

Spring 2021

控制系統  
Control Systems

Unit 40  
Feedback Analysis

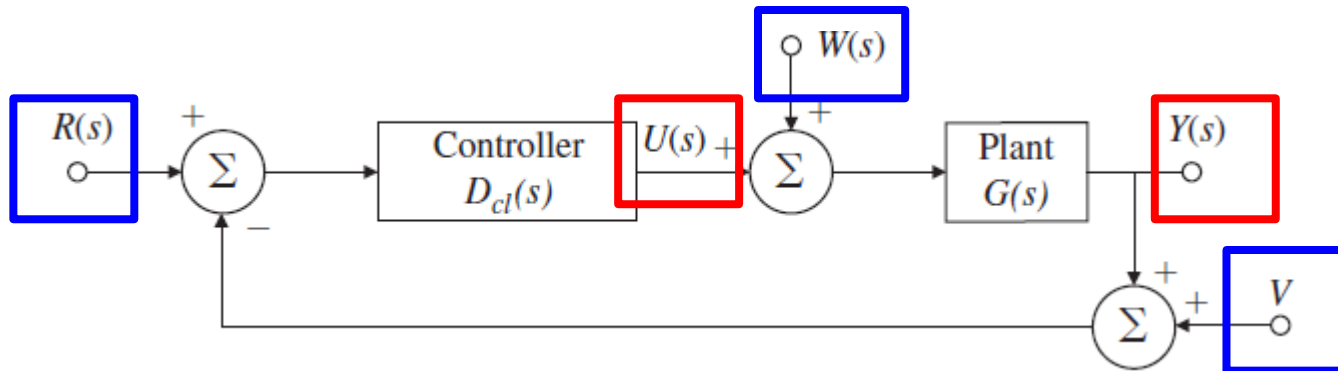
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NTU-EE

Feb – Jun, 2021

- **U4A The Basic Equations of Control**
  - Open-Loop and Closed-Loop Systems
  - Stability, Tracking, Regulation, Sensitivity
- **U4B Control of Steady-State Error to Polynomial Inputs: System Type**
  - Control of Steady-State Error to Polynomial Inputs
  - System Type for Tracking
  - System Type for Regulation and Disturbance Rejection
- **U4C PID Control**
  - PID Control (Proportional, Integral, Derivative)
  - P, I, D, PI, PD, PID
  - Examples
- **U4D Ziegler–Nichols Tuning**
  - Ziegler–Nichols Tuning of the PID Controller,
  - Examples
- **U4E Feedforward Control by Plant Model Inversion**
  - Feedforward Control by Plant Model Inversion
  - Examples

- Closed-loop system showing reference,  $R$ , control,  $U$ , disturbance,  $W$ , output,  $Y$ , and sensor noise,  $V$



$$Y_{cl} = \mathcal{T} R + G S W - \mathcal{T} V \quad T_{cl} = \frac{Y_{cl}}{R} = \mathcal{T}$$

$$U = D_{cl} S R - \mathcal{T} W - D_{cl} S V$$

$$E_{cl} = S R - G S W + \mathcal{T} V = \frac{G D_{cl}}{1 + G D_{cl}}$$

$$S = \frac{1}{1 + G D_{cl}}$$

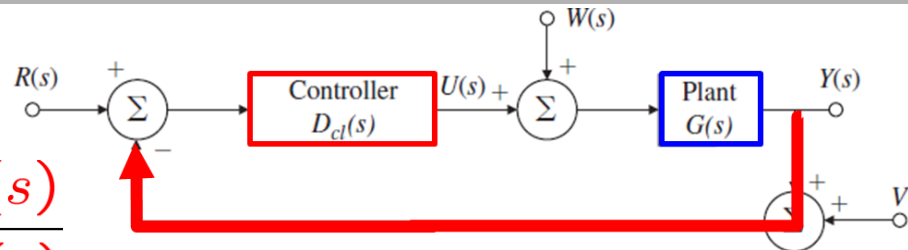
● Sensitivity Function

$$S + \mathcal{T} = 1$$

$$\mathcal{T} = \frac{G D_{cl}}{1 + G D_{cl}}$$

● Complementary Sensitivity Function

## Stability:



$$T_{ol} = \frac{Y_{ol}}{R} = G D_{ol} = \frac{b(s)}{a(s)} \frac{c(s)}{d(s)}$$

$$T_{cl} = \frac{Y_{cl}}{R} = \frac{G D_{cl}}{1 + G D_{cl}} = \frac{b(s) c(s)}{a(s) d(s) + b(s) c(s)}$$

$$G = \frac{b(s)}{a(s)}$$

$$D_{ol} = \frac{c(s)}{d(s)}$$

## Tracking & Regulation:

$$E_{cl} = \frac{1}{1 + G D_{cl}} R - \frac{G}{1 + G D_{cl}} W + \frac{G D_{cl}}{1 + G D_{cl}} V$$

## Sensitivity:

- For Open-Loop:  $S_G^T = 1$

- For Closed-Loop:  $S_G^T = \frac{1}{1 + G D_{cl}}$

$$\mathcal{T} = \frac{G D_{cl}}{1 + G D_{cl}}$$

$$\mathcal{S} = \frac{1}{1 + G D_{cl}}$$

$$\mathcal{S} + \mathcal{T} = 1$$

- In general case: 
$$e_{ss} = \lim_{s \rightarrow 0} \frac{s^n}{s^n + K_n} \frac{1}{s^k}$$

### Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1 + K_p}$	$\infty$	$\infty$
Type 1	0	$\frac{1}{K_v}$	$\infty$
Type 2	0	0	$\frac{1}{K_a}$

$$K_p = \lim_{s \rightarrow 0} GD_{cl}(s), \quad n = 0$$

$$K_v = \lim_{s \rightarrow 0} s GD_{cl}(s), \quad n = 1$$

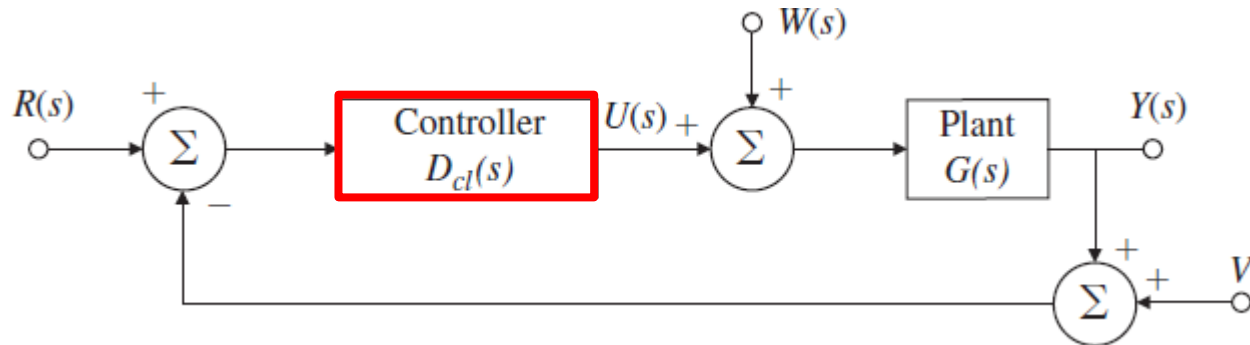
$$K_a = \lim_{s \rightarrow 0} s^2 GD_{cl}(s), \quad n = 2$$

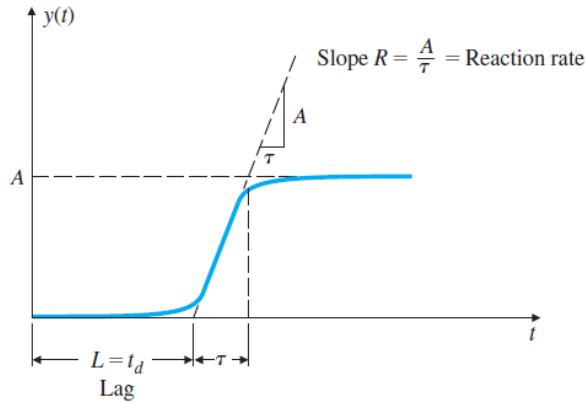
$$GD_{cl}(s) = \frac{GD_{cl}^0(s)}{s^n}$$

- System Type**: a robust property with respect to parameter changes in the unity feedback structure.

- PID Control:
  - Proportional: Simple Proportional Feedback
  - Integral: Eliminating Bias Offset
  - Derivative: Anticipatory

$$D_c(s) = k_P + \frac{k_I}{s} + k_D s$$

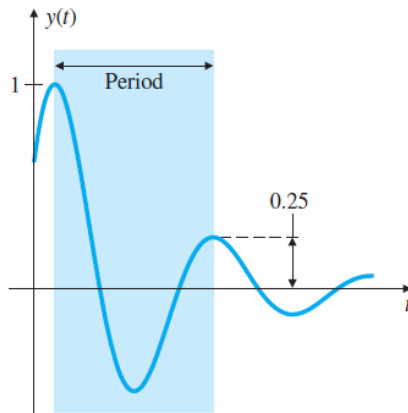




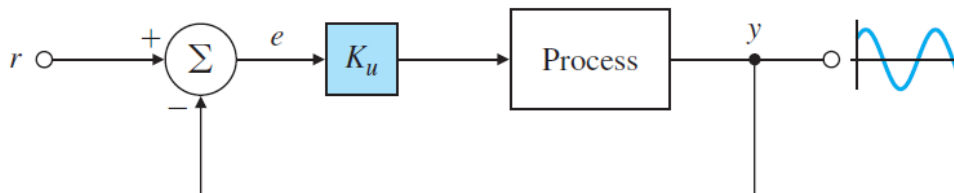
$$\frac{Y(s)}{U(s)} = \frac{A e^{-s t_d}}{\tau s + 1}$$

- A **first-order** system with a **time delay** (lag)

## ● Method 1: Quarter Decay Ratio



## ● Method 2: Ultimate Sensitivity Method:



### Ziegler–Nichols Tuning for the Regulator

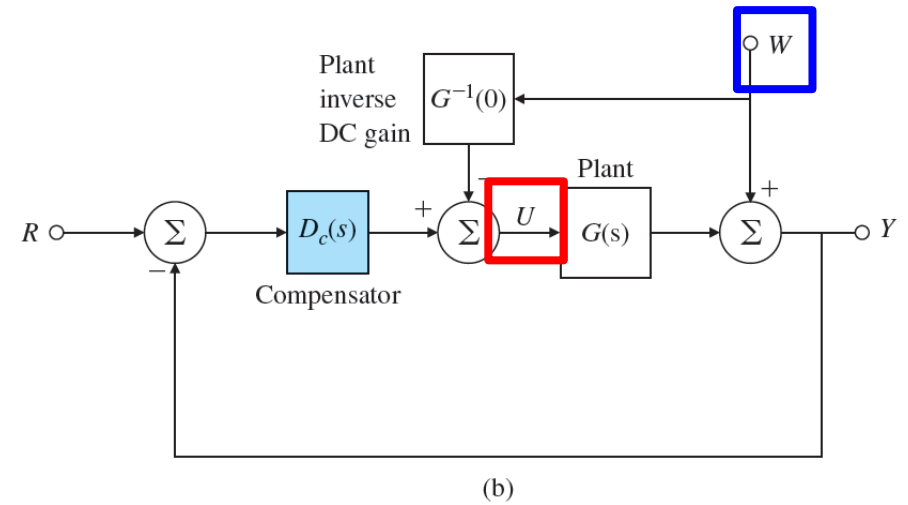
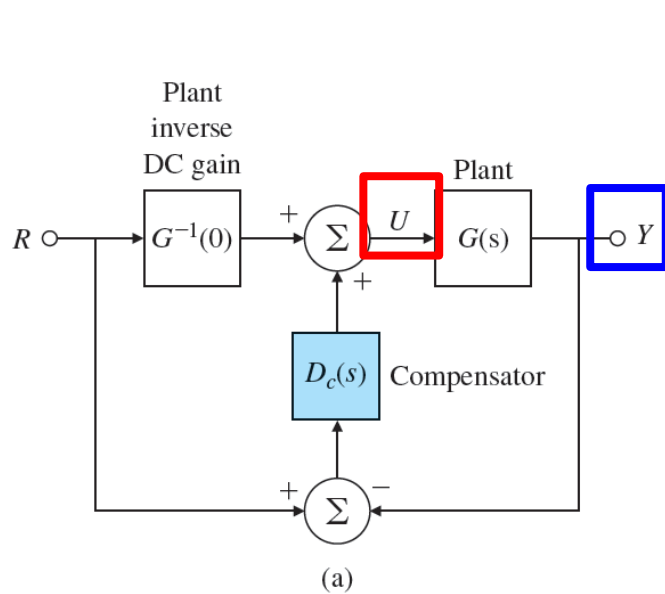
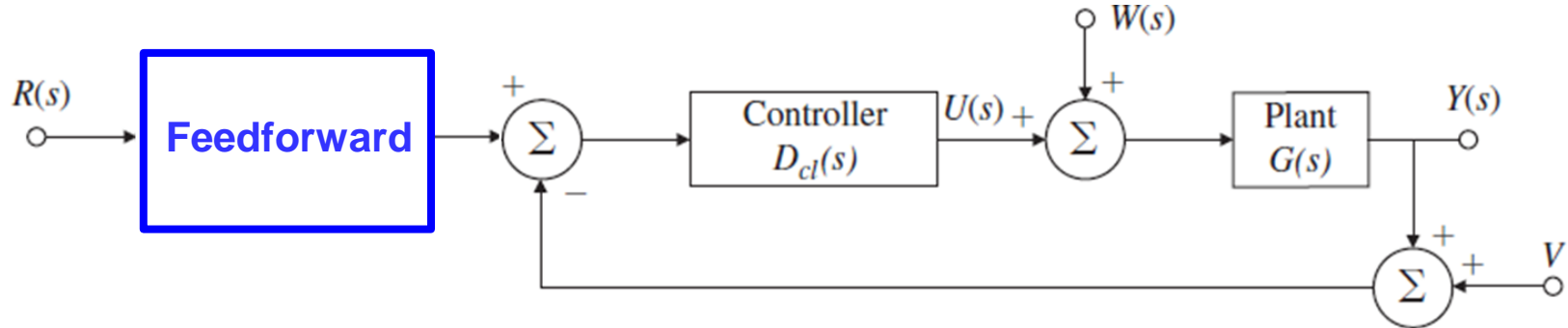
$D_c(s) = k_p(1 + 1/T_I s + T_D s)$ , for a Decay Ratio of 0.25

Type of Controller	Optimum Gain
P	$k_p = 1/RL$
PI	$\begin{cases} k_p = 0.9/RL \\ T_I = L/0.3 \end{cases}$
PID	$\begin{cases} k_p = 1.2/RL \\ T_I = 2L \\ T_D = 0.5L \end{cases}$

### Ziegler–Nichols Tuning for the Regulator

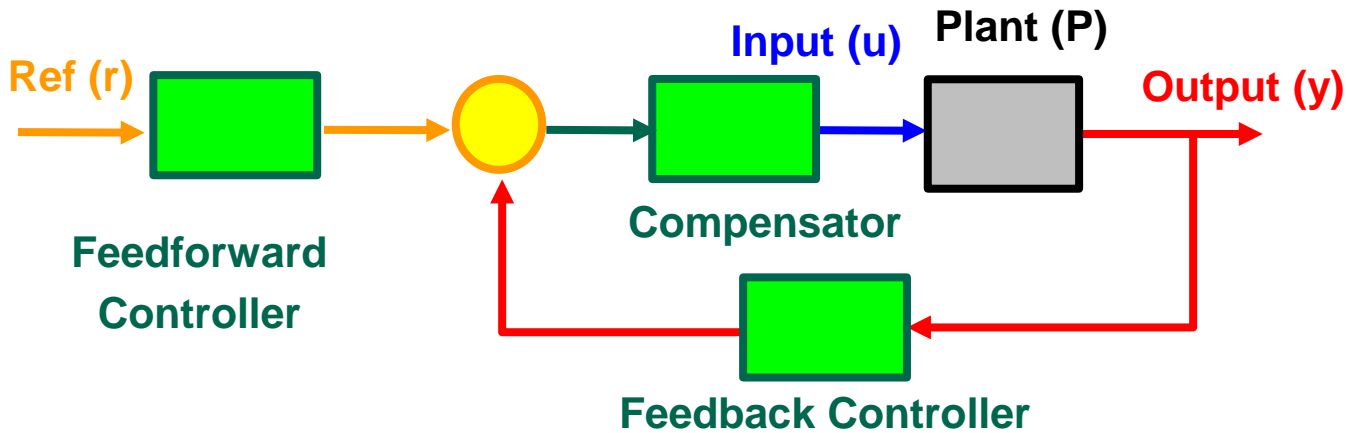
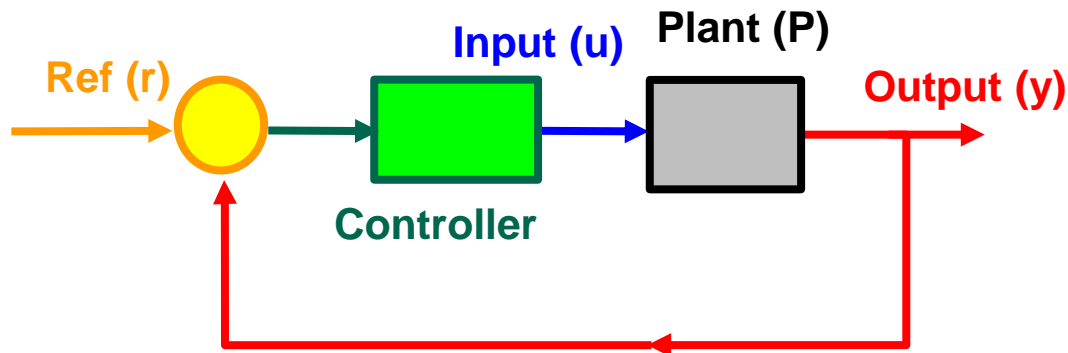
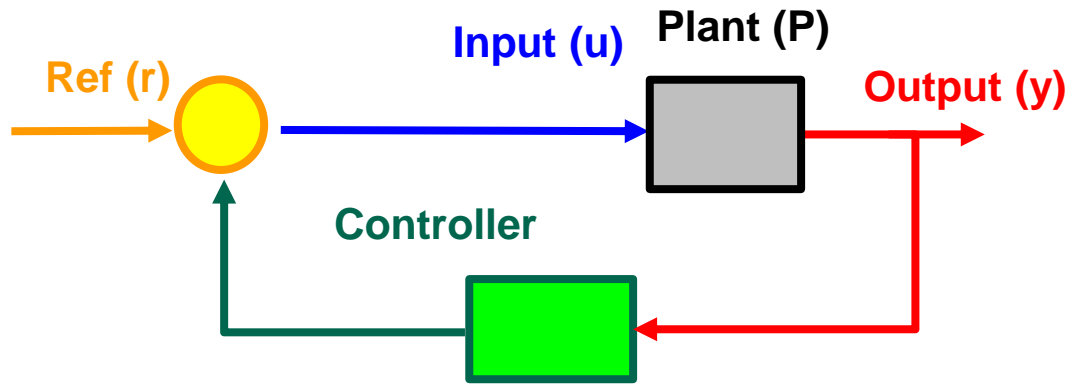
$D_c(s) = k_p(1 + 1/T_I s + T_D s)$ , Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_p = 0.5K_u$
PI	$\begin{cases} k_p = 0.45K_u \\ T_I = \frac{P_u}{1.2} \end{cases}$
PID	$\begin{cases} k_p = 0.6K_u \\ T_I = 0.5P_u \\ T_D = 0.125P_u \end{cases}$



- Feedforward control structure for: (a) tracking; (b) disturbance rejection





Steady-State Error from Reference Input

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{1}{1 + G D_{cl}} \frac{1}{s^{k+1}}$$

$$G D_{cl}(s) = \frac{G D_{cl}^0(s)}{s^n} \quad \begin{array}{l} G D_{cl}^0(s) \text{ no pole at } s = 0 \\ G D_{cl}^0(0) = K_n \end{array}$$

■ Type 0,  $k=0$ :  $e_{ss} = \frac{1}{1 + G D_{cl}(0)} = \frac{1}{1 + K_p} = \frac{1}{1 + K_0}$

■ Position Constant

■ Type 1,  $k=1$ :  $e_{ss} = \lim_{s \rightarrow 0} \frac{1}{s + s G D_{cl}} = \frac{1}{0 + G D_{cl}^0(0)} = \frac{1}{K_v} = \frac{1}{K_1}$

■ Velocity Constant

■ Type 2,  $k=2$ :  $e_{ss} = \lim_{s \rightarrow 0} \frac{1}{s^2 + s^2 G D_{cl}} = \frac{1}{0 + G D_{cl}^0(0)} = \frac{1}{K_a} = \frac{1}{K_2}$

■ Acceleration Constant

## Steady-State Value from Disturbance Input

$$\begin{aligned}
 y_{ss} &= \lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} s \mathcal{T}_w(s) W(s) = \lim_{s \rightarrow 0} s [ s^n \mathcal{T}_{w,0}(s) ] W(s) \\
 &= \lim_{s \rightarrow 0} s \mathcal{T}_w(s) \frac{1}{s^{k+1}} = \lim_{s \rightarrow 0} \mathcal{T}_{w,0}(s) \frac{s^n}{s^k} \\
 &\quad \text{with } \mathcal{T}_{w,0}(0) = \frac{1}{K_{w,n}}
 \end{aligned}$$

■ Type 0,  $k=0, n=0$ :  $y_{ss} = \mathcal{T}_{w,0}(0) = \frac{1}{K_{w,0}}$

$$\mathcal{T}_w(s) = s^0 \mathcal{T}_{w,0}(s)$$

■ Type 1,  $k=1, n=1$ :  $y_{ss} = \mathcal{T}_{w,0}(0) = \frac{1}{K_{w,1}}$

$$\mathcal{T}_w(s) = s^1 \mathcal{T}_{w,0}(s)$$

■ Type 2,  $k=2, n=2$ :  $y_{ss} = \mathcal{T}_{w,0}(0) = \frac{1}{K_{w,2}}$

$$\mathcal{T}_w(s) = s^2 \mathcal{T}_{w,0}(s)$$

- PID Control:

- Proportional: Simple Proportional Feedback
- Integral: Eliminating Bias Offset
- Derivative: Anticipatory

$$D_c(s) = k_P + \frac{k_I}{s} + k_D s$$

