Spring 2021

控制系統
Control Systems

Unit 20
Dynamic Models

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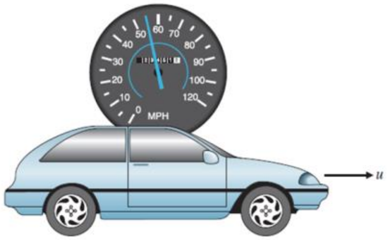
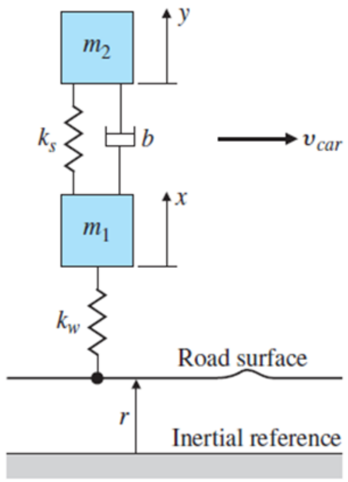
- For the plant to be analyzed and controlled
  - Dynamic Models
  - Mathematical Models
- Methodology
  - Based on Physics and By Differential Equations
  - From Experimental Data (System Identification)
- Key Ingredients:
  - Physics, Chemistry, Biology, Sociology, Economics, etc.
  - Differential Equations (Equations of Motion, Dynamic Equations)
  - Laplace Transforms, Fourier Transforms
  - Transfer Function (From Input to Output)

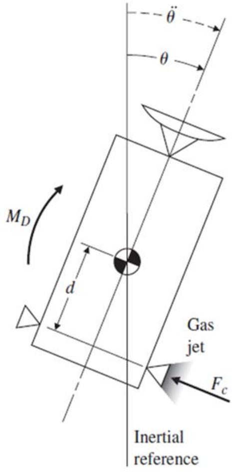
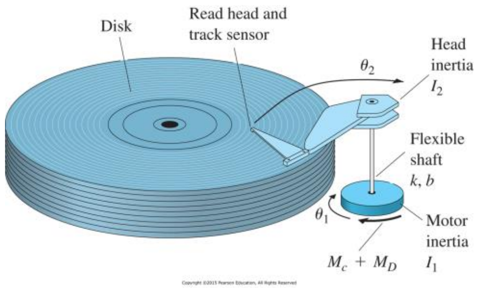
Dynamic Models

- Mechanical Systems
  - U2A: Translational Motion
  - U2B: Rotational Motion
  - U2C: Combined Rotation and Translation
  - U2D: Distributed Parameter Systems
- Electrical Circuits
  - U2E: Kirchhoff's Current Law (KCL)
  - U2E: Kirchhoff's Voltage Law (KVL)
  - U2E: Operational Amplifier
- Electromechanical Systems
  - U2F: Loudspeakers
  - U2F: Motors
  - U2F: Gears
- Heat and Fluid-Flow Models
  - U2G: Heat Flow
  - U2G: Incompressible Fluid Flow

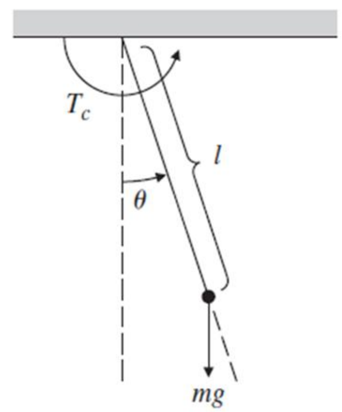
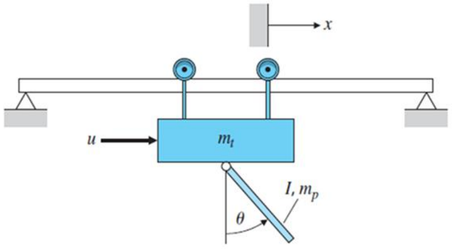
Summary of Unit 2: Mechanical Systems

<table>
<tr><th>System</th><th>Differential Equation</th><th>Transfer Function</th></tr>
<tr><td>Ex. 2.1: Cruise Control
</td><td>$$\dot{v} + \frac{b}{m} v = \frac{u}{m}$$</td><td>$$\frac{V(s)}{U(s)} = \frac{\frac{1}{m}}{s + \frac{b}{m}}$$</td></tr>
<tr><td>Ex. 2.2: Two-Mass System
</td><td>$$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}(x) = \frac{k_w}{m_1}(r)$$
$$\ddot{y} + \frac{b}{m_2}(\dot{y} - \dot{x}) + \frac{k_s}{m_2}(y - x) = 0$$</td><td>$$\frac{Y(s)}{R(s)},$$
$$Y(s) = \frac{k_w}{m_1}\frac{b}{m_2}(s + \frac{k_s}{b})$$
$$R(s) = s^4 + (\frac{b}{m_1} + \frac{b}{m_2})s^3 + (\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1})s^2 + \frac{k_w b}{m_1 m_2}s + \frac{k_w k_s}{m_1 m_2}$$</td></tr>
</table>

Summary of Unit 2: Mechanical Systems

System	Differential Equation	Transfer Function
Ex. 2.3: Satellite Control 	$F_c \cdot d + M_D = I \cdot \ddot{\theta}$	$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2}$ $(F_c \cdot d + M_D = u)$
Ex. Disk Read/Write Head 	$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_c + M_D$ $I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$	$\frac{\Theta_2(s)}{M_c(s)} = \frac{k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$ $\frac{\Theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$

Summary of Unit 2: Mechanical Systems

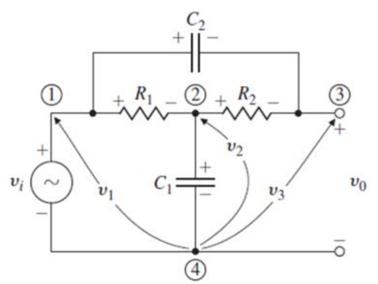
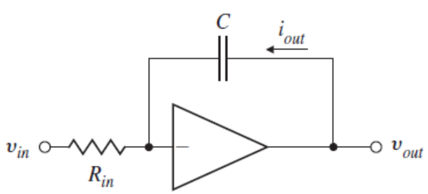
<table>
<tr><th>System</th><th>Differential Equation</th><th>Transfer Function</th></tr>
<tr><td>Ex. 2.6: Pendulum
</td><td>$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$</td><td>$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}}$</td></tr>
<tr><td>Ex. 2.8: Hanging Crane
</td><td>$(I + m_p l^2)\ddot{\theta} + m_p g l \theta$
$= -m_p l \ddot{x}$

$(m_t + m_p)\ddot{x} + b\dot{x} + m_p l \ddot{\theta}$
$= u$</td><td>$\frac{\Theta(s)}{U(s)}$,

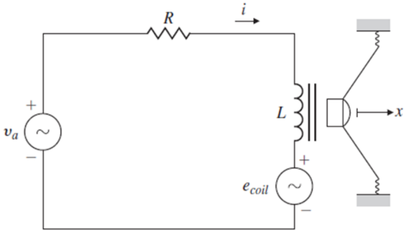
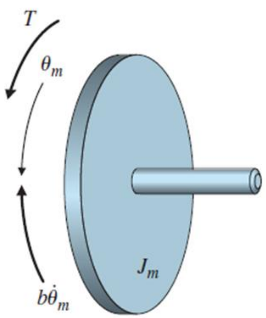
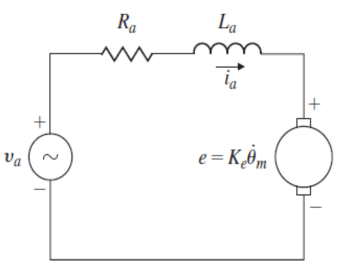
$\Theta(s) = -m_p l$

$U(s) = s^2\big((I + m_p l^2)(m_t + m_p) - m_p^2 l^2\big)$
$+ m_p g l (m_t + m_p)$</td></tr>
</table>

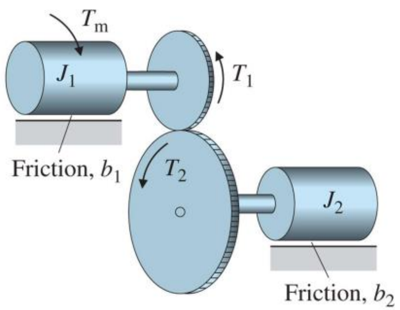
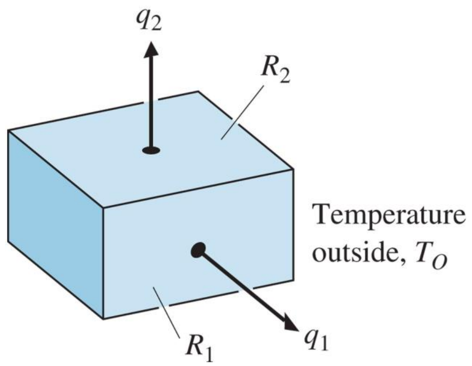
Summary of Unit 2: Electric Circuits

System	Differential Equation	Transfer Function
Ex. 2.9: Bridged Tee Circuit 	$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1 \frac{dv_2}{dt} = 0$ $\frac{v_3 - v_2}{R_2} + C_2 \frac{d(v_3 - v_1)}{dt} = 0$	$\frac{V_2(s)}{V_1(s)} = \frac{\cdots}{C_1 s + \cdots}$ $\frac{V_3(s)}{V_1(s)} = \frac{\cdots}{C_2 s + \cdots}$
Ex. 2.12: Integrator 	$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}(\tau)d\tau$ $+v_{out}(0)$	$V_{out}(s) = -\frac{1}{s}\frac{V_{in}(s)}{R_{in}C}$

Summary of Unit 2: Electromechanical Systems

System	Differential Equation	Transfer Function
Ex. 2.13 &14: Loudspeaker 	$L\frac{di}{dt} + Ri = v_a - e_{coil} = v_i$ $M\ddot{x} + b\dot{x} = Bli$	$\frac{I(s)}{V_i(s)} = \frac{1}{Ls + R}$ $\frac{X(s)}{I(s)} = \frac{Bl}{(Ms^2 + bs)}$
Ex. 2.15: Motors   (a)	$J_m\ddot{\theta}_m + b\dot{\theta}_m = T = K_t \cdot i_a$ $L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m$	$\frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{s\left[(J_m s + b)(L_a s + R_a) + K_t K_e\right]}$

Summary of Unit 2:

System	Differential Equation	Transfer Function
Ex. Gears 	$J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 = T_m - T_1$ $J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = T_2$	$\frac{\Theta_2(s)}{T_m(s)} = \frac{n}{J_{eq}s^2 + b_{eq}s}$
Ex. Heat flow 	$\dot{T}_I = \frac{1}{C_I}(\frac{1}{R_1} + \frac{1}{R_2})(T_O - T_I)$	$\frac{T_I(s)}{T_o(s)} = \frac{\cdots}{s + \cdots}$

Summary of Unit 2: Key equations

Key Equations for Dynamic Models

System	Important Laws or Relationships	Associated Equations	Equation Number(s)
Mechanical	Translational motion (Newton's law)	$F = ma$	(2.1)
	Rotational motion	$M = I\alpha$	(2.14)
Electrical	Operational amplifier		(2.46), (2.47)
Electromechanical	Law of motors	$F = Bli$	(2.53)
	Law of generators	$e = Blv$	(2.56)
	Torque developed in a rotor	$T = K_t i_a$	(2.60)
Back emf	Voltage generated as a result of rotation of a rotor	$e = K_e \dot{\theta}_m$	(2.61)
Gears	Effective inertia	$J_{eq} = J_2 + J_1 n^2$	(2.80)
Heat flow	Heat-energy flow	$q = {}^1/_R(T_1 - T_2)$	(2.81)
	Temperature as a function of heat-energy flow	$\dot{T} = \frac{1}{C} q$	(2.82)
	Specific heat	$C = mc_v$	(2.83)
Fluid flow	Continuity relation (conservation of matter)	$\dot{m} = w_{in} - w_{out}$	(2.88)
	Force of a fluid acting on a piston	$f = pA$	(2.90)
	Effect of resistance to fluid flow	$w = {}^1/_R(p_1 - p_2)^{1/\alpha}$	(2.91)