Spring 2020

控制系統
Control Systems

Unit 6G
Bode's Gain-Phase Relationship

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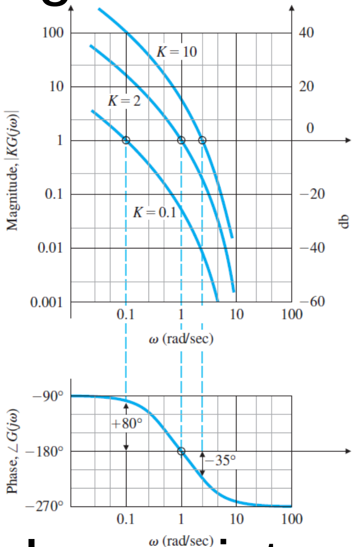
Bode's Theorem

- One of Bode's important contributions is the following theorem:

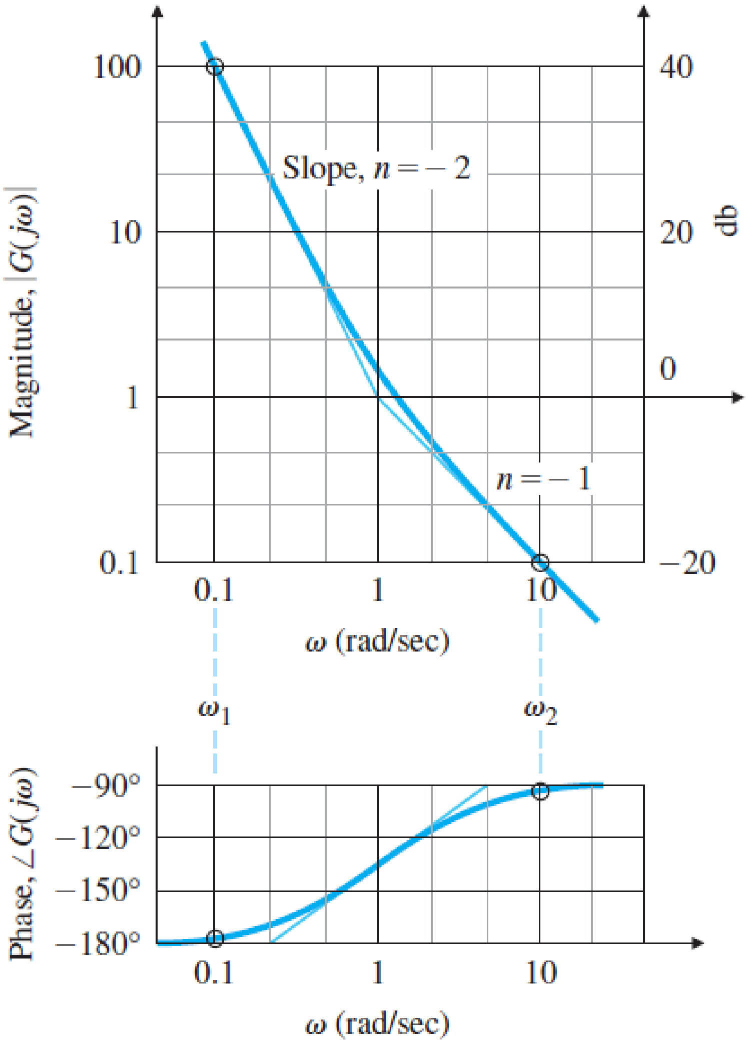


- For any stable minimum-phase system (no RHP zeros/poles),
- The phase of G(jw) is uniquely related to the magnitude of G(jw)
- When the slope of |G(jw)| versus ω on a log-log scale persists at a constant value for approximately a decade of frequency, the relationship is particularly simple and is given by:

$$\angle G(jw) \approx n \times 90^o$$

- *n*: the slope of |G(jw)| in units of decade of amplitude per decade of frequency

Bode's Theorem



- Slope:
  - At ω_1= 0.1, (n = -2)
  - At ω_2= 10, (n = -1)
- Phase:
  - At ω_1= 0.1, -180^o
  - At ω_2= 10, -90^o

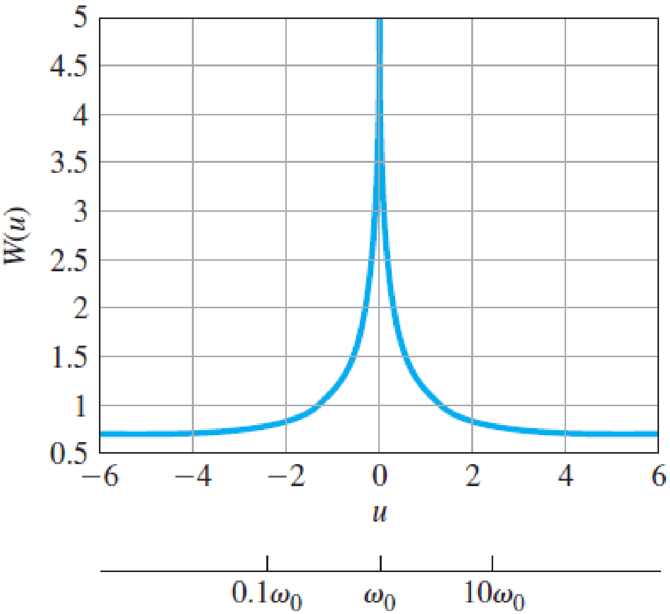
- An exact statement of the Bode Gain-Phase Theorem is:

$$\angle G(jw) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\left(\frac{dM}{du}\right) W(u)\, du \quad \text{in radians}$$

- Where
  - *M* = log magnitude = ln |G(jw)|
  - *u* = normalized frequency = ln(ω / ω_0)
  - *dM/du* ~= slope *n*
  - *W(u)* = weighting function

    = ln(coth|*u*|/2)

$$W(u) \approx \frac{\pi^2}{2}\delta(u)$$



Bode's Theorem

- But, usually use $\qquad \angle G(jw) \approx n \times 90^o$
- When | KG(jw) | = 1 $\qquad \angle G(jw) \approx -90^o \quad \text{if } n = -1$

$$\angle G(jw) \approx -180^o \quad \text{if } n = -2$$

- For stability, we want:

$$\angle G(jw) > -180^o \quad \text{for PM to be} > 0$$

- Therefore, adjust the |KG(jw)| curve
- So that it has a slope of –1 at the crossover frequency ω_c
- If the slope = -1 for a decade above/below ω_c, then PM ~= 90^o
- However, to ensure a reasonable PM,

  it is usually necessary only to insist that

  a -1 slope persist for a decade in frequency centered at ω_c

Bode's Theorem

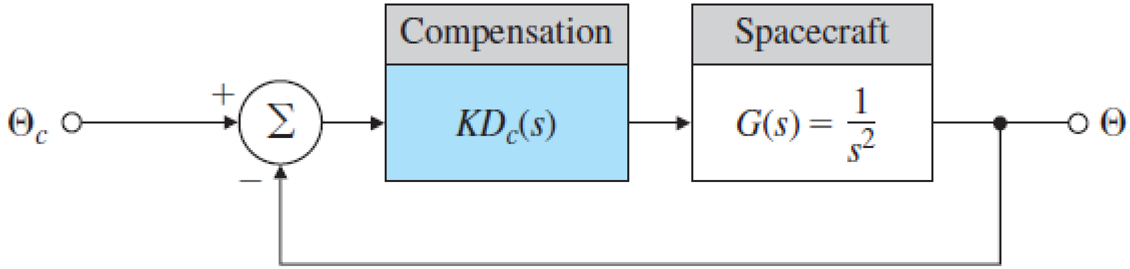
- A very simple design criterion:
- Adjust the slope of the magnitude curve |KG(jw)|
- So that it crosses over magnitude 1 with a slope of -1

  for a decade around ω_c
- This criterion will usually be sufficient

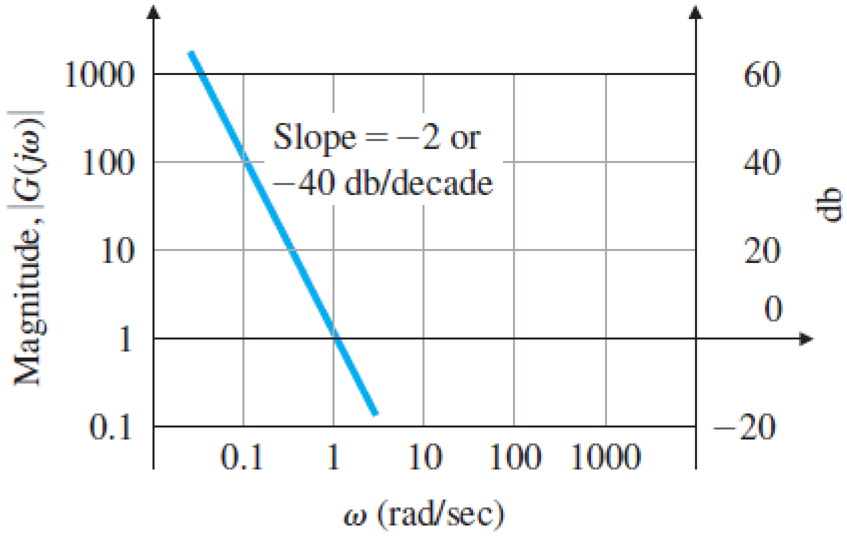
  to provide an acceptable PM and adequate system damping.
- To achieve the desired speed of response,
- the system gain is adjusted
- so that the crossover point is at a frequency

  that will yield the desired bandwidth or speed of response.
- Natural Freq ω_n ~= Bandwidth ω_{BW} ~= Crossover Freq ω_c

- Example 6.14: Use of Simple Design Criterion for Spacecraft Attitude Control



$$K\ D_c(s) \ = \ K\ (\ T_D\ s + 1\ )$$



- Adjust K
- to provide desired bandwidth
- Adjust break point $\omega_1 = 1/T_D$
- to provide the -1 slope

  at the crossover frequency

  0.2 rad/sec

Example

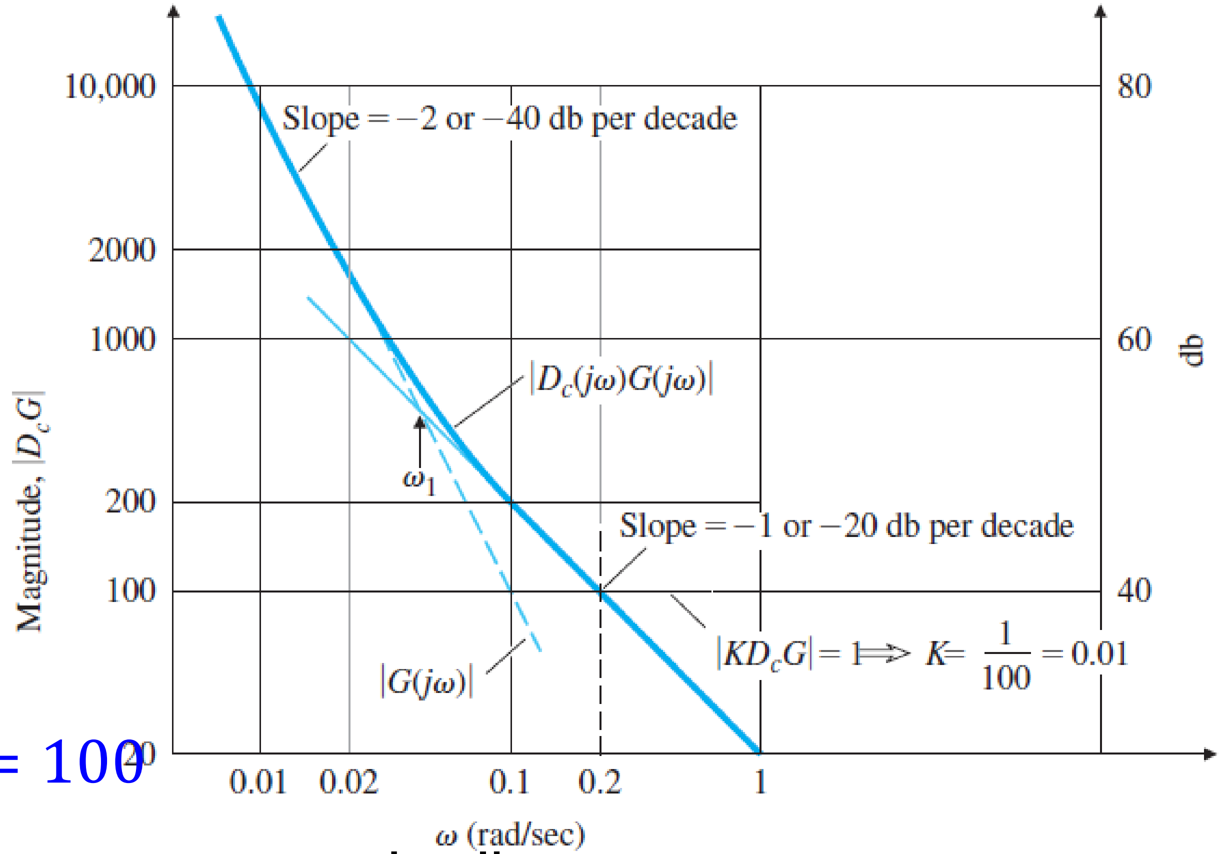
1. Plot |G(jw)|
2. Modify the plot

   to include $|D_c(jw)|$

   with $\omega_1 = 0.05$

   $T_D = 20$

   → slope=-1 at 0.2



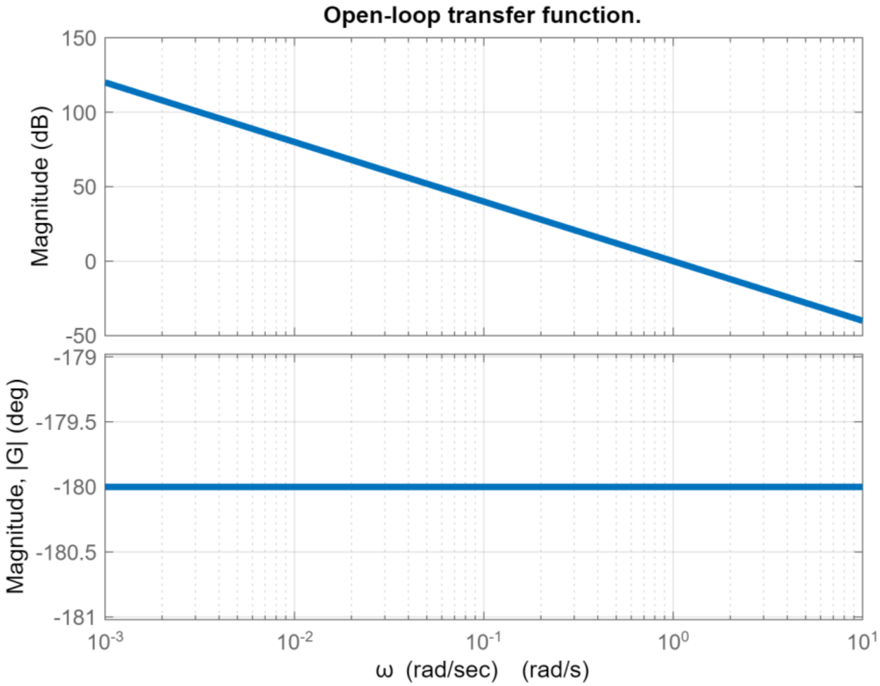
3. Determine $|D_c\ G| = 100$

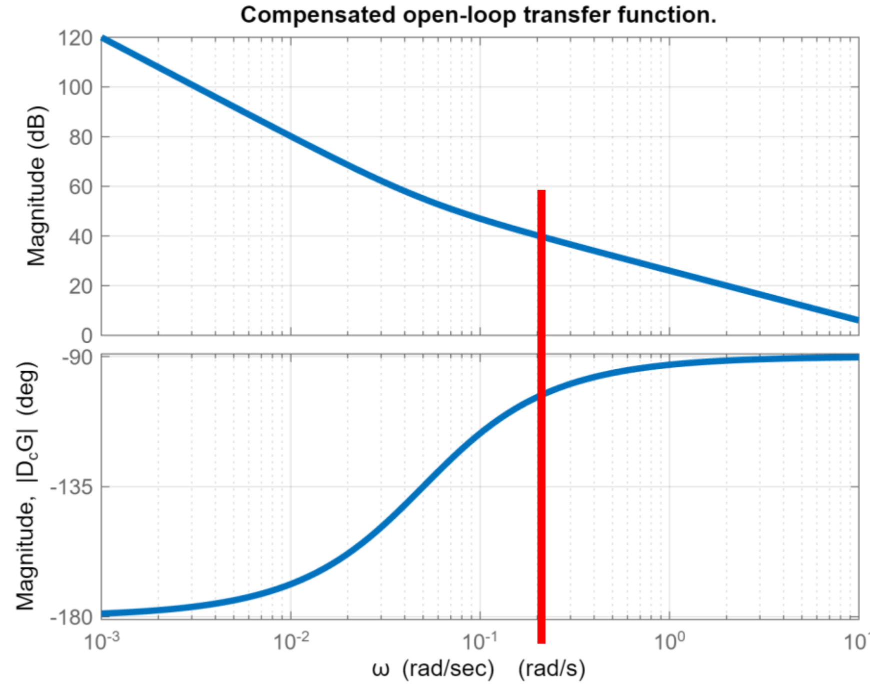
   where the $|D_c\ G|$ curve crosses the line $\omega = 0.2$

4. Compute $K = \dfrac{1}{|D_c G|_{w=0.2}} = \dfrac{1}{100} = 0.01$

$$\Rightarrow\ K\ D_c(s) = 0.01\ (\ 20\ s + 1\ )$$

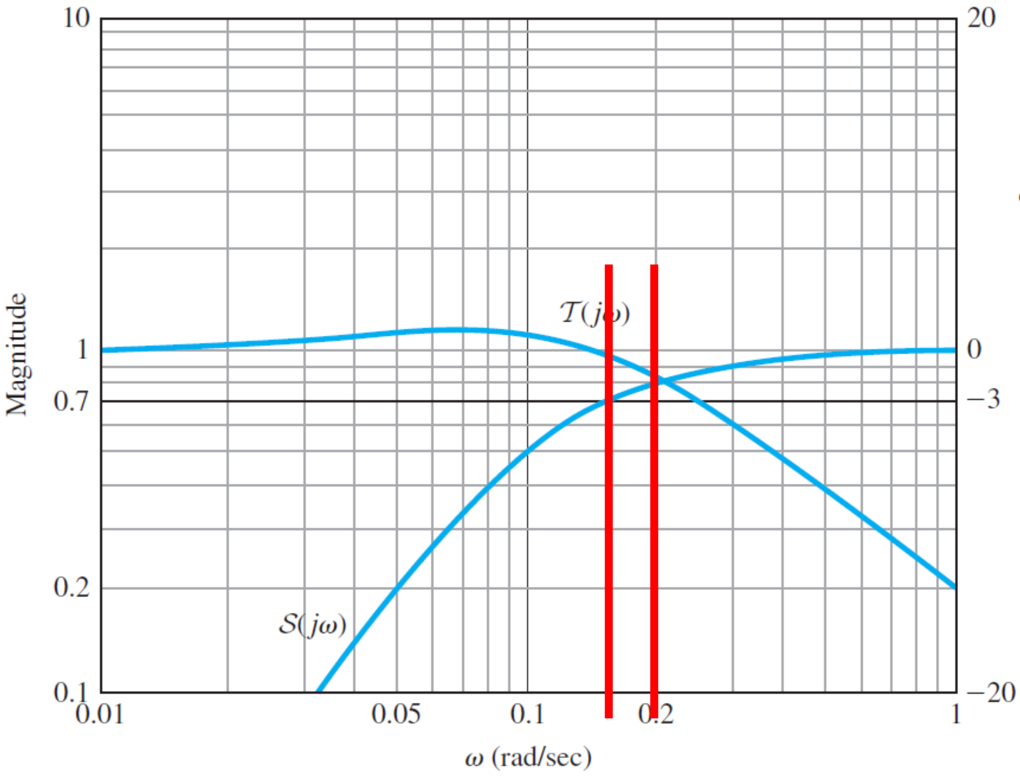
Example


Open-loop transfer function.
Magnitude (dB)
150
100
50
0
-50
-179
-179.5
-180
-180.5
-181
Magnitude, |G| (deg)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
ω (rad/sec) (rad/s)


Compensated open-loop transfer function.
Magnitude (dB)
120
100
80
60
40
20
0
-90
-135
-180
Magnitude, $|D_cG|$ (deg)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
ω (rad/sec) (rad/s)

Example

- The closed-loop frequency-response magnitude *T(jw)* and the sensitivity function *S(jw)*
- Desired Bandwidth = 0.2 rad/sec
- Disturbance Rejection (-3db) at 0.15 rad/sec



- Overshoot = 14%