

Spring 2021

控制系統  
Control Systems

Unit 30  
Dynamic Response

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NTU-EE

Feb – Jun, 2021

- **U3A Review of Laplace Transforms**
  - Laplace Transforms: From Differential Eqns to Algebraic Eqns
  - Transfer Functions, Frequency Response, Poles and Zeros
  - Step, Ramp, Impulse Functions
  - Laplace Transforms and Inverse LT, The Final Value Theorem
  - Using Laplace Transforms to Solve Differential Equations
- **U3B System Modeling Diagrams: Model Visualization**
- **U3C Effect of Pole Locations**
- **U3D Time-Domain Specifications**
  - Rise Time
  - Overshoot and Peak Time
  - Settling Time
- **U3E Effects of Zeros and Additional Poles**
- **U3F Stability**
  - Bounded Input–Bounded Output Stability
  - Stability of LTI Systems
  - Routh's Stability Criterion

### Table of Laplace Transforms

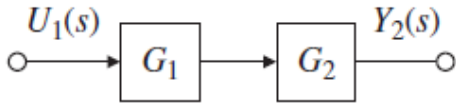
Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	$t$
4	$2!/s^3$	$t^2$
5	$3!/s^4$	$t^3$
6	$m!/s^{m+1}$	$t^m$
7	$\frac{1}{s+a}$	$e^{-at}$
8	$\frac{1}{(s+a)^2}$	$te^{-at}$
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{s(s+a)}{a}$	$1 - e^{-at}$
12	$\frac{1}{s^2(s+a)}$	$\frac{1}{a}(at - 1 + e^{-at})$

Number	$F(s)$	$f(t), t \geq 0$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt} - ae^{-at}$
17	$\frac{s}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1 - e^{-at} \left( \cos bt + \frac{a}{b} \sin bt \right)$

### Properties of Laplace Transforms

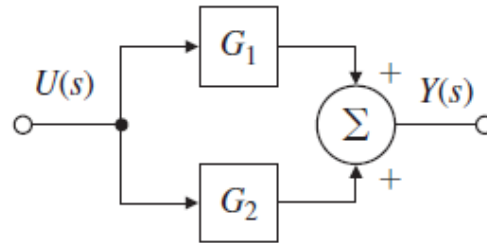
Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t - \lambda)$	Time delay ( $\lambda \geq 0$ )
3	$\frac{1}{ a } F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \dots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s)$	$\int_0^t f(\zeta) d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
8	$\lim_{s \rightarrow \infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s \rightarrow 0} sF(s)$	$\lim_{t \rightarrow \infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F_1(\zeta)F_2(s - \zeta) d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi} \int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega) d\omega$	$\int_0^\infty y(t)u(t) dt$	Parseval's Theorem
12	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

- Elementary block diagrams: (a) series; (b) parallel; (c) feedback



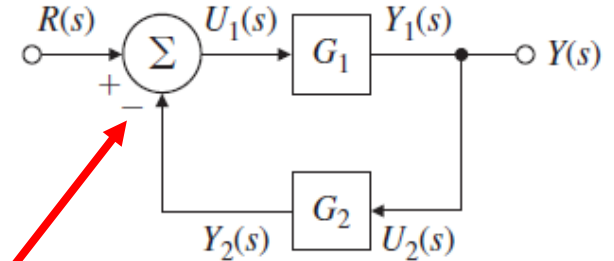
$$\frac{Y_2(s)}{U_1(s)} = G_2 G_1$$

(a)



$$\frac{Y(s)}{U(s)} = G_2 + G_1$$

(b)



$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_2 G_1}$$

(c)

- The **gain** of a **single-loop negative feedback** system

is given by

the **forward gain** divided by the sum of **1 plus the loop gain**.

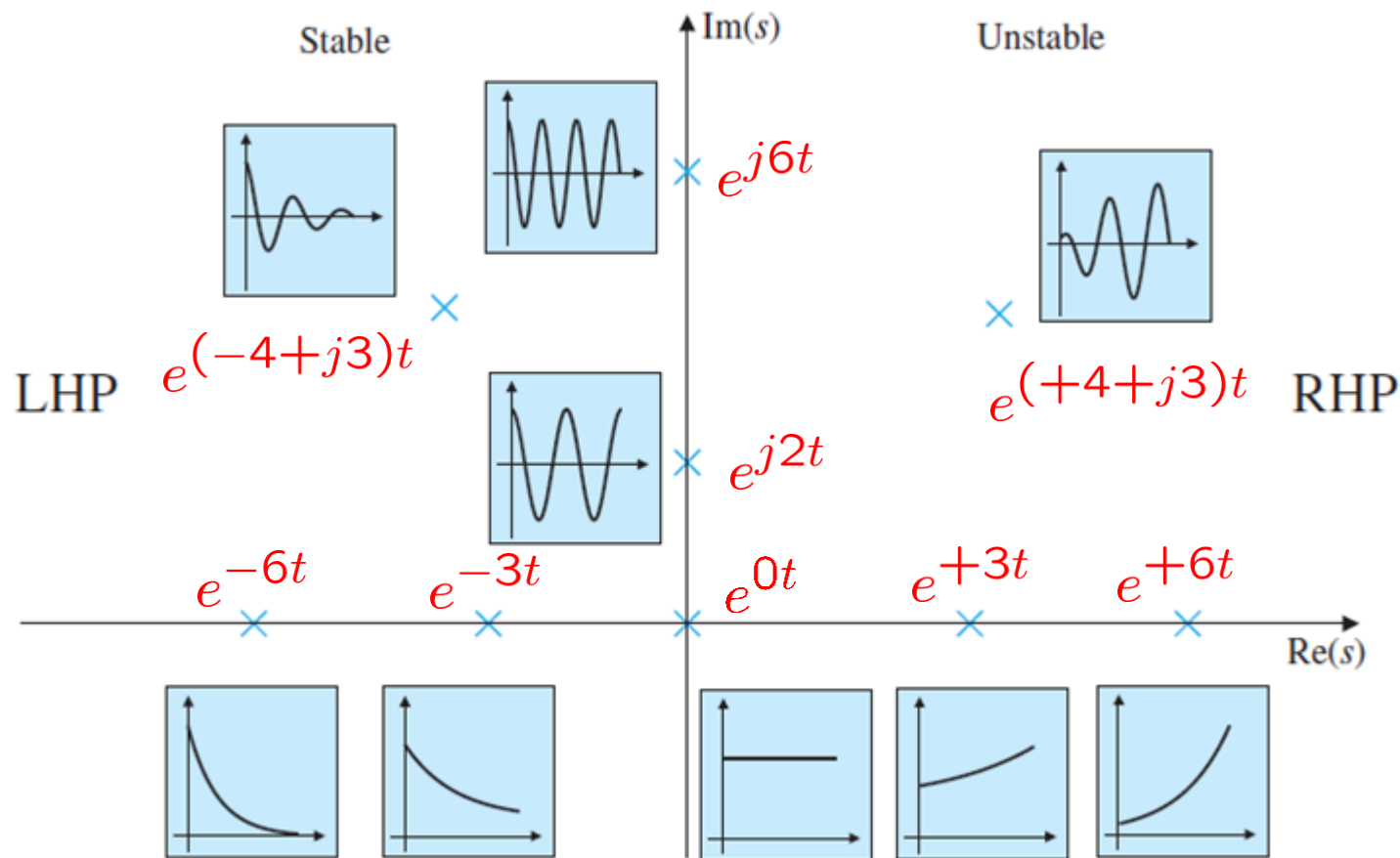
- Negative Feedback**
- Positive Feedback**
- Unity Feedback System**

$$\frac{Y}{R} = \frac{G_1}{1 + G_2 G_1}$$

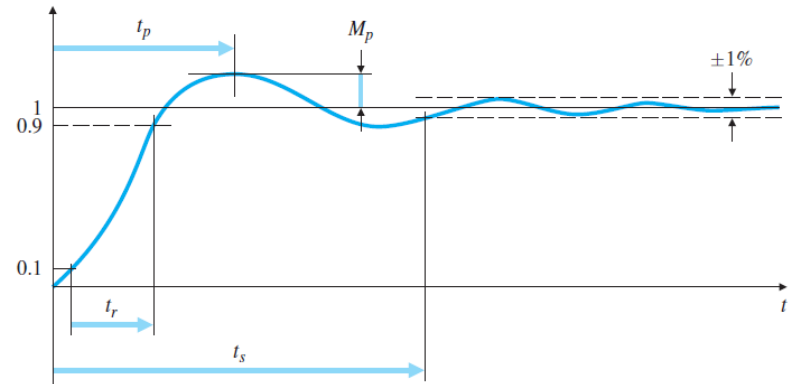
$$\frac{Y}{R} = \frac{G_1}{1 - G_2 G_1}$$

$$G_2 = 1$$

- Time functions associated with points in the s-plane  
(LHP, left half-plane; RHP, right half-plane)



- Rise time  $t_r$
- Settling time  $t_s$
- Overshoot  $M_p$
- Peak time  $t_p$



$$\Rightarrow t_r \approx \frac{1.8}{\omega_n}$$

$$\Rightarrow \omega_n \geq \frac{1.8}{t_r}$$

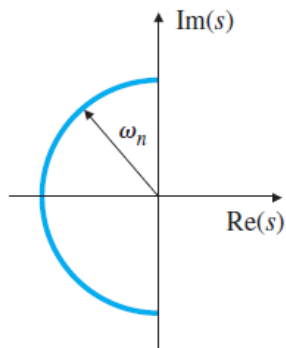
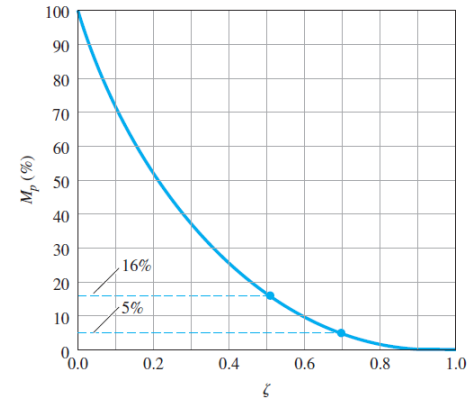
$$\Rightarrow t_s = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\sigma}$$

$$\Rightarrow \zeta \geq \zeta(M_p)$$

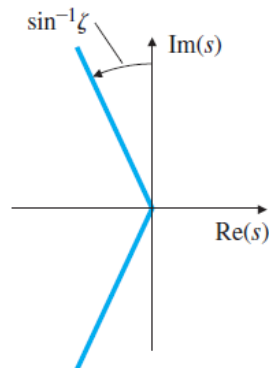
$$\Rightarrow M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}}$$

$$0 \leq \zeta < 1$$

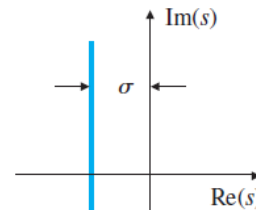
$$\Rightarrow \sigma \geq \frac{4.6}{t_s}$$



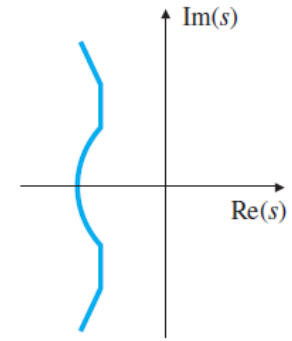
(a)



(b)

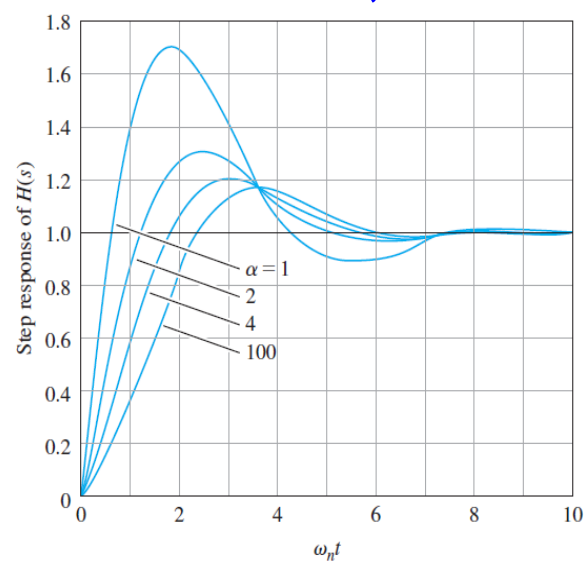


(c)

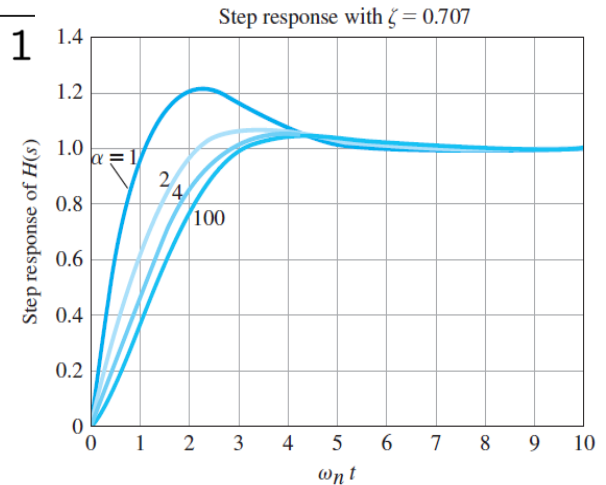


(d)

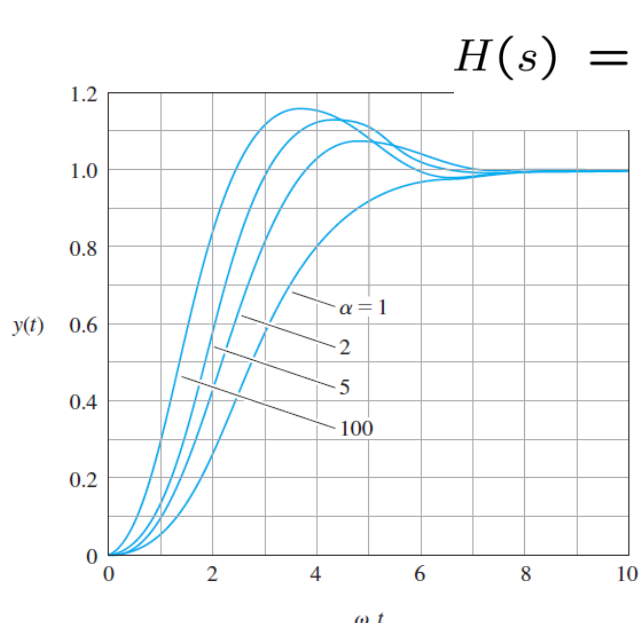
## Same Poles, Different Zeros



$$H(s) = \frac{\frac{s}{\alpha\zeta\omega_n} + 1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$$



## Effects of Pole-Zero Patterns on Dynamic Response



$$H(s) = \frac{1}{\left(\frac{s}{\alpha\zeta\omega_n} + 1\right)\left[\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1\right]}$$

