Spring 2021

# 控制系統
# Control Systems

## Unit 2F
## Electromechanical Systems –
## Loudspeakers, Motors, Gears

Feng-Li Lian

NTU-EE

Feb – Jun, 2021

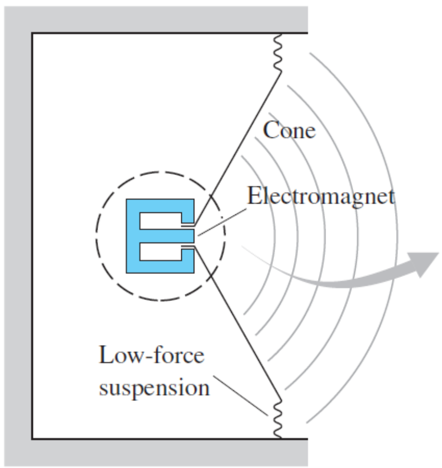
# Example 2.12: Loudspeaker

## Law of motors

- A current of $i$ amp in a conductor of $l$ m, arranged at right angles in a magnetic field of $B$ teslas.

- There is a force $F$ on the conductor at right angles to the plane of $i$ and $B$, with magnitude

$$F = BliN$$



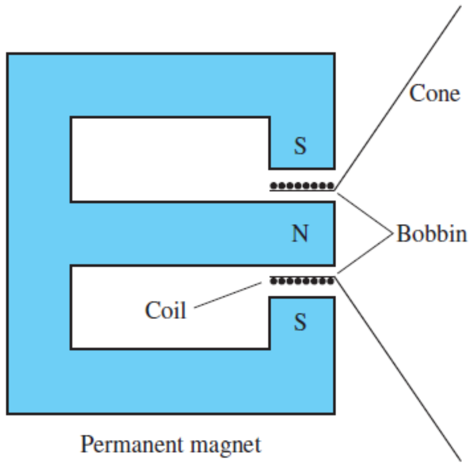

The loudspeaker configuration

## Model (Motion of loudspeaker cone)

$$M\ddot{x} + b\dot{x} = F$$

$x$ : the motion of the loudspeaker cone

$b$ : the friction coefficient



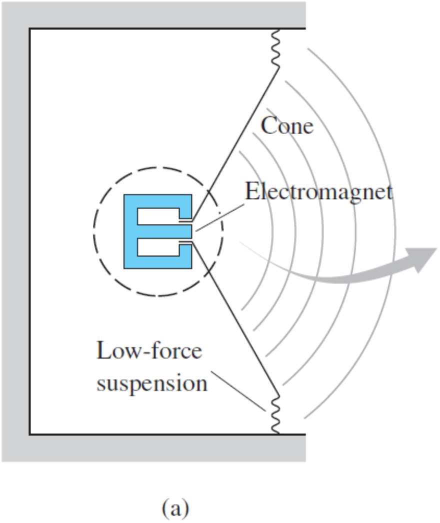

The electromagnet and voice coil

# Example 2.13: Loudspeaker with Circuit

## Law of generators

- A conductor of length $l$ m is moving in a magnetic field of $B$ teslas at a velocity of $v$ m/sec at mutually right angles
- an electric voltage is established across the conductor with magnitude

$$e = BlvV$$



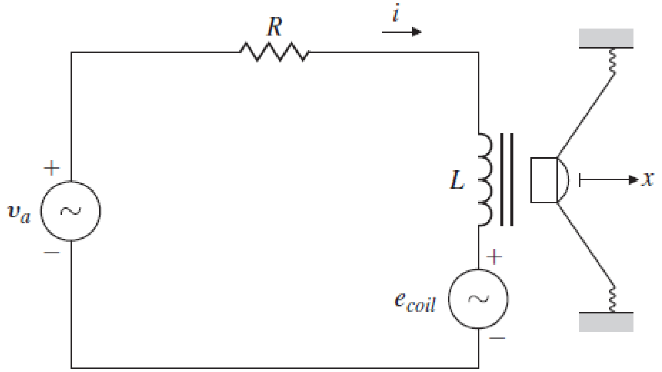

The loudspeaker configuration

## Model (Motion of electric circuit)

$$L\frac{di}{dt} + Ri = v_a - e_{coil}$$

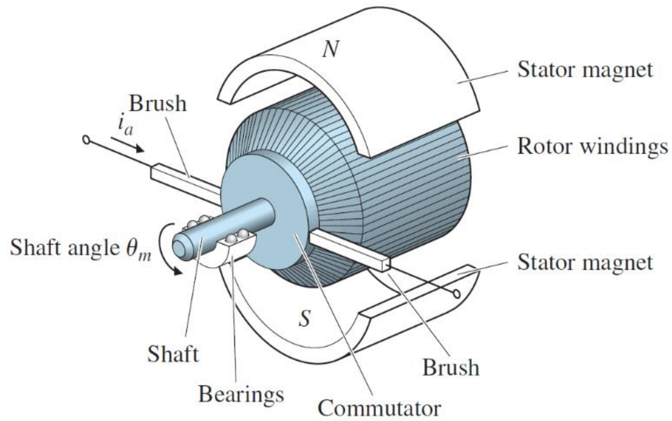
$L$ : the inductance

$R$ : the effective circuit resistance




The electric circuit of the loudspeaker

- The direct current (DC) motor




## Motor equations

- Motor equations give the torque $T$ on the rotor in terms of the armature current $i_a$
- Express the back emf voltage in terms of the shaft's rotational velocity $\dot{\theta}_m$

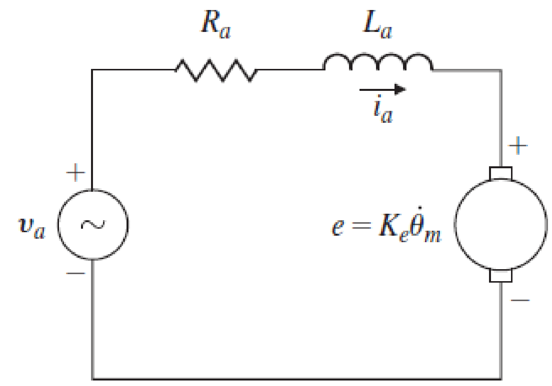
## Model (Equations of Motion)

$$T = K_t \cdot i_a$$

$$e = K_e \cdot \dot{\theta}_m$$

$K_t$ : the torque constant

$K_e$ : the electric constant



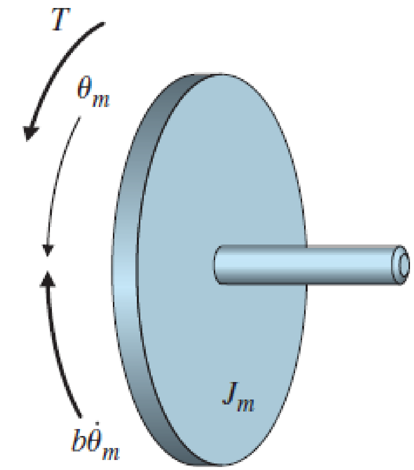

The electric circuit of the armature

- Model (DC motors)

$$J_m\ddot{\theta}_m + b\dot{\theta}_m = T = K_t \cdot i_a$$

$J_m$ : the inertia of rotor

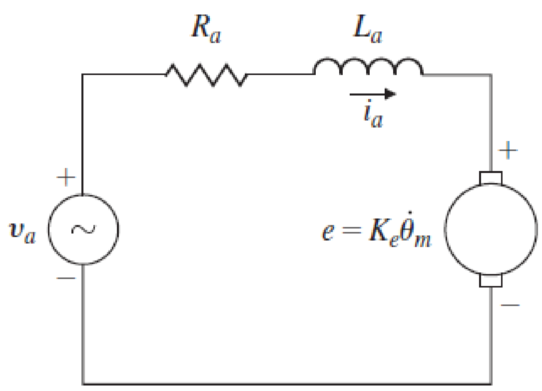
$b$ : the viscous friction coefficient




Free-body diagram of the rotor

- Model (Electric circuit)

$$L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m$$




The electric circuit of the armature

- Transfer function (Electric circuit)

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{s\left[(J_m s + b)(L_a s + R_a) + K_t K_e\right]}$$

# Example 2.14: Motors

## Model (DC motors)

- Sometimes, the effect of inductance is negligible

$$\cancel{L_a \frac{di_a}{dt}}^{\,0} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m$$

- In this case, we have: by $\left( J_m \ddot{\theta}_m + b\dot{\theta}_m = K_t \cdot i_a \right)$

$$J_m \ddot{\theta}_m + (b + \frac{K_t K_e}{R_a})\dot{\theta}_m = \frac{K_t}{R_a} v_a$$

$$\frac{\Omega(s)}{V_a(s)} = s\frac{\Theta_m(s)}{V_a(s)} = \frac{K}{\tau s + 1}.$$

## Transfer function (Electric circuit)

$$K := \frac{K_t}{bR_a + K_t K_e},$$

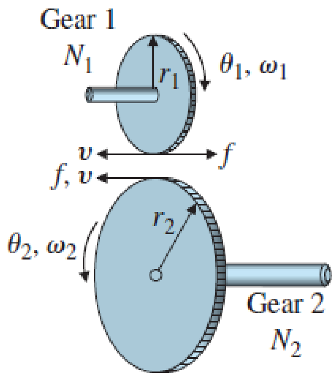
$$\tau := \frac{R_a J_m}{bR_a + K_t K_e}.$$

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{\frac{K_t}{R_a}}{J_m s^2 + s\left(b + \frac{K_t K_e}{R_a}\right)} = \frac{K}{s(\tau s + 1)}$$

## Transfer function (input $v_a$ to output $\dot{\theta}_m := \omega$ )

$$\frac{\Omega(s)}{V_a(s)} = s\frac{\Theta_m(s)}{V_a(s)} = \frac{K}{(\tau s + 1)}$$

- Forces of gears



(a)

$$\frac{T_1}{r_1} = \frac{T_2}{r_2} = f \quad \text{(force applied to teeth)}$$

$$\frac{T_2}{T_1} = \frac{r_2}{r_1} = \frac{N_2}{N_1} := n \quad (N_i : \text{number of gears})$$

$$\omega_1 r_1 = \omega_2 r_2 = v$$

$$\frac{\omega_1}{\omega_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = n$$

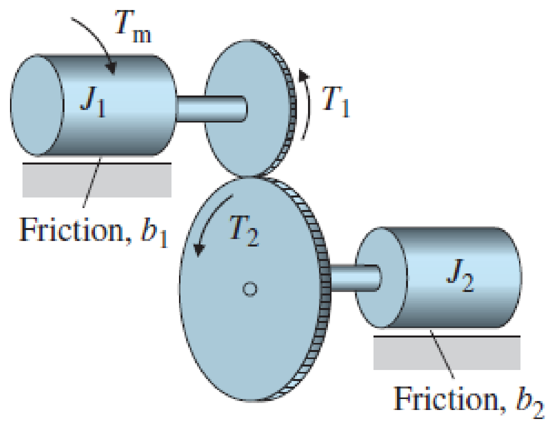
$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = n$$

## Model (Equations of Motion)

$$J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 = T_m - T_1$$

$(T_m : \text{ output torque of the servo motor})$

$$J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = T_2$$



(b)

# Example: Gears

## Model (Equations of Motion)

$$\frac{T_2}{T_1} = \frac{r_2}{r_1} = \frac{N_2}{N_1} := n$$

- Substitute $\theta_2$ for $\theta_1$ and replace $T_2$ with $T_1$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = n$$

$$(J_2 + J_1 n^2)\ddot{\theta}_2 + (b_2 + b_1 n^2)\dot{\theta}_2 = nT_m$$

## Transfer function (Electric circuit)

$$\frac{\Theta_2(s)}{T_m(s)} = \frac{n}{J_{eq}s^2 + b_{eq}s}$$

$$\begin{pmatrix} J_{eq} & = J_2 + J_1 n^2 \\ b_{eq} & = b_2 + B_1 n^2 \end{pmatrix}$$