Spring 2021

控制系統
Control Systems

Unit 2E
Electric Circuits

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Model of Electric Circuits

- The basic equations of electric circuits are the Kirchhoff's laws

- Kirchhoff's Current Law (KCL):

  The algebraic sum of the currents leaving a node

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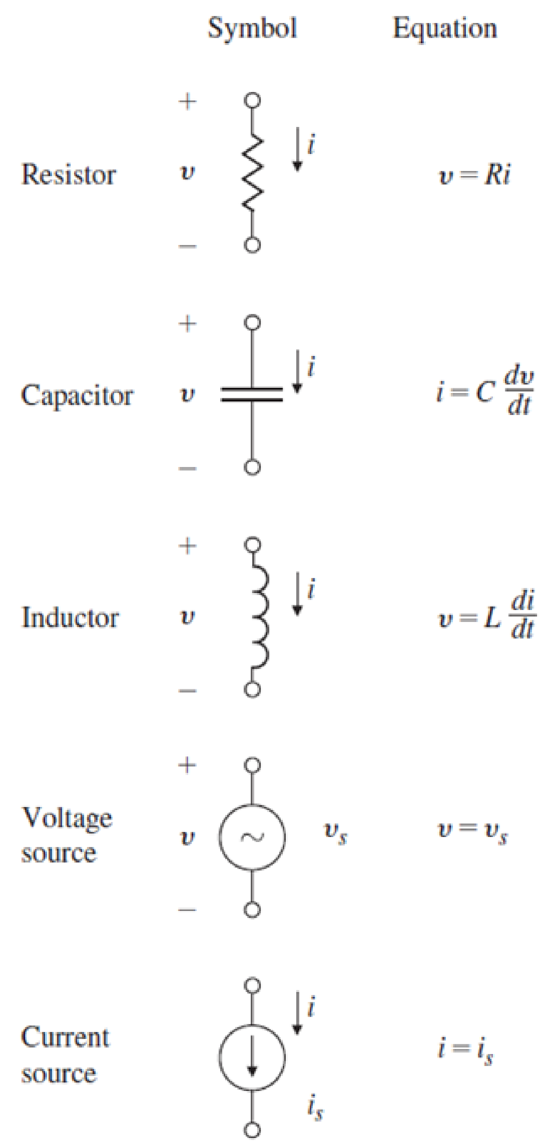
  The algebraic sum of the currents entering that node

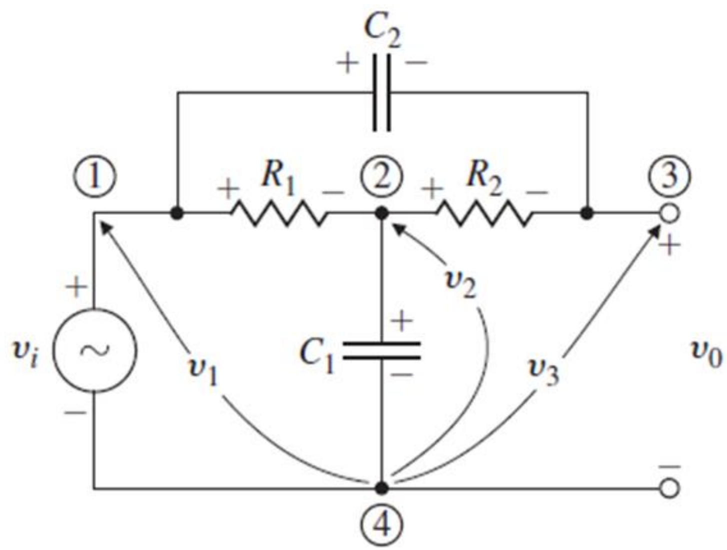
- Kirchhoff's Voltage Law (KVL):

  The algebraic sum of all voltages taken around a closed path in a circuit is zero



Example 2.8 the Bridged Tee Circuit

- Bridged Tee Circuit

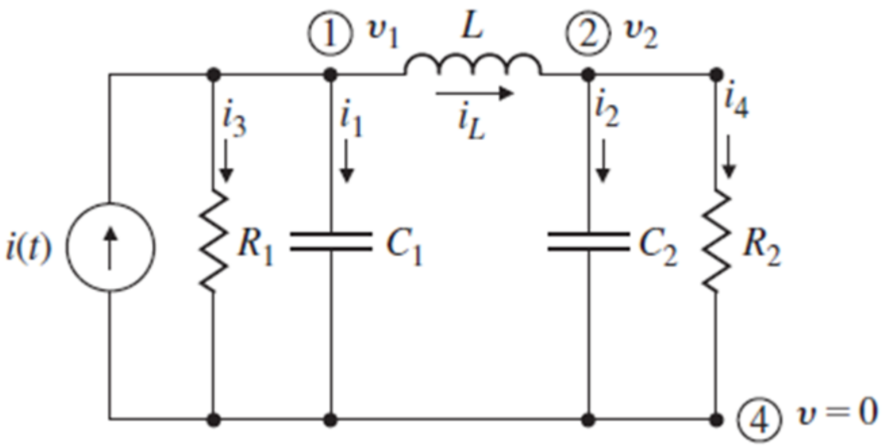


Model (Equations of Motion)

- Select node 4 as the reference
- $v_1,\ v_2,\ v_3$ as the unknowns
- By KVL, $v_1 = v_i$
- At node 2, the KCL is $$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1\frac{dv_2}{dt} = 0$$
- At node 3, the KCL is $$\frac{v_3 - v_2}{R_2} + C_2\frac{d(v_3 - v_1)}{dt} = 0$$
- Transfer function from input v_i to output v_o can be derived

Example 2.9 a Circuit with a Current Source

- Circuit with a current source



Model (Equations of Motion)

- Select node 4 as the reference
- $v_1, \; v_2, \; i_L$ as the unknowns
- At node 1, the KCL is

$$i(t) = i_3 + i_1 + i_L$$

- At node 2, the KCL is

$$i_L = i_2 + i_4$$

- We also have the relations

$$i_3 = \frac{v_1}{R_1}, \quad i_1 = C_1 \frac{dv_1}{dt},$$

$$i_2 = C_2 \frac{dv_2}{dt}, \quad i_4 = \frac{v_2}{R_2},$$

$$v_1 - v_2 = L \frac{di_L}{dt},$$

$$\Rightarrow$$

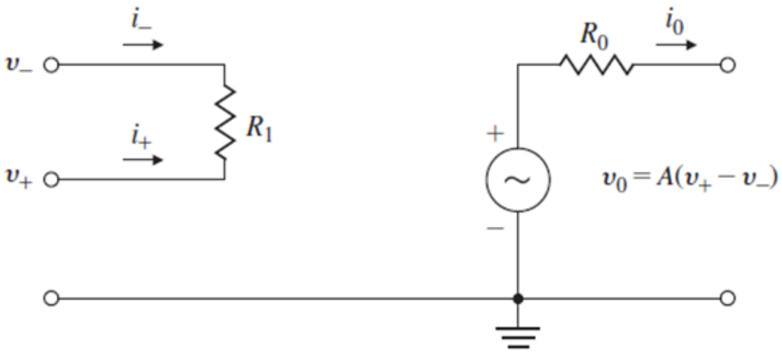
$$i(t) = \frac{v_1}{R_1} + C_1 \frac{dv_1}{dt} + i_L,$$

$$i_L = C_2 \frac{dv_2}{dt} + \frac{v_2}{R_2}$$

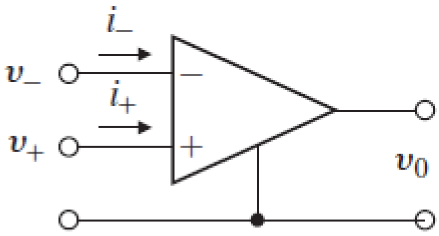
$$v_1 = L \frac{di_L}{dt} + v_2$$

Example 2.10 Operational Amplifier Summer

- Simplified circuit of op-amp
- op-amp schematic symbol

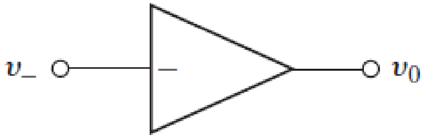


(a)



(b)

- Assume connected to ground, $v_+ = 0$

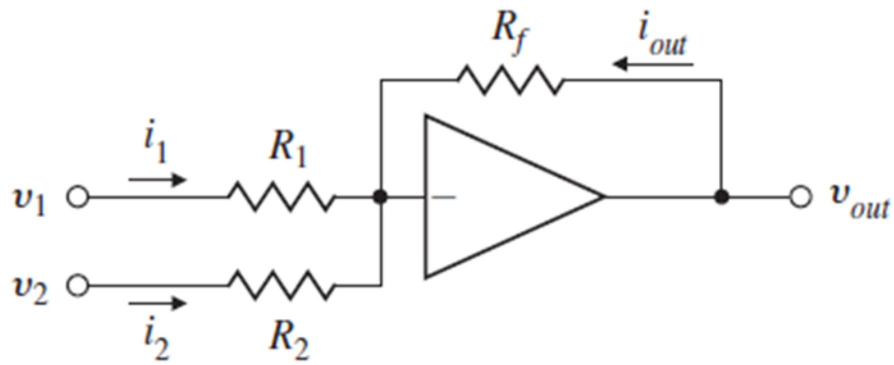


(c)

- Assume ideal op-amp, $R_1 = \infty,\ R_0 = 0,\ A = \infty$

$$i_+ = i_- = 0, \quad v_+ - v_- = 0$$

- The op-amp summer

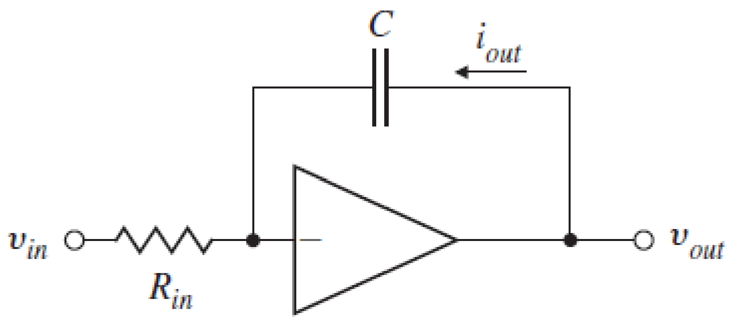


- From $v_+ - v_- = 0$, we have $v_- = 0$
- Thus, $i_1 = \dfrac{v_1}{R_1},\ i_2 = \dfrac{v_2}{R_2},\ i_{out} = \dfrac{v_{out}}{R_f}$
- From $i_+ = i_- = 0$, we have $i_1 + i_2 + i_{out} = 0$, $\dfrac{v_1}{R_1} + \dfrac{v_2}{R_2} + \dfrac{v_{out}}{R_f} = 0$

Model (Equations of Motion)

$$v_{out} = -[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2]$$ (Output is the weighted sum of input voltages)

- The op-amp integrator



$$i_{in} + i_{out} = 0$$

$$\frac{v_{in}}{R_{in}} + C\frac{dv_{out}}{dt} = 0$$

Model (Equations of Motion)

$$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}(\tau)d\tau + v_{out}(0)$$

Transfer Function

$$V_{out}(s) = -\frac{1}{s}\frac{V_{in}(s)}{R_{in}C}$$

(Assume zero initial condition)

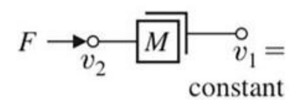
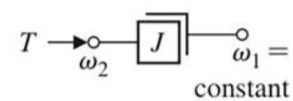
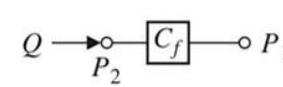
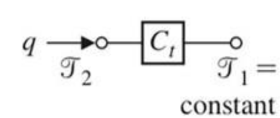
- Table 2.1 [Dorf & Bishop 2017]

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

- Table 2.2-1 [Dorf & Bishop 2017]

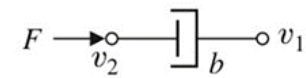
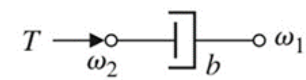
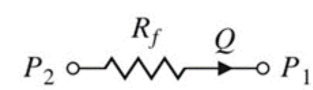
Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\dfrac{di}{dt}$	$E = \dfrac{1}{2}Li^2$	
	Translational spring	$v_{21} = \dfrac{1}{k}\dfrac{dF}{dt}$	$E = \dfrac{1}{2}\dfrac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \dfrac{1}{k}\dfrac{dT}{dt}$	$E = \dfrac{1}{2}\dfrac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\dfrac{dQ}{dt}$	$E = \dfrac{1}{2}IQ^2$	

- Table 2.2-2

Capacitive storage	Electrical capacitance	$i = C\dfrac{dv_{21}}{dt}$	$E = \dfrac{1}{2}Cv_{21}{}^2$	
	Translational mass	$F = M\dfrac{dv_2}{dt}$	$E = \dfrac{1}{2}Mv_2{}^2$	
	Rotational mass	$T = J\dfrac{d\omega_2}{dt}$	$E = \dfrac{1}{2}J\omega_2{}^2$	
	Fluid capacitance	$Q = C_f\dfrac{dP_{21}}{dt}$	$E = \dfrac{1}{2}C_fP_{21}{}^2$	
	Thermal capacitance	$q = C_t\dfrac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	

Summary of Governing Differential Equations for Ideal Elements

- Table 2.2.3 [Dorf & Bishop 2017]

Type of Element	Physical Element	Governing Equation	Energy *E* or Power $\mathscr{P}$	Symbol
	Electrical resistance	$i = \frac{1}{R} v_{21}$	$\mathscr{P} = \frac{1}{R} {v_{21}}^2$	
	Translational damper	$F = b v_{21}$	$\mathscr{P} = b {v_{21}}^2$	
Energy dissipators	Rotational damper	$T = b \omega_{21}$	$\mathscr{P} = b {\omega_{21}}^2$	
	Fluid resistance	$Q = \frac{1}{R_f} P_{21}$	$\mathscr{P} = \frac{1}{R_f} {P_{21}}^2$	
	Thermal resistance	$q = \frac{1}{R_t} \mathscr{T}_{21}$	$\mathscr{P} = \frac{1}{R_t} \mathscr{T}_{21}$