

Spring 2021

控制系統
Control Systems

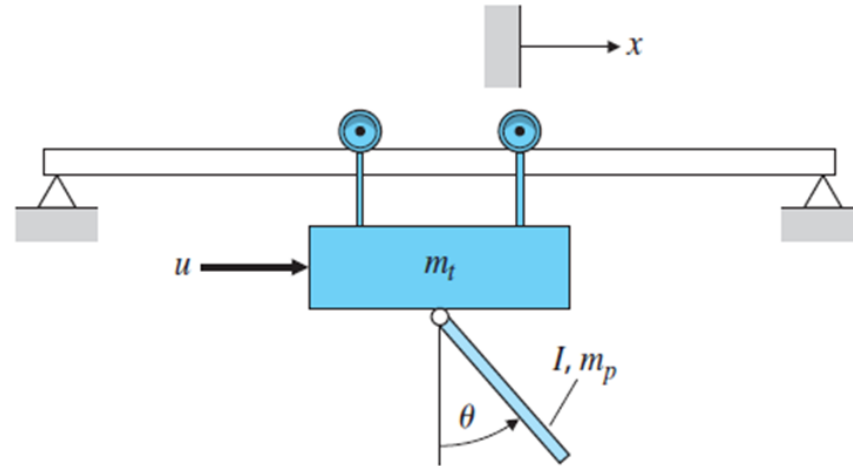
Unit 2C
Mechanical Systems –
Combined Rotation and Translation

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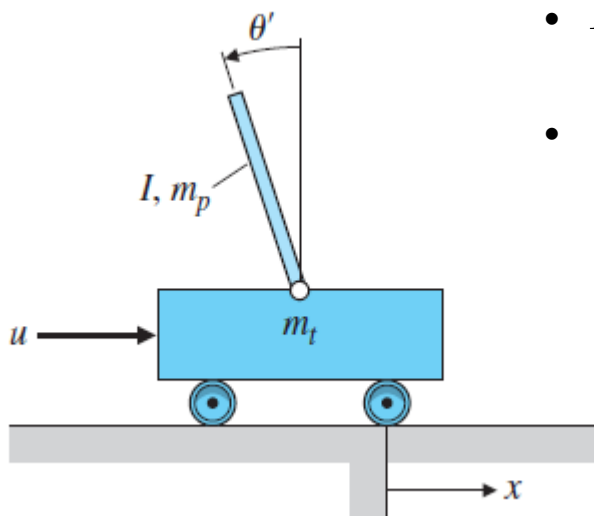
Feb – Jun, 2021

- Crane with hanging load

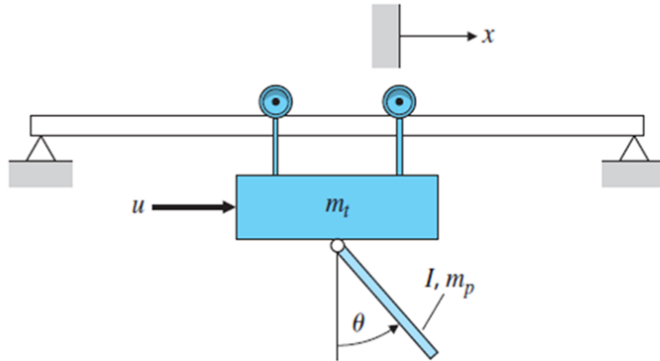


- Linearize the system at $\theta = \pi$ will result to the inverted pendulum

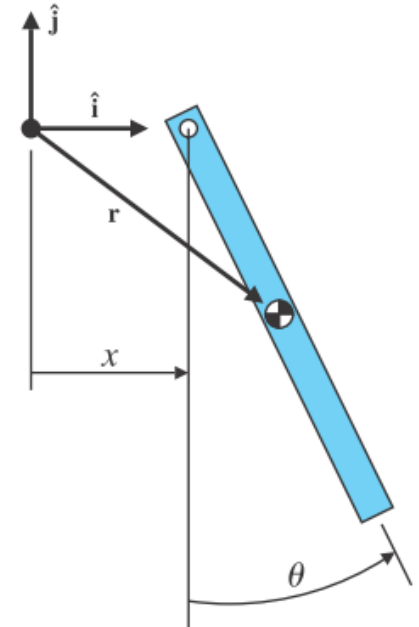
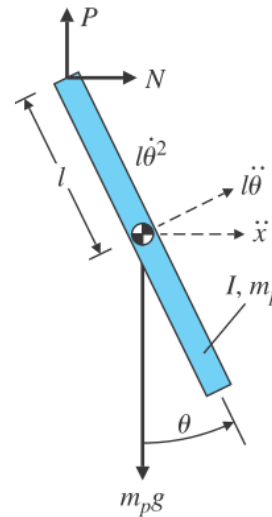
- I : the inertia about mass center of the pendulum
- $I + m_p \cdot l^2$: Moments of inertia of the pendulum about the pivot point



● Free-body diagram



The free-body diagram of the trolley and pendulum



Position vector of the pendulum

■ The Equations of Motion (see APPENDIX W2.1.4)

$$(I + m_p l^2) \ddot{\theta} + m_p g l \sin \theta = -m_p l \ddot{x} \cos \theta$$

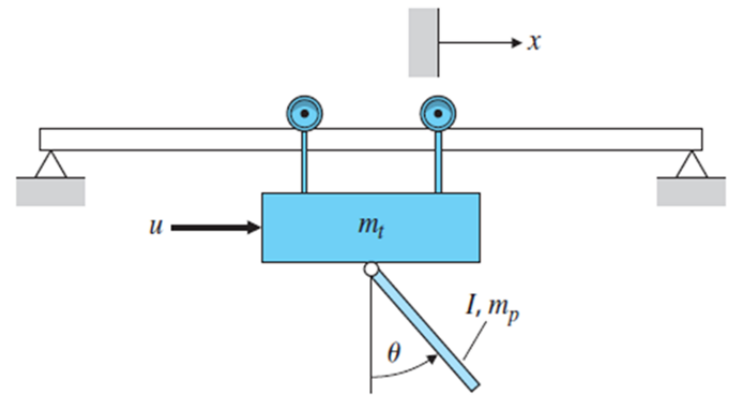
$$(m_t + m_p) \ddot{x} + b \dot{x} + m_p l \ddot{\theta} \cos \theta - m_p l \dot{\theta}^2 \sin \theta = u$$

Model (Equations of Motion)

- Assume small angles. For the small motion $\theta \approx 0$ we consider $\cos \theta \approx 1$, $\sin \theta \approx \theta$, $\dot{\theta}^2 \approx 0$

$$(I + m_p l^2) \ddot{\theta} + m_p g l \theta = -m_p l \ddot{x}$$

$$(m_t + m_p) \ddot{x} + b \dot{x} + m_p l \ddot{\theta} = u$$



Transfer Function

- Neglecting the friction term, let $b = 0$

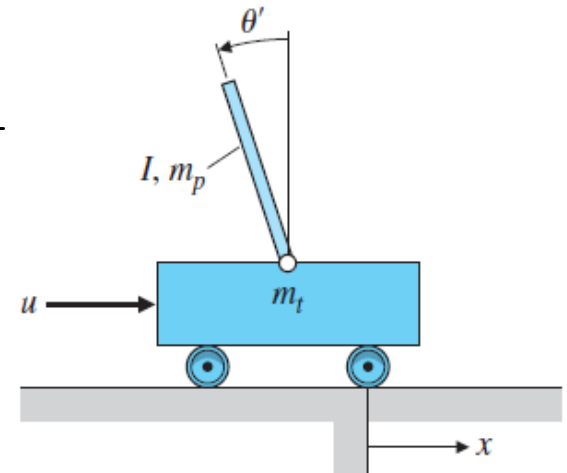
$$\frac{\Theta(s)}{U(s)} = \frac{-m_p l}{s^2 \left((I + m_p l^2)(m_t + m_p) - m_p^2 l^2 \right) + m_p g l (m_t + m_p)}$$

Model (Equations of Motion)

- For inverted pendulum, we consider $\theta \approx \pi$
- Assume $\theta = \pi + \theta' \Rightarrow \sin \theta' \approx -\theta', \cos \theta' \approx -1$

$$(I + m_p l^2) \ddot{\theta}' + m_p g l \theta' = -m_p l \ddot{x}$$

$$(m_t + m_p) \ddot{x} + b \dot{x} + m_p l \ddot{\theta}' = u$$



Transfer Function

- Neglecting the friction term, let $b = 0$

$$\frac{\Theta'(s)}{U(s)} = \frac{-m_p l}{s^2 \left((I + m_p l^2)(m_t + m_p) - m_p^2 l^2 \right) + m_p g l (m_t + m_p)}$$