Spring 2021

控制系統
Control Systems

Unit 2B
Mechanical Systems – Rotational Motion

Feng-Li Lian
NTU-EE

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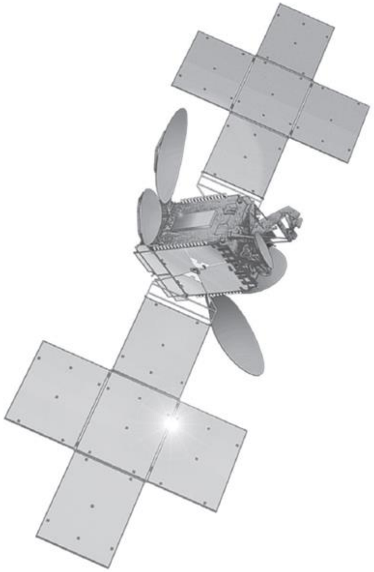
Example 2.3 (Rotational motion): Satellite Attitude Control Model

- Communication satellite



Source: Courtesy Thaicom PLC and Space Systems/Loral

- The purpose is to control the attitude of the satellite, such as
  - ✓ Antennas point toward earth
  - ✓ Solar panels orient toward the sun

▪ Model (Equations of Motion: Rotational motion)

$$M = I \alpha$$

- M ($N \cdot m^2$): the sum of all external moments about the center of mass,
- I ($Kg \cdot m^2$): the body's mass moment of inertia about its center of mass,
- α (rad/sec^2): the angular acceleration of the body

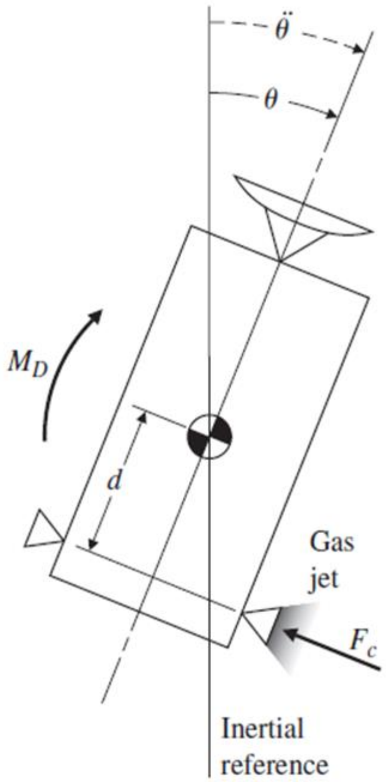
Example 2.3 (Rotational motion): Satellite Attitude Control Model

▪ Model (Equations of Motion)

- Three axes, consider one axis at a time

$$F_c \cdot d + M_D = I \cdot \ddot{\theta}$$

- $F_c \cdot d$: Moments of control force
- M_D : Moments of small disturbance



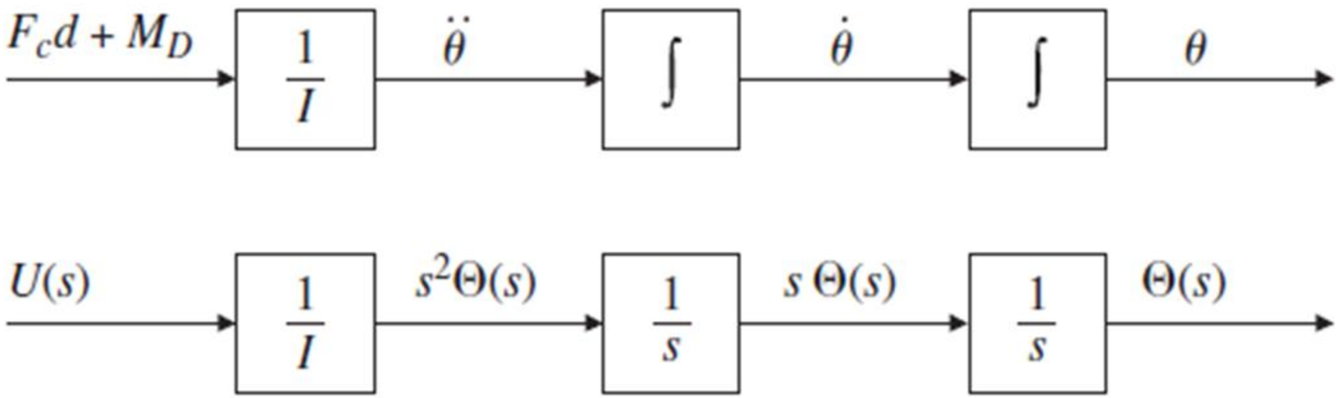
▪ Transfer Function

- Let $F_c \cdot d + M_D = u$

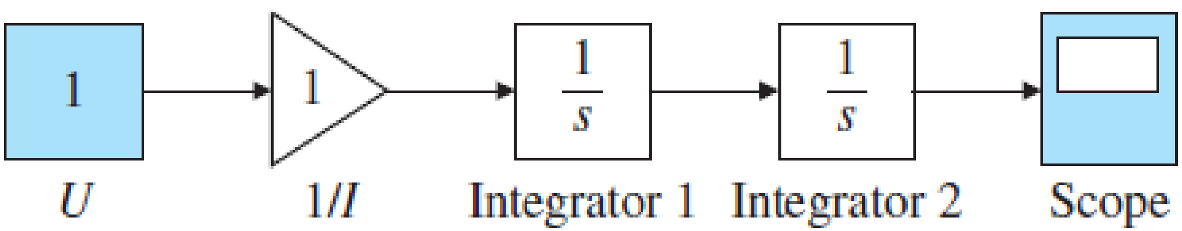
$$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2}$$ (Double-Integrator plant)

Example 2.3 (Rotational motion): Satellite Attitude Control Model

- Block diagram $\dfrac{\Theta(s)}{U(s)} = \dfrac{1}{I} \cdot \dfrac{1}{s^2}$ (Double-Integrator plant)

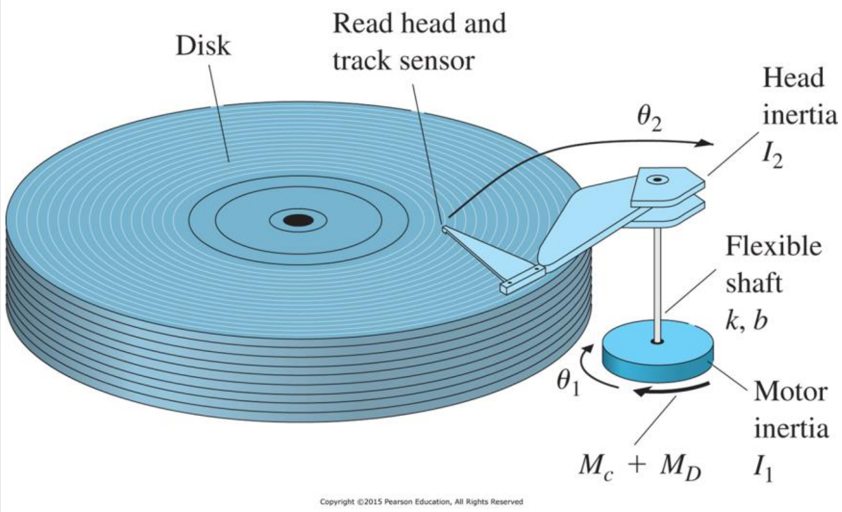


- Simulink

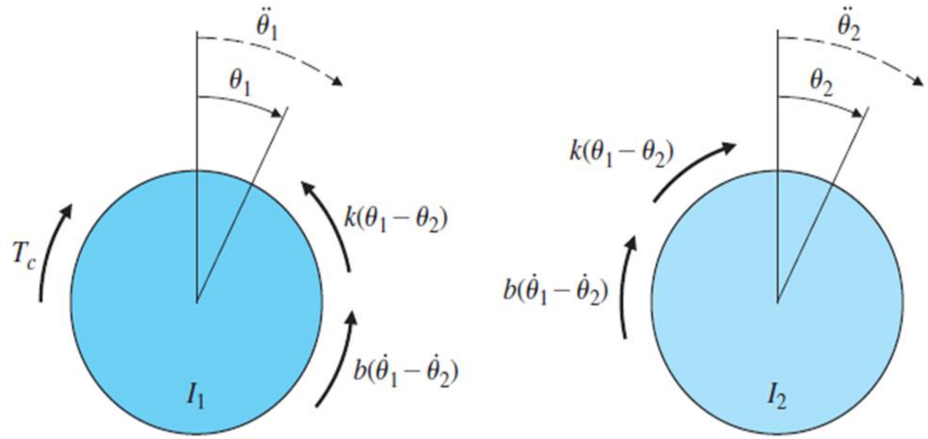


Example 2.4 Flexible Read/Write for a Disk Drive

- Disk Read/Write Head



- The moment of each body: free body diagram



- Model (Equations of Motion: Rotational motion)

$$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_c + M_D$$

$$I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$$

- M_c : Moments of applied control
- M_D : Moments of small disturbance

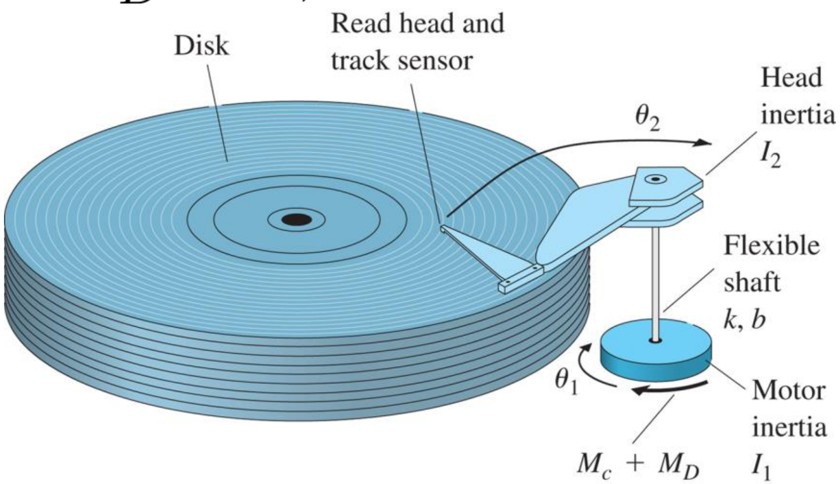
Example 2.4 Flexible Read/Write for a Disk Drive

▪ Model (Equations of Motion)

- Simplify the model, consider the case $M_D = 0, b = 0$

$$I_1\ddot{\theta}_1 + k(\theta_1 - \theta_2) = M_c$$

$$I_2\ddot{\theta}_2 + k(\theta_2 - \theta_1) = 0$$



▪ Transfer Function

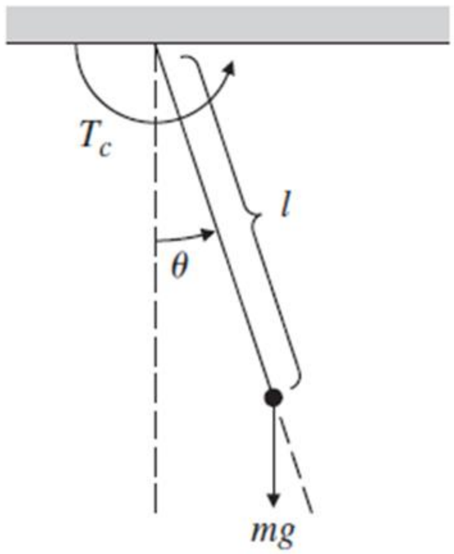
$$\frac{\Theta_2(s)}{M_c(s)} = \frac{k}{I_1 I_2 s^2\left(s^2 + \frac{k}{I_1} + \frac{k}{I_2}\right)}$$

$$\frac{\Theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + k}{I_1 I_2 s^2\left(s^2 + \frac{k}{I_1} + \frac{k}{I_2}\right)}$$

- **"Noncollocated case":** there is **flexibility** between the sensor and the actuator
- **"Collocated case":** the sensor and the actuator are **rigidly attached** to one another

Example 2.5 Pendulum

- Pendulum



Model (Equations of Motion)

$$T_c - mgl\sin\theta = I\ddot{\theta}$$

- The moments of inertia about the pivot point is

$$I = ml^2$$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

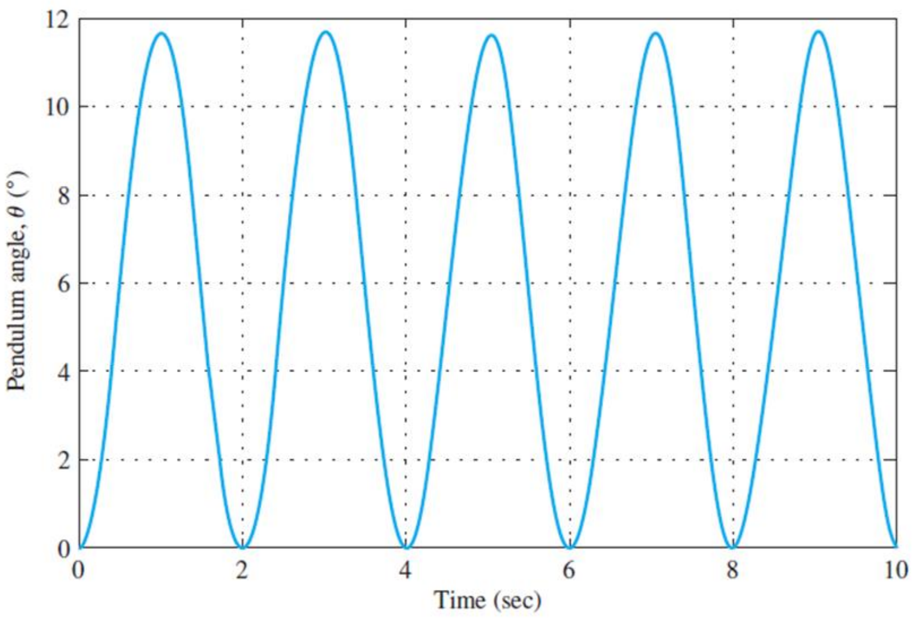
- The model is nonlinear due to $\sin\theta$
- When the motion is small, i.e., θ small, $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$ (Linearization model)

Example 2.5 Pendulum

- Transfer Function

$$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}}$$



- Matlab code

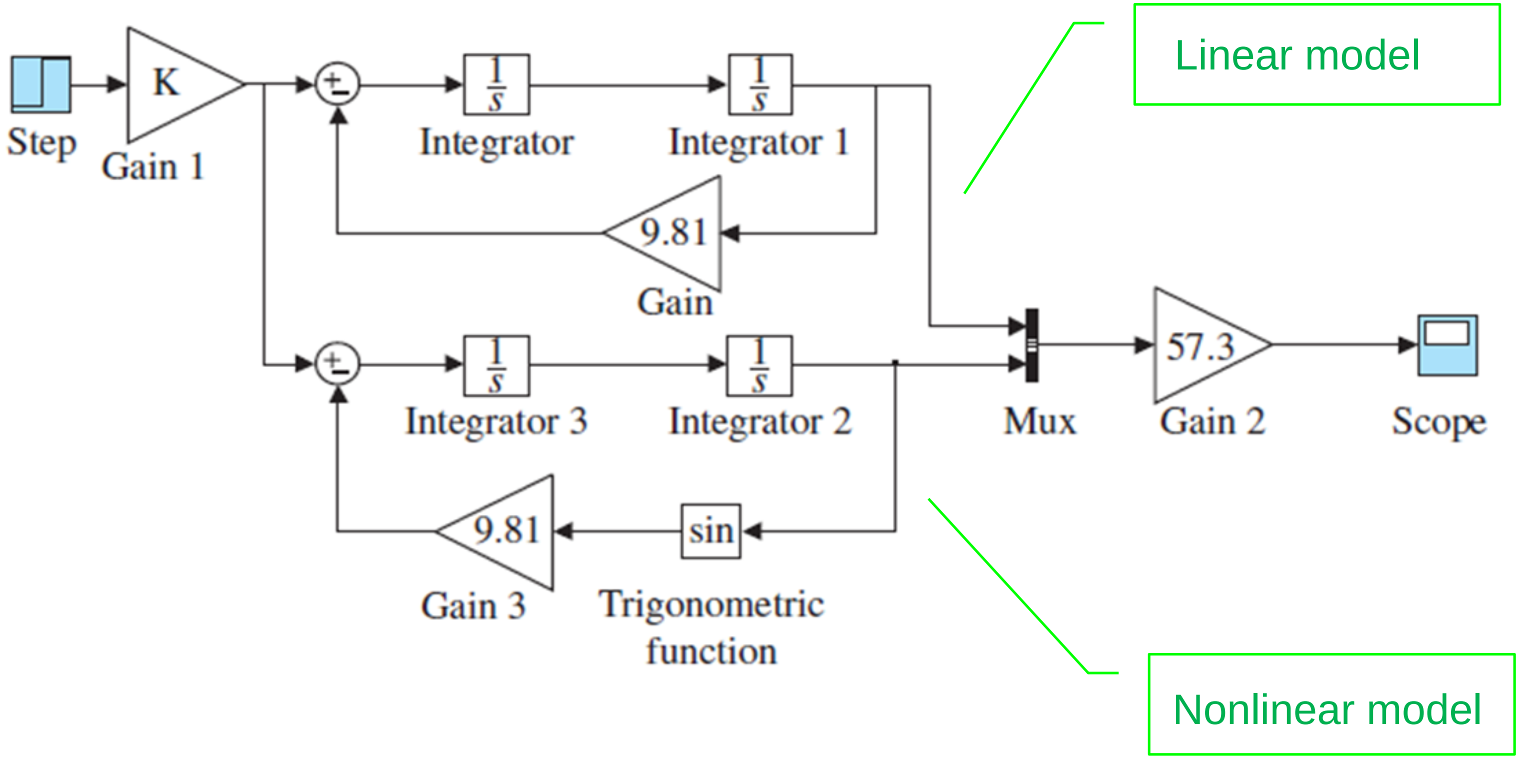
```
– t = 0:0.02:10;
– m = 1; L = 1; g = 9.81;
– s = tf( 's' );
– sys = (1/(m*L^2))/(s^2+g/L) ;
– y = step(sys,t);

– Rad2Deg = 57.3;
– Plot( t, Rad2Deg*y )       %converts output from radians to degrees
```

Example 2.6 Pendulum (Simulink for nonlinear motion)

- Matlab Simulink (m=1; L=1; g=9.81)

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$



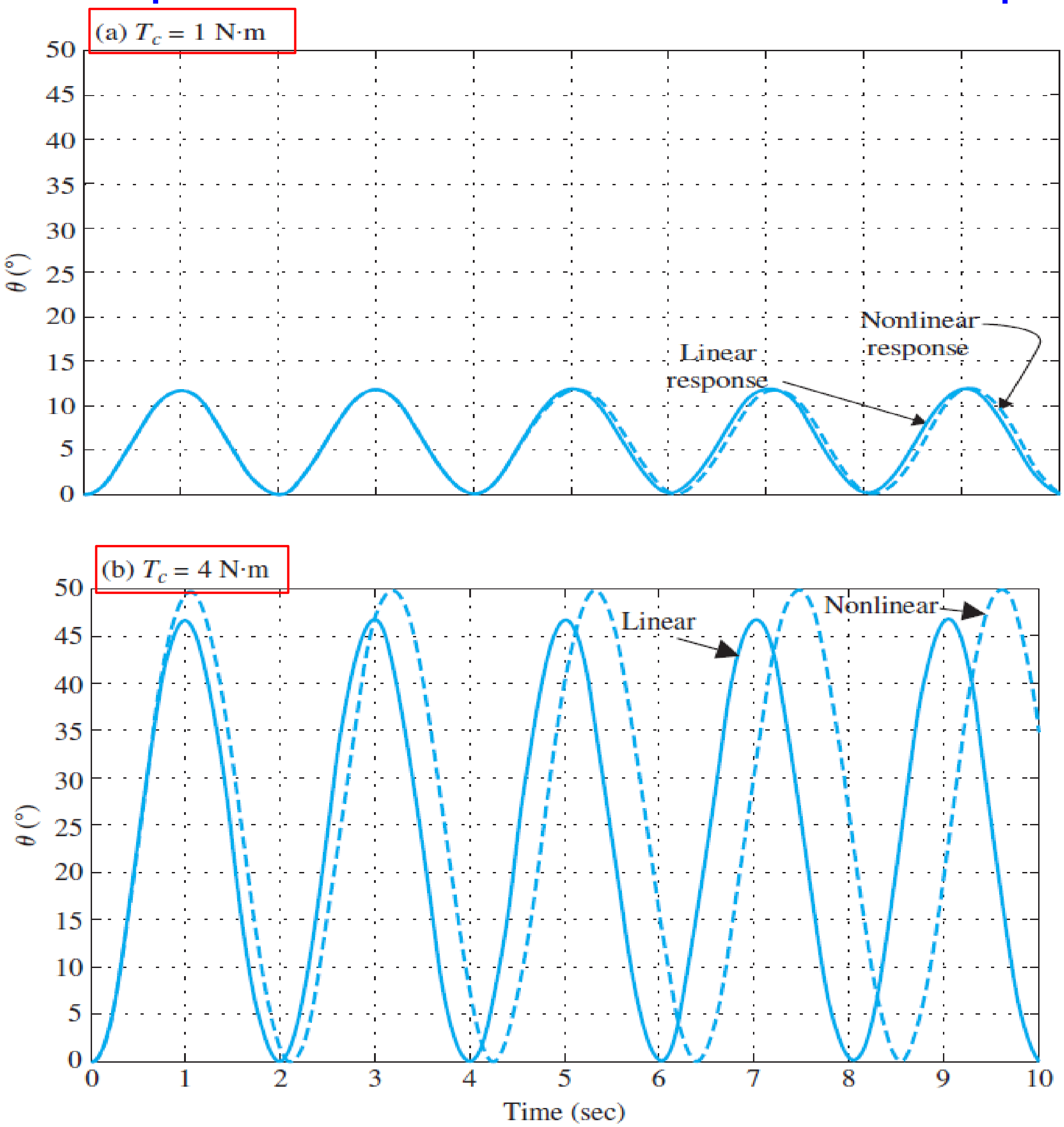
$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

Example 2.6 Pendulum (Simulink for nonlinear motion)

- Comparisons of linear & nonlinear responses

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$



- When T_c=1, the output θ remains small, thus the approximation is still good ($\sin\theta \approx \theta$)

- When T_c=4, the output θ becomes large, thus the approximation is not good ($\sin\theta \approx \theta$ does not hold)