl}}}\,R + \boxed{\frac{-\,G\,D_{cl}}{1 + G\,D_{cl}}}\,W + \boxed{\frac{-\,D_{cl}}{1 + G\,D_{cl}}}\,V$$

$$\mathcal{S} = \frac{1}{1 + G\,D_{cl}} \qquad \mathcal{T} = \frac{G\,D_{cl}}{1 + G\,D_{cl}}$$

$$U = D_{cl}\,\mathcal{S}\,R \qquad -\,\mathcal{T}\,W \qquad -\,D_{cl}\,\mathcal{S}\,V$$

Open-Loop and Closed-Loop Systems

- Closed-loop system showing reference, R, control, U, disturbance, W, output, Y, and sensor noise, V



$$Y_{cl} = \mathcal{T}\, R + G\,\mathcal{S}\,W - \mathcal{T}\,V$$

$$U = D_{cl}\,\mathcal{S}\,R - \mathcal{T}\,W - D_{cl}\,\mathcal{S}\,V$$

$$E_{cl} = \mathcal{S}\,R - G\,\mathcal{S}\,W + \mathcal{T}\,V$$

$$T_{cl} = \frac{Y_{cl}}{R} = \mathcal{T} = \frac{G\,D_{cl}}{1 + G\,D_{cl}}$$

$$\mathcal{S} = \frac{1}{1 + G\,D_{cl}}$$

- Sensitivity Function

$$\mathcal{S} + \mathcal{T} = 1$$

$$\mathcal{T} = \frac{G\,D_{cl}}{1 + G\,D_{cl}}$$

- Complementary Sensitivity Function

Stability, Tracking, Regulation, Sensitivity

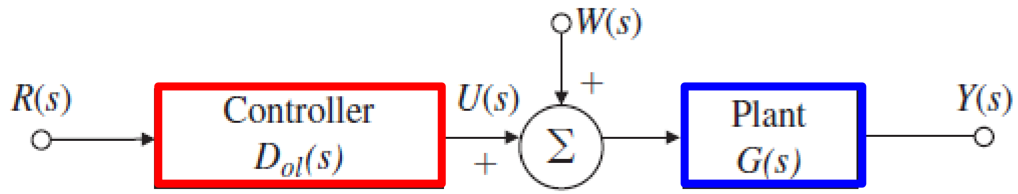
- Stability:
  - All poles of the transfer function must be in the left-hand s-plane.
- Tracking:
  - To cause the output to follow the reference input as closely as possible.
- Regulation:
  - To keep the error small

    when the reference is at most a constant set point and

    disturbances are present.
- Sensitivity:
  - The change of plant transfer function

    affects the change of closed-loop transfer function.

- Stability:
  - All poles of the transfer function must be in the left-hand s-plane.
- Open-loop system:



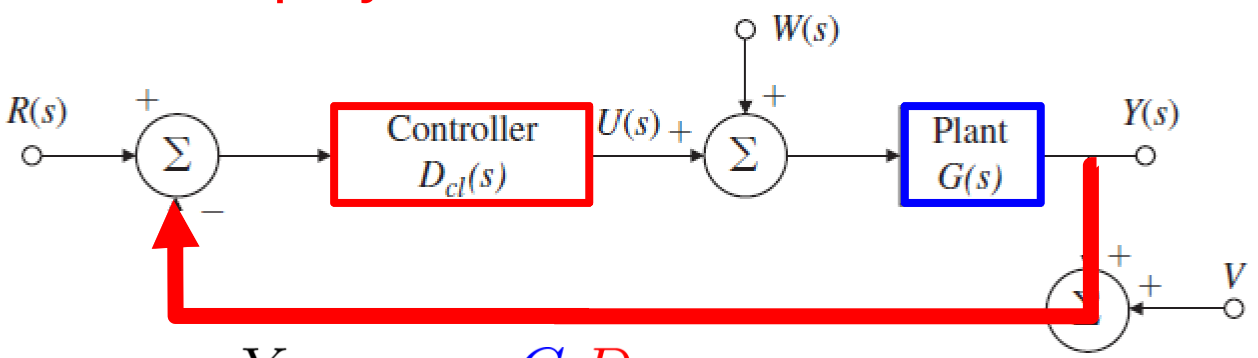
$$T_{ol} = \frac{Y_{ol}}{R} = G\ D_{ol} = \frac{b(s)}{a(s)}\ \frac{c(s)}{d(s)}$$

$$G = \frac{b(s)}{a(s)}$$

$$D_{ol} = \frac{c(s)}{d(s)}$$

- IF unstable poles in plant:
- IF poor zeros in plant:

- Stability:
  - All poles of the transfer function must be in the left-hand s-plane.
- Closed-loop system:



$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G\ D_{cl}}{\boxed{1 + G\ D_{cl}}}$$

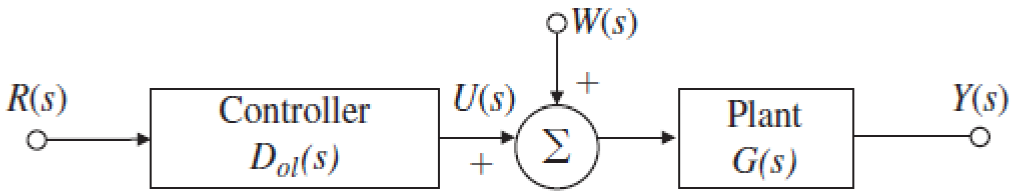
- The characteristic equation:

$$1 + G\ D_{cl} = 0 \qquad G = \frac{b(s)}{a(s)}$$

$$1 + \frac{b(s)}{a(s)}\frac{c(s)}{d(s)} = 0 \qquad D_{cl} = \frac{c(s)}{d(s)}$$

$$a(s)\ d(s) + b(s)\ c(s) = 0$$

- Tracking:
  - To cause the output to follow the reference input as closely as possible.
- Open-loop system:



$$T_{ol} = \frac{Y_{ol}}{R} = G\,D_{ol} = \frac{b(s)}{a(s)}\,\frac{c(s)}{d(s)}$$

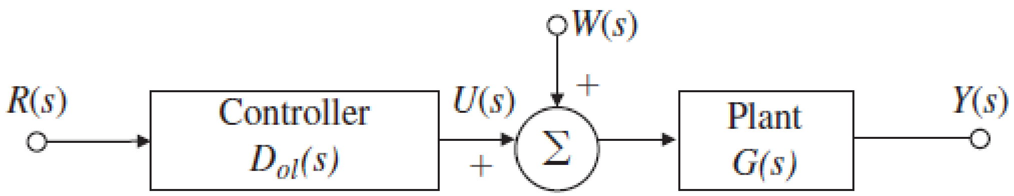
- Three caveats:
  - Controller transfer function must be proper
  - Must not get greedy and request unrealistically fast design
  - Pole-zero cancellation cause unacceptable transient

- Regulation:
  - To keep the error small

    when the reference is at most a constant set point and

    disturbances are present.

- Open-loop system:



  - The controller has no influence at all on the system response

    to the disturbances,

    so this structure is useless for regulation

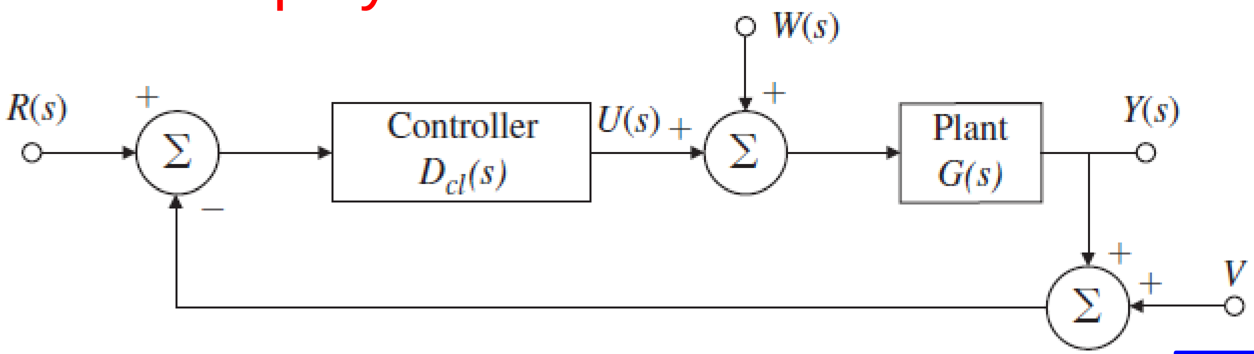
- Regulation:
  - To keep the error small

    when the reference is at most a constant set point and

    disturbances are present.

$$\mathcal{T} = \frac{G\ D_{cl}}{1 + G\ D_{cl}}$$

- Closed-loop system:



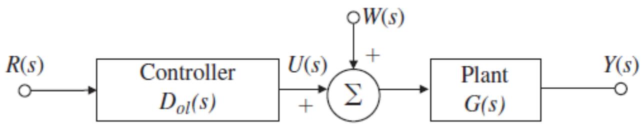
$$\mathcal{S} = \frac{1}{1 + G\ D_{cl}}$$

$$\mathcal{S} + \mathcal{T} = 1$$

$$E_{cl} = \frac{1}{1 + G\ D_{cl}} R - \boxed{\frac{G}{1 + G\ D_{cl}}} W + \boxed{\frac{G\ D_{cl}}{1 + G\ D_{cl}}} V$$

- The dilemma for the impact from W, V
- The resolution is to design controller for different frequencies

- Sensitivity:
  - The change of plant transfer function affects the change of closed-loop transfer function.
- The sensitivity of a transfer function to a plant gain is defined as follows (Open-Loop):



$$S_G^T = \frac{\frac{\delta T_{ol}}{T_{ol}}}{\frac{\delta G}{G}} = \frac{G}{T_{ol}} \frac{\delta T_{ol}}{\delta G} = 1$$

$$T_{ol} = \frac{Y_{ol}}{R} = G\, D_{ol}$$

$$T_{ol} + \delta T_{ol} = D_{ol}\,(\, G + \delta G\,) = D_{ol}\, G + D_{ol}\, \delta G$$

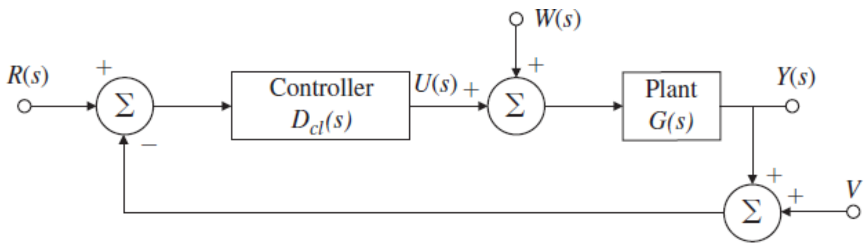
$$\delta T_{ol} = D_{ol}\, \delta G$$

$$\frac{\delta T_{ol}}{T_{ol}} = \frac{D_{ol}\, \delta G}{D_{ol}\, G} = \frac{\delta G}{G}$$

- Sensitivity:
  - The change of plant transfer function

    affects the change of closed-loop transfer function.

- For Closed-Loop:



$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G\ D_{cl}}{1 + G\ D_{cl}}$$

$$T_{cl} + \delta T_{cl} = \frac{(G + \delta G)\ D_{cl}}{1 + (G + \delta G)\ D_{cl}}$$

$$\delta T_{cl} = \frac{dT_{cl}}{dG}\ \delta G$$

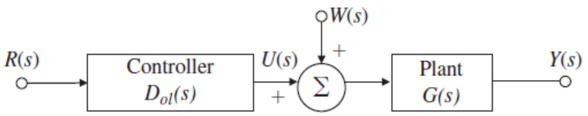
$$S_G^T = \frac{\frac{\delta T_{cl}}{T_{cl}}}{\frac{\delta G}{G}} = \frac{G}{T_{cl}}\ \frac{\delta T_{cl}}{\delta G} = \frac{1}{1 + G\ D_{cl}}$$

$$= \frac{G}{\frac{G\ D_{cl}}{1 + G\ D_{cl}}}\ \frac{D_{cl}(1 + GD_{cl}) - (GD_{cl})D_{cl}}{(1 + GD_{cl})^2}$$

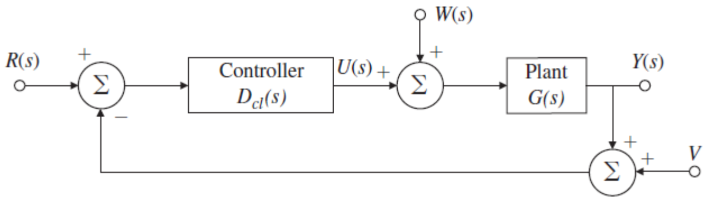
- Sensitivity:
  - The change of plant transfer function

    affects the change of closed-loop transfer function.

- For Open-Loop: $S_G^T = 1$



- For Closed-Loop: $S_G^T = \dfrac{1}{1 + G\,D_{cl}}$



- A major advantage of feedback
  - In feedback control,

    the error in the overall transfer function gain

    is less sensitive to variation in the plant gain

    by a factor S compared to errors in open-loop control gain.

- Sensitivity Function

$$\mathcal{S} = \frac{1}{1 + G\, D_{cl}}$$

- Complementary Sensitivity Function

$$\mathcal{T} = \frac{G\, D_{cl}}{1 + G\, D_{cl}}$$

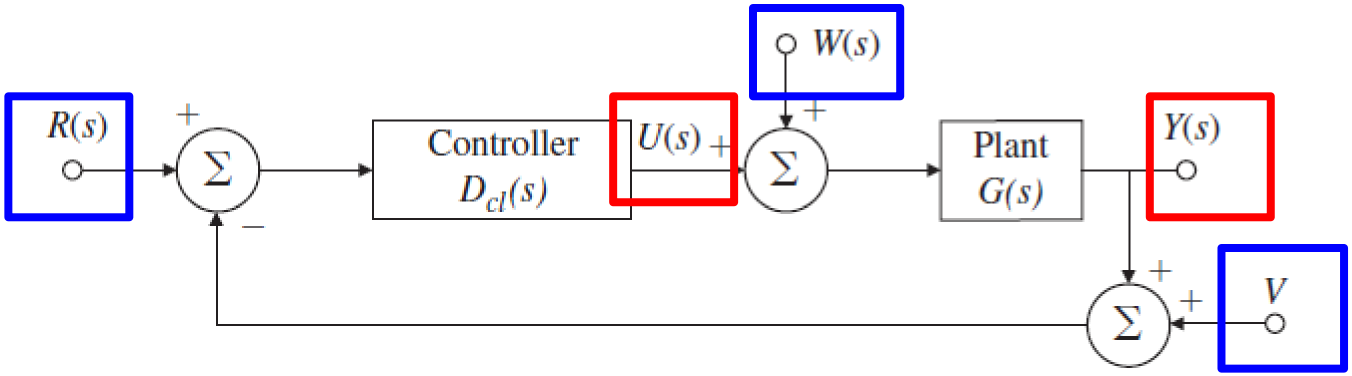
$$\mathcal{S} + \mathcal{T} = 1$$

$$E_{cl} = \frac{1}{1 + G\, D_{cl}} R - \frac{G}{1 + G\, D_{cl}} W + \frac{G\, D_{cl}}{1 + G\, D_{cl}} V$$

$$E_{cl}(jw_0) = \frac{1}{1 + G(jw_0)\, D_{cl}(jw_0)} R(jw_0) - \frac{G(jw_0)}{1 + G(jw_0)\, D_{cl}(jw_0)} W(jw_0) + \frac{G(jw_0)\, D_{cl}(jw_0)}{1 + G(jw_0)\, D_{cl}(jw_0)} V(jw_0)$$

Summary: The Basic Equations of Control

- Closed-loop system showing
  reference, R, control, U, disturbance, W, output, Y, and sensor noise, V



$$Y_{cl} = \mathcal{T}\ R + G\,\mathcal{S}\,W - \mathcal{T}\ V \qquad T_{cl} = \frac{Y_{cl}}{R} = \mathcal{T}$$

$$U = D_{cl}\,\mathcal{S}\,R - \mathcal{T}\ W - D_{cl}\,\mathcal{S}\,V \qquad = \frac{G\ D_{cl}}{1 + G\ D_{cl}}$$

$$E_{cl} = \mathcal{S}\ R - G\,\mathcal{S}\,W + \mathcal{T}\ V$$

$$\mathcal{S} = \frac{1}{1 + G\ D_{cl}}$$

- Sensitivity Function

$$\mathcal{S} + \mathcal{T} = 1$$

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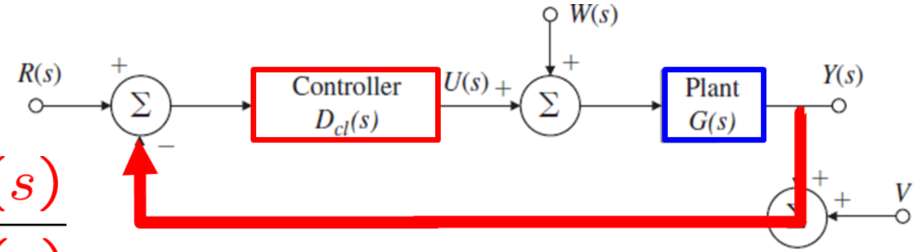
- Complementary Sensitivity Function

Summary: The Basic Equations of Control



- Stability:

$$T_{ol} = \frac{Y_{ol}}{R} = G\,D_{ol} = \frac{b(s)}{a(s)}\,\frac{c(s)}{d(s)}$$

$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G\,D_{cl}}{1 + G\,D_{cl}} = \frac{b(s)\,c(s)}{a(s)\,d(s) + b(s)\,c(s)}$$

$$G = \frac{b(s)}{a(s)} \qquad D_{ol} = \frac{c(s)}{d(s)}$$

- Tracking & Regulation:

$$E_{cl} = \frac{1}{1 + G\,D_{cl}}\,R - \frac{G}{1 + G\,D_{cl}}\,W + \frac{G\,D_{cl}}{1 + G\,D_{cl}}\,V$$

- Sensitivity:
  - For Open-Loop: $S_G^T = 1$
  - For Closed-Loop: $S_G^T = \dfrac{1}{1 + G\,D_{cl}}$

$$\mathcal{T} = \frac{G\,D_{cl}}{1 + G\,D_{cl}}$$

$$\mathcal{S} = \frac{1}{1 + G\,D_{cl}}$$

$$\mathcal{S} + \mathcal{T} = 1$$